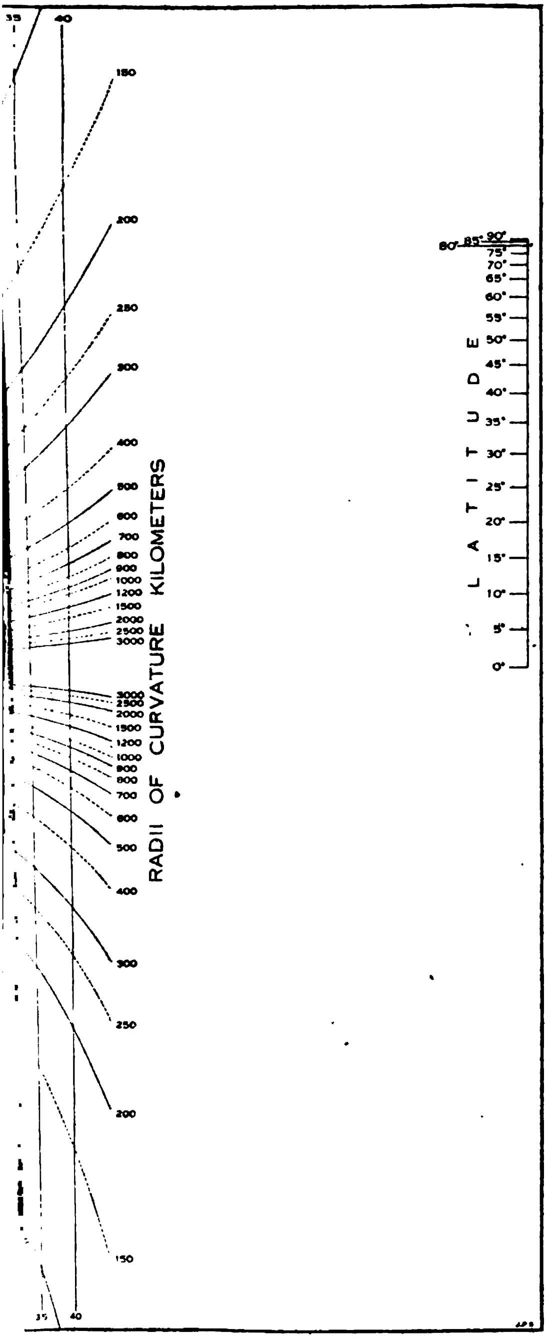
PHYSICS OF THE AIR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65°
70°
75°
80°
85°
90°

gram.

p. 3-4

PHYSICS OF THE AIR

BY

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INTRODUCTION

THE physical phenomena of the earth's atmosphere are exceedingly numerous and of great importance. Nevertheless, the explanations, even of those well understood, still remain scattered through many books and numerous journals. Perhaps this is because some of the phenomena have never been explained, and others but imperfectly so, but, however that may be, it is obvious that an orderly assemblage of all those facts and theories that together might be called the *Physics of the Air* would be exceedingly helpful to the student of atmospherics. An attempt to serve this useful purpose, begun in a course of lectures at the San Diego Aviation School (Rockwell Field) in 1914, led to the production of the following chapters—revised and reprinted from the *Journal of The Franklin Institute*, 1917, 1918, 1919, 1920.

The author begs to express his indebtedness to Prof. C. F. Marvin, Chief of the United States Weather Bureau, for numerous helpful criticisms; to Dr. C. F. Brooks, Editor of the *Monthly Weather Review*, for many excellent suggestions; to Prof. C. F. Talman, Librarian of the United States Weather Bureau, for valuable aid in locating original sources; and to Major R. B. Owens, D. S. O., Secretary of The Franklin Institute, for his encouraging interest in the work and invaluable attention to the details of its publication.

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PART I.

MECHANICS AND THERMODYNAMICS OF THE ATMOSPHERE.

CHAPTER I.

OBSERVATIONS.

BEFORE discussing any of the physical laws of the atmosphere it will be instructive briefly to consider the observational data upon which they are based; that is, to enumerate the meteorological phenomena which commonly are measured, and to indicate in each case the type of instrument generally used. No effort will be made to describe apparatus in detail, nor to discuss the minutiæ of every correction. These important matters are fully taken care of by observers' instructions issued by the United States Weather Bureau and other meteorological services. Besides, they pertain to the technique of the collection of data rather than to the science deduced therefrom, which latter, and not the former, is the object of the present discussion.

MEASURED PHENOMENA.

Temperature.—Probably the most obvious, satisfactory definition of temperature describes it as *that thermal state of an object which enables it to communicate heat to other objects.* Whenever the heat interchange that always takes place between two objects in thermal communication results in a net loss to the one and gain to the other, the temperature of the former is said to have been higher than that of the latter. If, however, there is no net loss or gain by either, the objects are said to have the same temperature.

Detection of net loss or gain of heat may be accomplished in any one of many ways, some of which are: change of volume; change of state; change of electromotive force; and change of electrical resistance. All these, according to circumstances, afford convenient means of comparing the temperatures of different objects, and of establishing a scale for ready reference. Thus the

ordinary mercury thermometer, the alcohol thermometer, adapted to low temperatures, and others of this nature, are based on the fact that the coefficient of volume expansion of the vessel is not the same as that of the contained fluid. Such thermometers, though capable of a high degree of accuracy, are not adapted to cheap and convenient registration, except of extremes; that is, the maximum or minimum temperature reached since last adjustment. Nevertheless, differential expansion does afford several means of obtaining continuous mechanical registration of temperature. The most compact and satisfactory apparatus of this kind in general use consists essentially of a curved closed tube of oval cross-section—a Bourdon tube—completely filled with a suitable liquid. Inequality of expansion between tube and liquid in this case demands change of volume, and that in turn changes the curvature of the tube.[1] Hence by making one end of the tube fast and connecting the other with a tracing point it at once becomes possible to obtain on a moving surface a complete record of temperature changes. The unequal expansion of the two sides of a bimetallic strip is also utilized in obtaining temperature registration.

Variation with temperature of electrical resistance, and of the electromotive force at a thermal junction, both provide means of measuring temperature changes to a high degree of accuracy.

In the case of the atmosphere, however, temperature commonly is measured at stated intervals, and whenever desired, by the readings (corrected when necessary) of a good mercurial, or, in very cold regions, alcohol, thermometer exposed to full circulation of the air, but protected from both solar and sky radiation. An excellent shelter for this purpose, with maximum and minimum thermometers in place, is shown in Fig. 1. Normally, of course, the door is closed. A less accurate but continuous record of atmospheric temperature usually is secured by the use of either a bimetallic or a Bourdon tube thermograph (Fig. 2). The connection between the thermal element and the tracing point may be either mechanical, as shown, or electrical. In the latter case the two may be separated any desired distance, the first placed outdoors, say, and the second conveniently located in an office. Other methods of measuring and even continuously recording the tem-

[1] For the theory of this extensively used gauge, see H. Lorenz; *Zeit. des Ver. Deutsch. Ingen.*, 54, p. 1865, 1910.

As a rule, the mercurial barometer is read by eye and only as occasion may require, but it also has been so constructed as to give excellent continuous records.

The aneroid, or, as its name implies, non-liquid, barometer, though involving many sources of error, is conveniently portable and capable of fairly satisfactory use in many places—on kites, aeroplanes, etc.—where the mercurial barometer would be wholly impracticable. It consists essentially of a disk-like vacuum cell, or series of such cells, a few centimetres in diameter, whose corrugated, flexible top and bottom are held apart by a short, stiff spring. Any change in the atmospheric or external pressure obviously must lead to a corresponding flexure of the spring, which motion may be communicated to either an index hand or a recording pen. In the ordinary barograph (Fig. 3) the pen commonly is actuated by a number of aneroid cells placed in series.

FIG. 3.

Barograph.

Most aneroids, whether single- or multiple-celled, require careful attention and frequent comparison with a standard mercurial instrument. They also are inherently subject to lag errors due to the imperfect elasticity of the cells that vary according

to the pressure conditions and the characteristics of the particular instrument, and which, for accurate readings, must always be allowed for.

Wind Velocity.—The velocity of the wind may be determined by triangulation on clouds, balloons, and other floating objects; by noting the speed of rotation, easily automatically recorded, of a windmill anemometer, air meter, or other similar device, and applying the necessary corrections; by the pressure on a flat surface squarely facing the wind; by the difference in level between the two free surfaces of a liquid in a U-tube or equivalent vessel when one surface is protected and the other exposed to the full force of the wind; and by a great many other but generally less accurate methods.

The Robinson cup anemometer (Fig. 4) appears to be the most convenient and reliable instrument wherever it can be properly exposed. The theory of its action, however, is but imperfectly understood.[1a] Rapid velocity changes, manifesting themselves in irregular puffs and of great importance to the aviator, the architect, and the engineer, generally are observed and recorded by some quick-acting pressure apparatus, such as the Dines pressure tube anemometer, or the Pitot tube, or Venturi tube.

The Pitot tube, of which the Dines anemometer is only a modification, consists of a tube with a "dynamic" opening facing the wind, or current of other fluid, and one or more "static" openings facing at right angles to the direction of the flow. When the respective openings communicate with closed chambers, obviously the head h of the fluid in question that would balance the difference between the dynamic and the static pressures in a perfect instrument (the best gives very nearly theoretical values) is given by the equation,

$$h = \frac{V^2}{2g}$$

in which V is the velocity of the current, and g gravity acceleration; all in C.G.S. units.

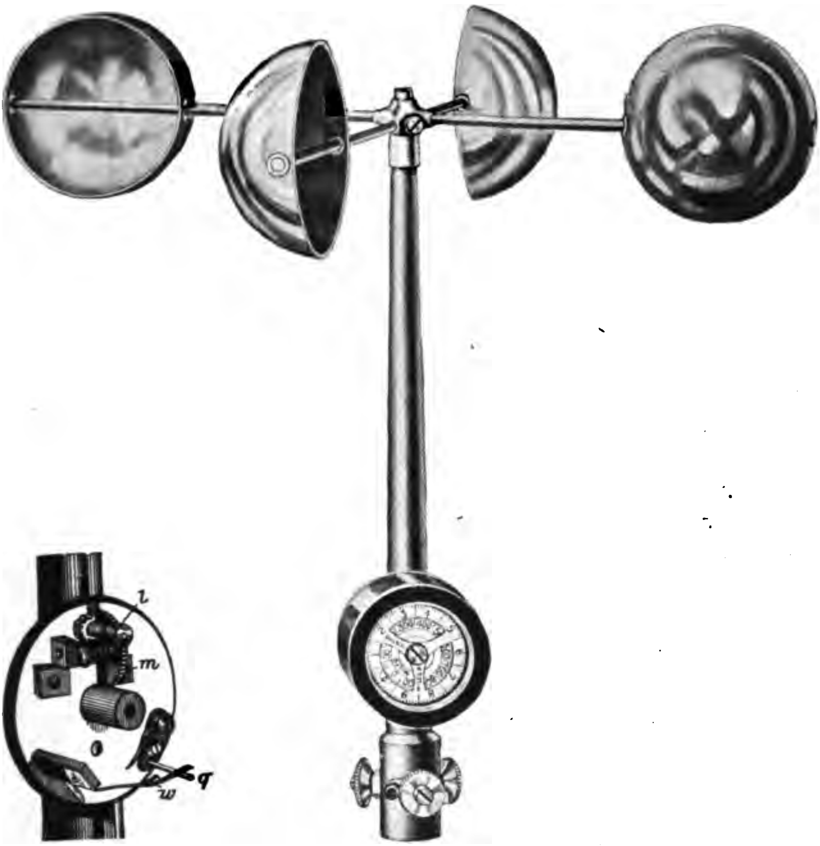
In practice the pressure difference is given by a column of liquid, a compressed spring, or other device, differentially connected with the two chambers, dynamic and static. In each case

[1a] Chree, *Phil. Mag.* 40, p. 63, 1895.

$$V = \sqrt{2gcp}$$

when p is the corrected reading of the indicator in whatever terms, and c the value of h per unit of p. If, for instance, p is dynes per square centimetre, c is the thickness in centimetres of a

FIG. 4.



Robinson's cup anemometer.

horizontal layer of the air, say, that would produce a gravity pressure of one dyne per square centimetre; and similarly for other types of graduation.

The Venturi tube which measures velocity of flow in exactly the reverse manner from that of the Pitot tube, that is, by decrease of pressure, consists of two oppositely directed hollow

cones joined together coaxially by a short throat of uniform cross-section. The angular opening of the receiving cone, which may have a short cylindrical mouth, is relatively large, while the discharge cone is comparatively long and tapering.

Let this tube be mounted parallel to the wind whose velocity V it is proposed to measure, and let r be the ratio of the cross-section of the mouth to that of the throat, in which the velocity, therefore, is rV. Clearly, then, if the flow through the tube is smooth (in good tubes it is very nearly so) the pressure against the wall of the mouth cylinder and that against the throat are each less than the outside static pressure. Furthermore, if h_1, h_2, and h_3 are the heads of the current atmosphere that would give the static, mouth, and throat pressures, respectively, then, neglecting the effect of the wind outside the tube,

$$h_2 + \frac{V^2}{2g} = h_3 + \frac{r^2 V^2}{2g} = h_1,$$

and

$$V = \sqrt{2g(h_1 - h_2)} = \frac{1}{r}\sqrt{2g(h_1 - h_3)} = \sqrt{\frac{2g(h_2 - h_3)}{r^2 - 1}}$$

To determine V by this method it clearly is only necessary to connect the mouth and throat cylinders through small openings to the opposite sides of a manometer, or either opening to one side of a manometer the other side of which is connected to a static chamber. If, as in the Pitot tube, p is the manometer reading and c the value of h per unit of p,

$$V = \sqrt{2gcp_1} = \frac{1}{r}\sqrt{2gcp_2} = \sqrt{\frac{2gcp_3}{r_2 - 1}}$$

for the several connections, as indicated.

Obviously, a Pitot and a Venturi tube can easily be combined by connecting the dynamic opening of the first and the throat of the second to opposite sides of a manometer, and the reading of the latter thereby made approximately double that given by either tube alone.

Wind velocities at considerable heights in the free air commonly are obtained by triangulation on clouds, or, less satisfactorily, owing to constantly changing altitude, on small free balloons.

Wind Direction.—The direction of the wind, as the term

is used in meteorological literature, always means the direction *from* which the wind is blowing at the point in question. It may be determined approximately by the course of smoke, clouds, or other floating objects, by the set of a wind vane (Fig. 5), drift of a pennant, flexure of trees, or other simple methods. Various devices for automatically recording this direction, either in its entirety or for selected points only, are possible, the simplest, perhaps, being electrical and under control of contacts made by a rod connected to and rotated by the wind vane. In common practice only a small number of directions, usually eight, are registered, each covering an angle of 45 degrees. That is, a wind from any point between W. 22.5° S. and W. 22.5° N. is registered as a west wind; and similarly for the other octants. This division may seem very coarse, but it is sufficient for most meteorological uses.

Humidity, Definitions.—The mixture of water vapor with the permanent gases of the atmosphere has occasioned a number of "humidity problems" over which the student is in danger of becoming more or less confused. And this danger is increased by the use in this connection of the same word by recognized authorities to connote quite different ideas. For the sake of clearness, therefore, this subject will be briefly discussed under several sub-heads.

I. *Absolute Humidity.*

Two entirely different definitions are in use for the common expression "absolute humidity":

a. The mass of water vapor per unit volume.

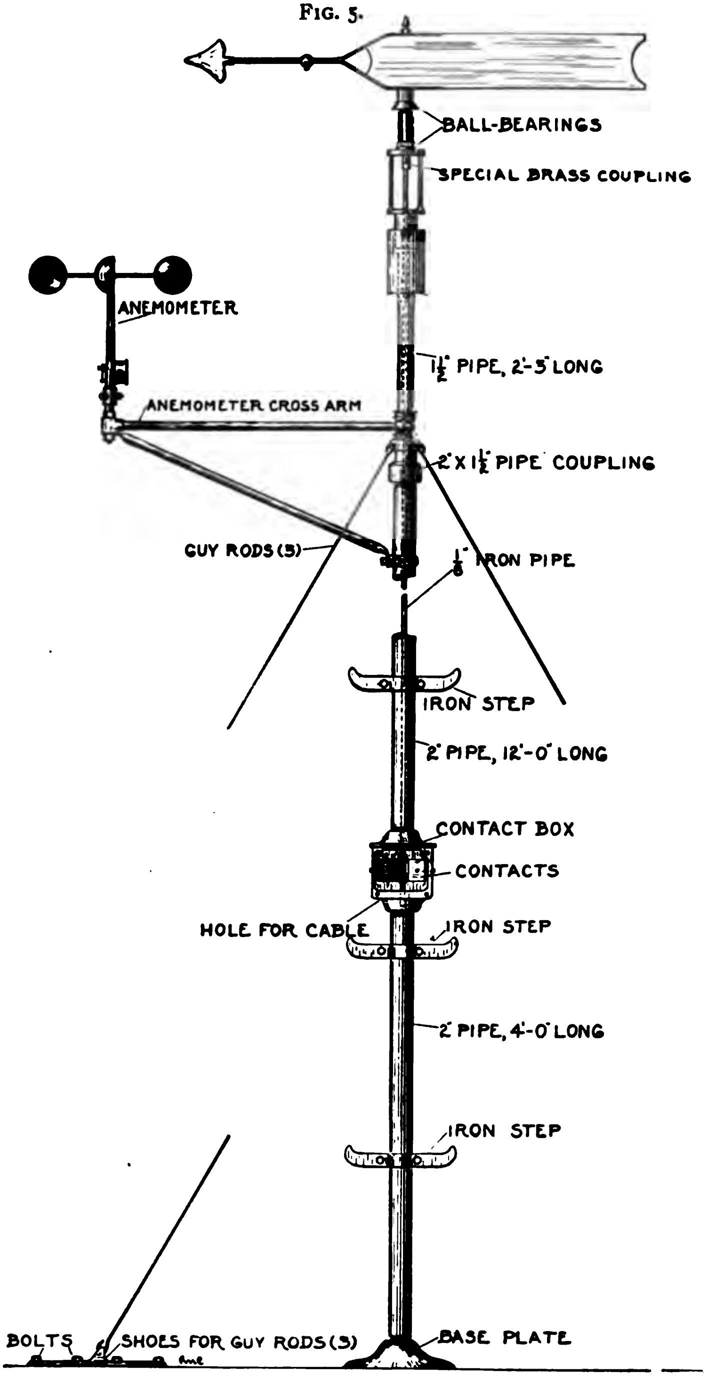
b. The gas pressure exerted by the water vapor per unit area.

According to the first definition, the absolute humidity may be expressed in terms of any units of mass and volume, as, for instance, grammes per cubic metre.

According to the second definition, it may be expressed in terms of any units of force and area—dynes, say, per square centimetre; or any measurable pressure effect, such as height of the mercury column the vapor pressure would sustain.

Accepting the simple definition *a* as being correct, as every one does, it remains to show the equivalence to it of definition *b*. But this follows at once from the well-known fact that the pressure exerted by any constituent in a uniform mixture of gases is to the total pressure as the number of its molecules

FIG. 5.



Wind vane and anemometer support, pattern 1913 (showing 4-foot wind vane on ball bearings).

per given volume is to the total number in the mixture. Vapor pressure, therefore, varies directly as vapor density, or mass per unit volume. Hence the two definitions, *a* and *b*, of absolute humidity are equivalent to each other, for any given temperature.

II. *Relative Humidity.*

Different definitions are also in use for the expression "relative humidity":

a. The ratio of the actual to the saturation quantity of water vapor, at the same temperature, per unit volume.

b. The ratio of the actual to the saturation pressure of water vapor at the same temperature.

In these definitions the expressions "saturation quantity" and "saturation pressure" refer to the maximum quantity of water vapor per unit volume and maximum pressure of water vapor per unit area, respectively, that can exist in the presence of a flat water surface, at the given temperature.

III. *Specific Humidity.*

The term "specific humidity," occasionally found in meteorological literature, means the weight of water vapor per unit weight of moist air.

IV. *Dew Point.*

The expression "dew point," as used in humidity tables and elsewhere, means simply that temperature at which, without change of pressure, saturation is just reached. It might also be defined as that temperature at which the saturation pressure is the same as the existing vapor pressure.

V. *Saturation Deficit.*

"Saturation deficit," a term much used by plant physiologists, is susceptible of several definitions, especially: (1) Amount of water vapor, in addition to that already present, per unit volume, grammes per cubic metre, say, necessary to produce saturation at the existing temperature and pressure. (2) Difference between actual and saturation pressure. (3) Ratio of the vapor pressure deficit to the saturation pressure at the existing temperature. The third is relative, the others absolute.

Humidity, Instrumentation.—The absolute humidity, in the sense of mass of water vapor per unit volume, can be determined

by noting the increase in weight of phosphorus pentoxide or other suitable drying agent on absorbing a known volume of the vapor. This direct determination of the humidity, however, is impracticable for routine observations.

On the other hand, as partial pressure ratios are independent of temperature, the determination of the absolute humidity in the sense of vapor pressure merely requires measuring the loss of pressure due to absorption of the vapor in a closed space, for which there are several devices;[1b] or, as more commonly practised, finding the dew point and referring it to a table of predetermined saturation pressures. Similarly, the difference between the current and dew-point temperatures is sufficient to determine, from suitable tables, the relative humidity.

The dew point may be found by any one of several slightly different methods, all of which have for their basis the determination of that temperature at which moisture just begins to collect on a cooling surface. A thin-walled silver tube, burnished on the outside, is an excellent vessel for the cooling mixture. The temperature of the liquid, if well stirred, and that of such a tube will be very nearly the same; and, besides, the dulling of the surface promptly reveals the slightest condensation.

It should be noted, however, that if carefully taken the observed temperatures of the silver hygrometer will be slightly below the actual "dew point." This is because the initial deposit is in the form of minute droplets, whose vapor pressure is greater than that of a flat surface at the same temperature, in accordance with the equation,[1c]

$$\Delta p = \frac{2\,T\rho_v}{R(\rho_w - \rho_v)}$$

which may be derived as follows:

Let R be the radius of a capillary tube standing in a vessel of water, Fig. 6; h the height of the water in the tube when saturation is attained; T the surface tension; ρ_w and ρ_v the densities of the water and the saturated vapor, respectively; and g the gravity acceleration. Then, obviously,

$$2\pi RT = \pi R^2 (\rho_w - \rho_v)\, gh;$$

and

$$\Delta p = \rho_v\, gh,$$

[1b] A. N. Shaw, *Trans. Roy. Soc. Can.*, 10, p. 85, 1916.
[1c] Sir William Thomson, *Proc. Roy. Soc., Ed.* 7, p. 63 (1870).

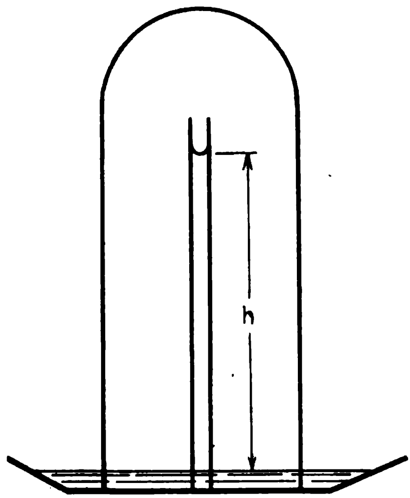
being the difference between the vapor pressures at the inner and outer surfaces.

Hence

$$h = \frac{\Delta p}{\rho_v g} = \frac{2T}{R(\rho_w - \rho_v)g}, \text{ and } \Delta p = \frac{2T\rho_v}{R(\rho_w - \rho_v)}.$$

At ordinary temperatures and for droplets whose radii are 10^{-4} cm. (a possible size) the temperature depression, or error, amounts roughly to 0.02° C. According to the equation, the error obviously might have any value, though actually it seems

FIG. 6.



Relation of curvature of surface to saturation vapor pressure.

always to be small, owing to limiting values of R, and presumably also to other causes; that is, this too, like many other physical equations, has its limitations.

In taking humidity measurements the observer must be careful that his presence does not affect the amount of moisture in the air under examination—he must stand to the lee of his apparatus.

Although the dew-point apparatus and other absolute hygrometers are extremely simple in theory, they generally are too complicated in structure and too difficult to manipulate to be suitable for routine observations. On the other hand, the psychrometer, presently to be explained, which depends on the maximum cool-

ing of water by evaporation when amply ventilated, is less obvious in theory,[1d] but both simple in construction and easy to use.

A convenient form of the psychrometric equation is:

$$e = e' - AB\,(t - t'),$$

in which

t = the air temperature.
t' = the temperature of a vigorously ventilated wet-bulb thermometer.
e = the vapor pressure.
e' = the saturation pressure at temperature t'.
B = barometric pressure.
A = a number that, in the case of ample ventilation, varies only with t', and with it but slowly.

Obviously, the relatively cool wet-bulb gains heat by conduction from the adjacent atmosphere and loses heat through evaporation. There is also absorption and emission of radiation, but when the ventilation is ample—3 metres per second or more—the net gain or loss of heat by this process is negligible in comparison with that by conduction or evaporation, respectively. Radiation effects may be still further reduced by surrounding the thermometer and its stem with a suitable wet-lined, reflecting shield, preferably of the Dewar bulb type. For most purposes, however, the use of a shield is unnecessary. It may be assumed, then, that the equilibrium or steady temperature of an amply ventilated wet-bulb thermometer is that at which the heat it gains by conduction from the passing air is equal to the heat it loses from evaporation.

The details of the processes of evaporation and heat conduction involved are not all fully known. But it is known that when radiation effects are excluded, the equilibrium temperature is measurably independent of the rate of ventilation, which, under these conditions, is only a convenient means of quickly attaining a steady state. In the extreme case of simple molecular diffusion in an otherwise stagnate atmosphere it seems safe to assume that

[1d] Ivory, *Phil. Mag.*, **60**, p. 81, 1822; August, *Ann. der Phys.*, **5**, 69, 1825; Apjohn. *Trans. Roy. Irish Acad.*, 1834; Regnault. *C. R.*, **20**, p. 1127;. 1220, 1845; 35, p. 930, 1852; Maxwell, Ency. Brit., 9th Ed., "Diffusion," 1878; Stefan. *Zeit. Ost. Gessell. für Meteorologie*, **16**, p. 177, 1881. Ferrel. Annual Report, Chief Signal Officer, Appendix 71, "Hygrometry," 1885; Carrier, *Trans. Amer. Soc. Mechan. Eng.* 33, p. 1005, 1912; Grossmann. *Ann. d. Hydr. usw.*, **44**, p. 577, 1916.

the space immediately adjacent to the wet surface is fully saturated at the temperature of the wet-bulb, and that heat is supplied by molecular bombardment of the air. Furthermore, if the temperature is not affected by ventilation (all radiation effects excluded) it seems that identically the same conditions just stated, or their equivalents, must hold whatever the ventilation.

In so far as these assumptions are true, it follows that during a steady state the gain of heat, Q, per unit time, say, is given by the equation

$$Q = m\, s\, (t - t')$$

in which m is the mass of previously free air, temperature t, that during the time in question comes into actual contact with the wet-bulb, temperature t', and s its average specific heat, at constant pressure, between these temperatures.

Similarly, the equal amount of heat simultaneously lost is given by the equation

$$Q = \frac{e' - e}{B}\, r\, m\, L_{t'}$$

in which e' is the saturation vapor pressure at the temperature t', e the vapor pressure in the free air, B the current barometric pressure, r the ratio of the molecular weight of water to the equivalent molecular weight of the free air, and $L_{t'}$, the latent heat of vaporization at the temperature t'.

Hence, equating the two values of Q.

$$e = e' - A\, B\, (t - t')$$

in which

$$A = \frac{s}{r\, L_{t'}}$$

The value of A, therefore, varies slightly, but to known amounts with e and t'. At $t' = 0°$ C. it also depends upon the phase, liquid or solid, of the evaporating water. But as vapor pressure at 0° C. is the same over ice as over water, it seems that the abrupt change of A at this temperature must be accommodated by an equivalent opposite change in the value of $t - t'$.

The general agreement between this theory and the best psychrometric observations, such as those of Hazen, Marvin and others of the Weather Bureau (Signal Corps),[1e] while not per-

[1e] "Annual Report Chief Signal Officer," Appendix 24, 1886.

FIG. 7—Sling psychrometer.

fect, is sufficiently close to make it doubtful which is in greater error, and thus to increase confidence in each.

When e, e', and B are expressed in millimetres of mercury under standard conditions and t and t' in centigrade values, observation gives $e = e' - 0.000660\ B\ (t - t')\ (1 + 0.00115\ t')$, approximately.

In practice t and t' commonly are obtained with a properly constructed and adequately ventilated (usually whirled) psychrometer carrying both a wet and a dry bulb thermometer. The sling psychrometer (Fig. 7), whirled by hand, is a simple device for this purpose. The observer has only to note the air temperature, the wet bulb depression (that is, difference between the wet and dry bulb temperatures), and barometric pressure. With these values he reads off from tables the vapor pressure and the dew point.

Assmann's aspiration psychrometer (Fig. 8), however, appears to be the most accurate instrument for this purpose. This consists of two parallel double-walled tubes, silvered to minimize radiation effects, containing a wet and dry bulb thermometer, respectively, united into a common stem and surmounted by a small ventilating fan.

Several kinds of instruments for automatically recording humidity have been devised, but of these only the hair hygrometer is in general use. It resembles some forms of thermograph except that the actuating member instead of being a thermal element is a strand of clean hairs, freed of oil, that increase in length with the humidity. With reasonable care the data thus obtained on a calibrated scale are both consistent and fairly reliable.

Cloudiness.—The degree of cloudiness generally is expressed in tenths (estimated) of the sky actually overcast.

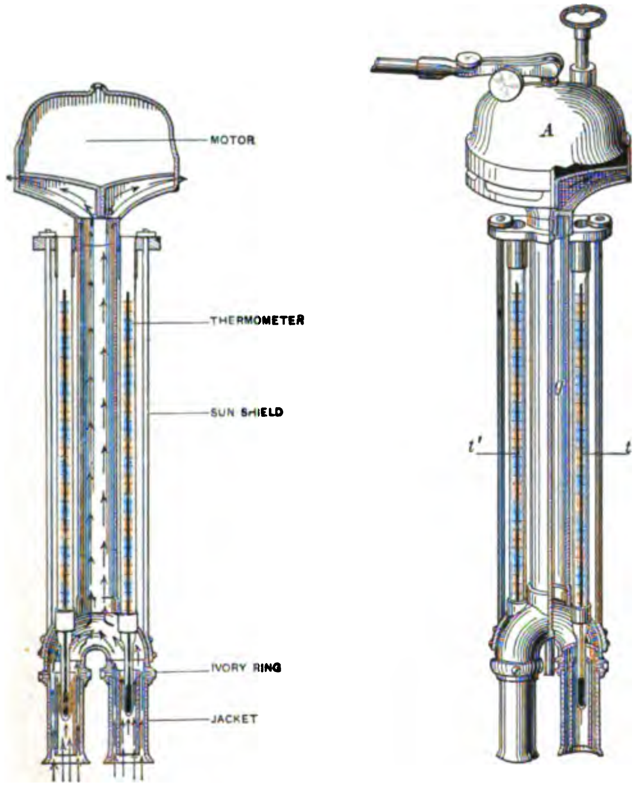
Kinds of Clouds.—As an indication of the approaching weather and general state of the atmosphere, the kind or kinds of clouds present

is more important than the mere total percentage of cloudiness. For convenient reference clouds have been divided into four primary and nine secondary (combination, alto, and fracto) forms.

These are:

PRIMARY FORMS: Cirrus—or curl cloud; stratus—or layer cloud; cumulus—or wool-pack cloud; and nimbus—or rain cloud.

FIG. 8.



Aspiration psychrometer.

COMBINATION FORMS: Cirro-stratus, cirro-cumulus, strato-cumulus, and cumulo-nimbus.

ALTO FORMS: Alto-stratus and alto-cumulus.

FRACTO FORMS: Fracto-stratus, fracto-cumulus, and fracto-nimbus.

The foregoing names are used in the International Cloud Classification, now generally accepted.

It will be noted that several names possible in accordance with this scheme of nomenclature are omitted, even though a few of them have occasionally been used in certain other schemes of classification. Thus there is no cirro-nimbus, for the reason that rain clouds never have the cirrus form; no strato-nimbus, because all rain clouds are flat-bottomed; no alto-cirrus, because cirri usually are high; no alto-nimbus, because rain clouds, except the cumulo-nimbus, are never high; and no fracto-cirrus, because cirri are always broken and detached.

Most of these clouds may be grouped according to their respective altitudes:

Upper Clouds: Cirrus, cirro-stratus.

Intermediate Clouds: Cirro-cumulus, alto-stratus, alto-cumulus.

Lower Clouds: Strato-cumulus, nimbus, fracto-nimbus, stratus, fracto-stratus.

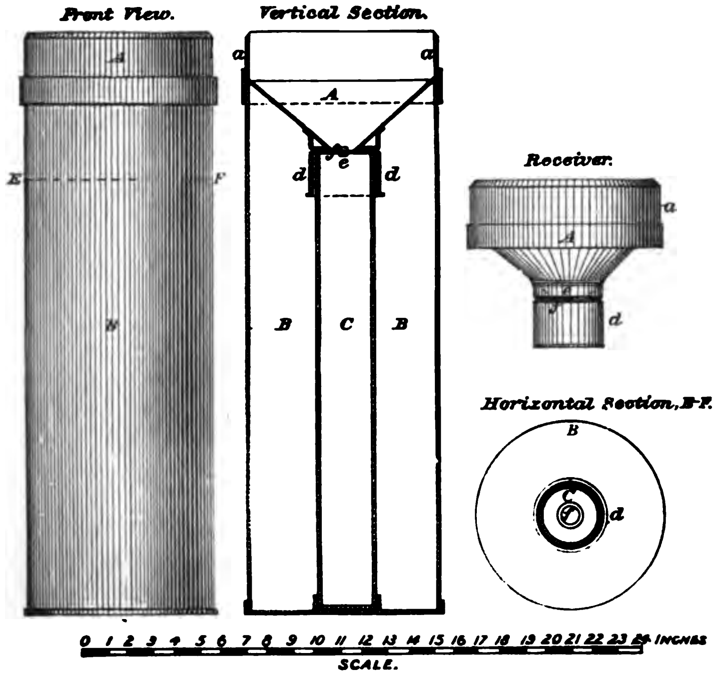
The cumulus, fracto-cumulus, and cumulo-nimbus (base), all caused by diurnal convection, vary in altitude from low to intermediate.

Precipitation.—The amount of precipitation is measured in the actual or, in case of snow, equivalent depth of horizontal water layer. The details in respect to the manner of catching and measuring precipitation have been greatly varied. The measuring, of course, is simple enough, but it is far from easy to secure a correct catch, owing chiefly to the influence of the vessel itself on the wind currents over and about its mouth and the consequent effect on the amount of precipitation actually caught. The details of a simple rain gauge are shown in Fig. 9, and its installation in Fig. 1. Many gauges are provided with a small tipping bucket just beneath the spout of the receiving funnel, by which the time of occurrence and rate of each rainfall are electrically recorded at any desired place.

Evaporation.—Evaporation is measured in terms of the depth of a flat layer of water, of area equal to that of the evaporating surface. This, too, like precipitation, has been measured by many kinds of apparatus, some of which have been designed with the view of simulating the surface of leaves, or meeting other special conditions. Many attempts have also been made to find from theoretical considerations a correct equation between rate of evaporation and the various factors upon which it depends,

such as shape of surface, extent of surface, temperature of the superficial layer, temperature of the air, humidity, barometric pressure, wind velocity, and anything else that might be considered of importance. A few special cases, such as evaporation from flush circular and elliptical water surfaces at constant temperature and in absolutely stagnant atmosphere, appear to have been completely analyzed.[2] But this work, however ingenious,

FIG. 9.



Rain gauge.

has contributed very little to the solution of the general problem, because in Nature water surfaces are of irregular outline, and all the factors that control evaporation are in such a maze of flux and reflex as to render equation-testing and evaluation of constants of doubtful accuracy and value. Evaporation, therefore, like most biological and many other phenomena, must be observed and measured; it cannot be computed very accurately as a function of given conditions. For discussion see Chapter XII.

Sunshine.—Sunshine generally is expressed in terms both of

[2] Stefan, *Sitz. der K. Akad.*, 73, 943, 954 (1881).

hours of its actual and percentage of its possible duration. It is recorded automatically, usually through electrical contact made or broken by the movement of a mercury piston in the stem of a vacuum-enclosed black bulb differential air-thermometer (Fig. 10); by charring on prepared cards in the focus of a glass sphere; or by photographic traces on sensitized paper.

FIG. 10

Sunshine recorder.

Radiation.—In relation to the atmosphere, radiation from three sources is of importance: from the sun, from the sky, and from the earth. Each may be measured integrally (that is, in terms of the amount of that energy delivered per minute, say, per unit normal area at the place of observation), or spectrally (that is, as distributed according to wave-length). The first kind of measurement, the integral, usually is made by some type of pyrheliometer, and the second (so far applied only to solar and sky radiation) by a bolometer.

Electrical Condition.—Measurements of the electrical condi-

tion of the atmosphere generally are confined to the vertical potential gradient, determined by any one of several methods, ionization and the consequent conductivity.

Optical Phenomena.—Various optical phenomena of the atmosphere are observed and recorded. These include, especially, mirages, sky colors, sky polarization, rainbows, coronas, and halos. For several of them—mirages, sky colors, and rainbows—mere eye observations are sufficient. Sky polarization, however, cannot be measured or even detected without the aid of suitable apparatus, while the data pertaining to halos and even coronas are far more valuable when they include accurate angular measurements.

Turbidity.—The turbidity or haze of the atmosphere, whether caused by dust particles (dust haze) or by irregular temperature distribution (optical haze), though often a matter of importance, seldom is measured, and even then only indirectly, since the usual method is to note the maximum distance at which a given object or certain of its details may be distinctly seen. Strictly speaking, this process measures only transparency, from which, however, the inverse, opacity, may be inferred.

Typical Installation.—A typical roof installation of the more common meteorological instruments is shown in Fig. 11. The wind vane is at the top of the tower, the whirling Robinson cup anemometer just below and to the left of the vane, and the sunshine recorder slightly lower, on the cage or platform railing. The thermometer shelter, with the door open, is in the lower portion of the tower. Finally, two rain gauges—one simple, the other tipping bucket—are shown in the lower left corner of the picture.

SOURCES OF METEOROLOGICAL INFORMATION.

As a further introduction to a discussion of the physics of the air, it will be helpful to consider a sort of vertical cross-section of the atmosphere as a whole with reference to the *sources* of meteorological information concerning each particular level. Other cross-sections that show its temperature, pressure, density, and composition at various elevations will be given later. Fig. 12, an adaptation of Wegener's profile of the atmosphere,[3] indicates the principal present sources of this information and the distribution of meteorological phenomena at various levels.

[3] *Phys. Zeitsch.*, **12**, Jahrg., 1911, p. 170.

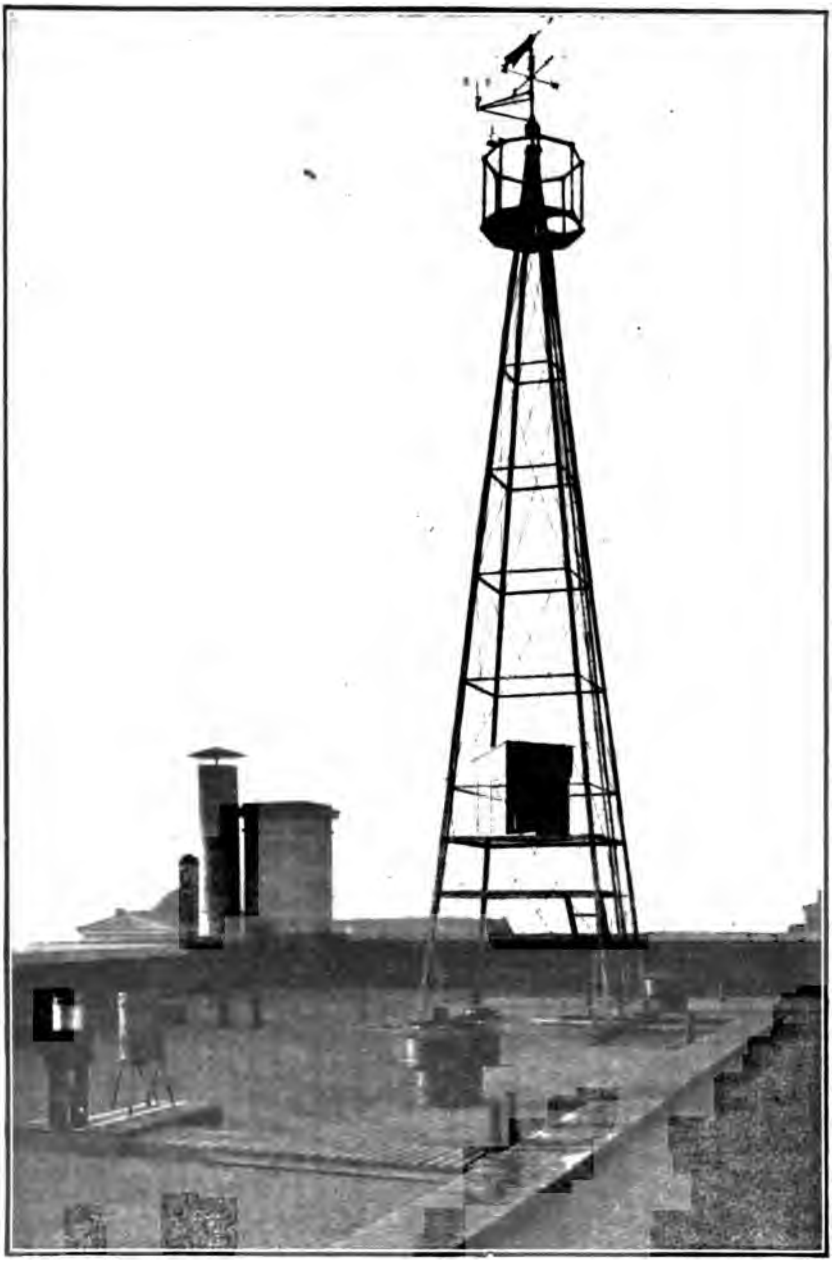
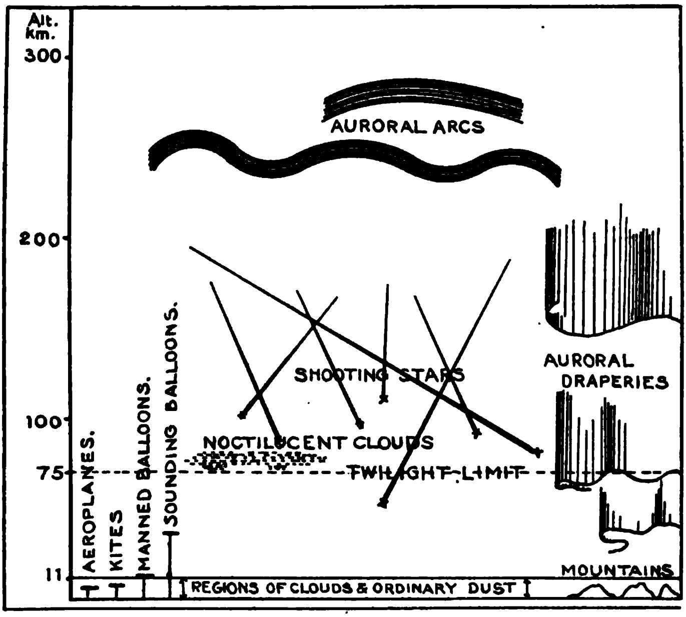


FIG. 11—Forty-foot steel wind instrument tower; typical installation for high office building (Lynchburg, Va., January, 1914.)

Mountains and other irregularities of the earth's surface make it practicable to examine the atmosphere minutely and to record continuously all its changes at every elevation from sea level up to nearly six kilometres. In fact, many continuous records have already been obtained at the summit station on El Misti, Peru, whose altitude is 5852 metres. Occasional and partial records have been obtained by this means as far up as about 7 kilometres, but no higher, since this is the limit to which any

FIG. 12.



Sources of meteorological information.

one has ever yet been able to climb. But all such records, whether obtained at high levels or low, of course are more or less affected by the surface conditions. Hence some means of obtaining observations and records other than apparatus carried about on the surface of the earth is essential to a knowledge of the conditions and movements of the free atmosphere. One obvious source of information as regards motion only, and which has been extensively used, is the observation of drifting clouds, which occur at all levels from the bottom of the atmosphere up to 11 kilometres, or thereabouts, in middle latitudes, and occasionally, in the tropics, even as high as 15 kilometres.

There are several methods of determining the elevation, direction of motion, and velocity of clouds, but all depend upon simple processes of triangulation. Thus, simultaneous theodolite observations made on the same spot in a cloud from two stations whose elevations and distance apart are known obviously furnish all the data necessary for an easy and fairly accurate determination of the height of the particular spot in question, while a single subsequent observation by either instrument on this spot, provided the time interval between the first and second observations is known, clearly gives all the additional data necessary to the determination of its velocity and direction of travel—assuming uniform motion and constancy of elevation. Excellent results, also, are gotten from cloud negatives simultaneously obtained with phototheodolites provided with fiducial lines. In this way, if several successive exposures are made, the height and movement of each distinguishable point in the cloud can be determined, and therefore not only the general height and drift of the cloud as a whole, but also its dimensions and something of its internal motions. However, the general motion of the wind at the point observed and time of observation, though interesting and often valuable, is by far the chief information about the atmosphere that clouds give, and, indeed, some, such as those formed by air billows over mountain crests and elsewhere, do not give even this. Besides, they are not always present, so that on clear days even this modicum of information about the upper air would be impossible to obtain if we had no other means of investigation. But there are others, the most fruitful of which is the carrying of self-registering thermometers, barometers, hygrometers, and the like into the free air by means of:

a. Kites (Fig. 13) to over 7 kilometres, the record being 7.26 kilometres.

b. Aeroplanes; present limit about 11 kilometres.

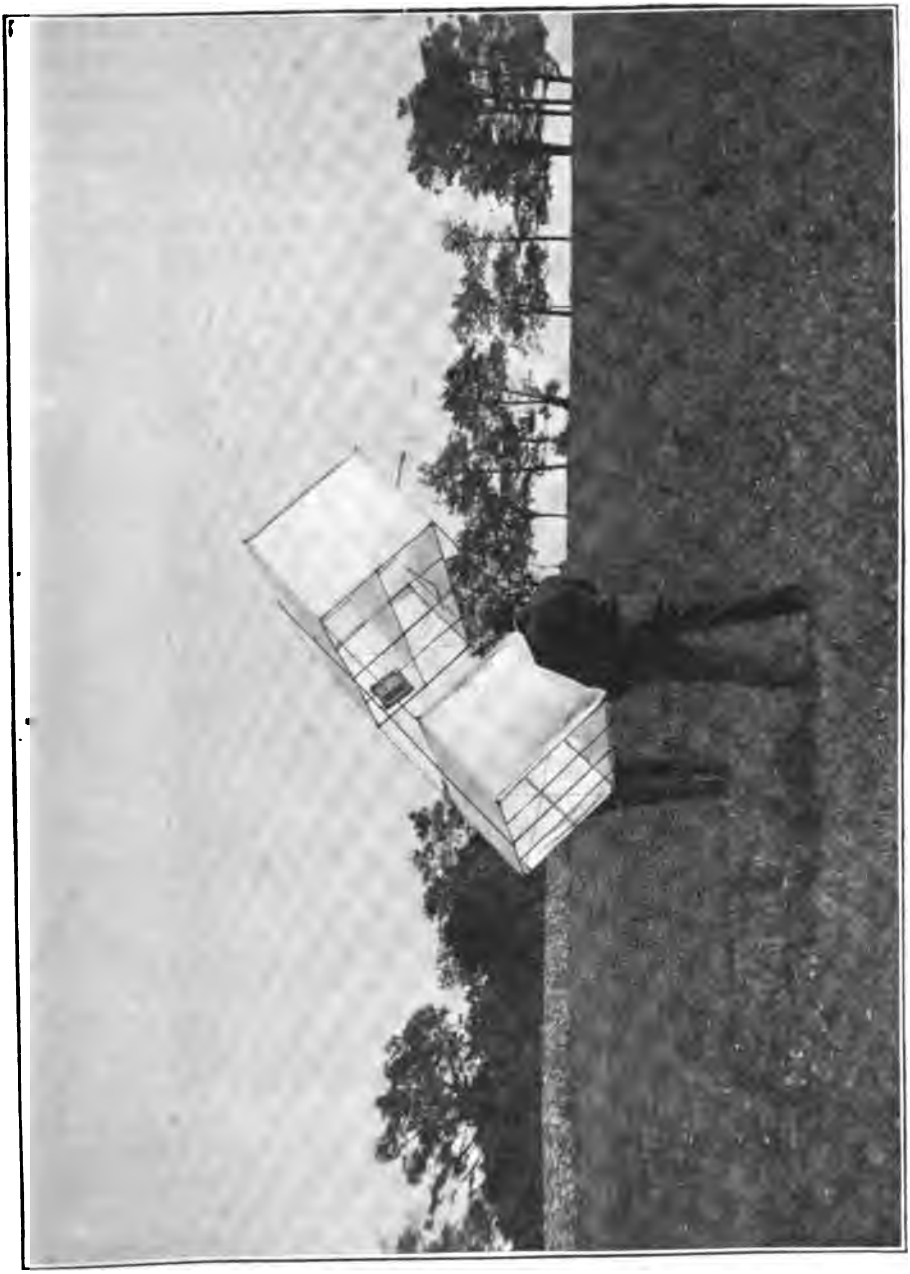
c. Manned balloons; maximum elevation, roughly, 11 kilometres.

d. Sounding balloons (Figs. 14 and 15), with a record of 35.08 kilometres.

Upper air movements are also shown by the flights of pilot balloons (small balloons without apparatus); record 39 kilometres.

The registering apparatus sent aloft by these various methods furnish reliable information concerning the composition (in-

Fig. 13.



Launching meteorological kite, Mount Weather.

cluding humidity), temperature, pressure, direction of motion, and, in some cases, velocity of the air, from the surface of the earth up to the greatest elevation reached. And it is this automatically recorded information, gathered, with but little exception, since the beginning of the twentieth century, that has so

FIG. 14.

Sounding balloons.

greatly extended our accurate knowledge of meteorology, and done so much to make of it an interesting and profitable branch of both theoretical and applied physics.

Beyond the reach of the pilot balloon, or, for the present, at elevations greater than 39 kilometres, our information of the atmosphere is limited to such deductions as properly may be

drawn from the height of the twilight arch—roughly, 75 kilometres; the paths of shooting stars, rarely, if ever, seen as high as 200 kilometres; and the phenomena of the auroras, those curious and but partially explained electrical discharges that seldom occur at a lower level than 90 kilometres or higher thân 300.

Fig. 15.

Sounding balloon.

The above, obviously, are all, or nearly all, the means by which our knowledge of the atmosphere has been obtained. Up to 35 kilometres it is comparatively well known, but beyond that level only deductions, growing less certain with increase of elevation, can possibly take us at present, or at any time until higher soundings have been made.

Chapter II.

SOME THEORETICAL TEMPERATURE RELATIONS OF THE ATMOSPHERE.

In order to acquire a clear understanding of the causes of the actual distribution of temperature in the atmosphere, it will be convenient, first, to consider some of the thermodynamic equations of gases, especially those that give relations between temperature, pressure, and volume.

If to a unit mass of air or other gas at constant pressure p a quantity of heat dQ be supplied, the energy so added will divide itself into two parts. One portion will change the temperature of the gas and the other will change its volume. Hence, if the work is expressed in its heat equivalent, or if each portion of the energy is expressed in heat units and not in units of work, then

$$dQ = C_v\, dT + A\, p\, dV \ldots\ldots\ldots\ldots (1)$$

in which C_v is the specific heat of the gas in question at constant volume, dT and dV the resulting changes in temperature and volume, respectively, and A the reciprocal of the mechanical equivalent of a unit of heat.

But to secure the relations desired, the relation of p to T, for instance, when both are variable, it is necessary to have an additional equation involving dT, dp, and dV. From Boyle's and Charles's laws, we have the equation,

$$pV = \frac{p_0 V_0}{T_0} T,$$

which expresses the fact that for a given quantity of gas the product of pressure and volume varies directly as the absolute temperature, T. So long, then, as the quantity of gas involved and its temperature are constant, so also is the product pV. But when this quantity is one gramme and the temperature 0° C., it is convenient to speak of the quantity, $\frac{p_0 V_0}{T_0}$, as the characteristic constant, R, of the gas in question. In general, then,

$$pV = RT,$$

in which the value of R depends solely upon the kind of gas.

Hence, differentiating,

$$p\, dV + V\, dp = R\, dT. \ldots\ldots\ldots\ldots (2)$$

To find the relation between dp and dT in an adiabatic process (that is, a process in the course of which no heat is either given to or taken from the gas involved, such as closely obtains in the case of rapidly rising or falling air), it is only necessary, by aid of equation (2), to eliminate dV from equation (1) and to put $dQ = 0$.

Thus

$$C_v\,dT + A\cdot(R\,dT - V\,dp) = 0$$

or

$$(C_v + AR)\,dT = AV\,dp. \qquad \ldots\ldots (3)$$

Also, since the excess of the specific heat at constant pressure over the specific heat at constant volume is simply the amount of heat necessary to perform the external work incident to expansion as a result of increasing the temperature 1° C., we have,

$C_p - C_v = AW$, the heat equivalent of the external work.

And from $pV = RT$, we get, with p constant,

$$p\,dV = R\,dT.$$

If $dT = 1°$ C., then $Ap\,dV$ is the heat equivalent of the external work done as a result of increasing the temperature of the unit mass of the gas in question 1° C. Hence,

$$C_p - C_v = AR.$$

On substituting this value of R in (3), we get,

$$C_p\,dT = A\,V dp,$$

or

$$\frac{dT}{dp} = \frac{AV}{C_p} = \frac{ART}{pC_p}. \qquad \ldots\ldots (4)$$

From this it appears that the ratio of the change of temperature to the change of pressure, in an adiabatic process, is directly proportional to the absolute temperature and inversely proportional to the pressure.

In the case of dry atmospheric air at ordinary temperatures $C_p = 0.241$, about.[4] Hence,

$$dT = dp\frac{ART}{p\ 0.241}.$$

But

$$R = \frac{p_0 V_0}{T_0} = \frac{p_0}{\rho_0 T_0},$$

from which, assuming p_0 to be the pressure in dynes per square

[4] Moody, *Phys. Rev.*, 34, p. 275 (1912); used by Bureau of Standards.

centimetre when the barometer under gravity $g=981$ and at 0° C. stands at 760 mm., and ρ_0 the corresponding density of dry air at 0° C., it follows that, numerically,

$$R = \frac{1033.2 \times 981}{0.001293 \times 273} = 2.871 \times 10^6;$$

and

$$A = \frac{1}{4.19 \times 10^7}.$$

Therefore,

$$dT = \frac{dp}{p} \frac{T}{3.5172},$$

and

$$\frac{dT}{T} = .2843 \frac{dp}{p}.$$

In the special case where the pressure is one atmosphere (barometer reading 760 mm.) and the temperature 0° C. (273° absolute), such as often happens on the surface of the earth, an adiabatic change of pressure represented by 1 mm. of the barometer produces a temperature change given by the equation,

$$dT = \frac{1}{760} \frac{273}{3.5172} = 0°.10213 \text{ C.}$$

From equation (4) we get

$$\frac{dT}{T} = \frac{dp}{p} \frac{AR}{C_p}$$

Hence

$$\log_e \frac{T_1}{T_2} = \frac{AR}{C_p} \log_e \frac{p_1}{p_2}$$

or,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{AR}{C_p}} = \left(\frac{p_1}{p_2}\right)^{\frac{C_p - C_v}{C_p}} = \left(\frac{p_1}{p_2}\right)^{0.286},$$

or,

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{3.50}.$$

If we wish to find the rate of adiabatic cooling with change of elevation, dh, a matter of great meteorological importance, it is necessary to find the value of dp in terms of dh and substitute it in equation (4). It must be remembered, too, that pressure p decreases as the height h increases.

But $-dp = 981\,\rho\,dh$, where ρ is the density of the gas in question, or

$$-dp = \frac{981\,dh}{V} = \frac{981\,p\,dh}{RT}.$$

Hence, by substitution in equation (4), as explained,

$$-\frac{dT}{dh} = \frac{981\,A}{C_p} = \frac{1}{10293} \quad\text{.........................(5)}$$

Clearly, then, when the atmosphere is dry and its temperature decreases with increase of altitude at the above adiabatic rate of 1° C. per 102.93 metres, any portion of it transferred without gain or loss of heat from one level to another has, at every stage, the same temperature and density as the adjacent air, and therefore, if abandoned at rest, will neither rise nor fall. If, however, the temperature decreases with altitude at a less rate than the above, an isolated mass of air, on being adiabatically lifted or depressed, becomes colder and denser or warmer and rarer, respectively, than the adjacent air, and consequently, if abandoned, will return to its initial level. Finally, if the temperature decrease with altitude is more rapid than the above rate, an isolated mass of air, on being elevated or depressed, will become warmer and lighter or colder and denser than the adjacent air, and, if permitted, will continue to rise or fall, respectively, until arrested by a change in the temperature gradient, or, if descending, perhaps even by the surface of the earth.

In short, the atmosphere is in neutral, stable, or unstable equilibrium according as the temperature decrease with increase of altitude is the same as, less than, or greater than the adiabatic rate of 1° C. per 102.93 metres (more or less, as g is less or more).

From equation (5) it is obvious that the adiabatic rate of temperature decrease of the atmosphere with increase of altitude is independent alike of altitude (except as gravity is so modified), temperature, and pressure, unless, possibly, its specific heat at constant pressure may slightly vary with temperature or pressure, or both. When, however, the composition of the atmosphere is changed, it is obvious that its specific heat, and therefore its temperature gradient, must also change. Now the chief variable constituent of the atmosphere is water vapor, and, since its specific heat is approximately 1.95 that of dry air, it follows that the greater the amount of water vapor present the less rapid will

be the adiabatic decrease of temperature with increase of altitude. If, for instance, 3 per cent. of the pressure is due to water vapor, the adiabatic decrease of temperature, before saturation is reached, will be at the rate of about 1° C. per 103.59 metres increase of elevation, and proportionately altered for other percentages.

Water vapor also frequently causes another and most important change in the temperature gradient. As soon as condensation sets in the latent heat of vaporization, and, if ice is formed, of fusion, is liberated, and thus the rate of temperature decrease with altitude is reduced. The amount of this reduction, often at least half the original value, depends, of course, slightly upon what becomes of the condensed vapor. If it is carried along with the rising air the process remains adiabatic, except as modified by conduction and radiation, but if, as in great measure must happen, it is left behind as precipitation, then the process becomes that special case of the nonadiabatic which von Bezold, followed by others, has called pseudoadiabatic. This whole subject has been more or less discussed by several writers, but most fully, first, by Hertz[5] and, later, by Neuhoff.[6]

Undoubtedly much of the condensation drops out, or begins to drop out, as soon as formed, so that the actual temperature gradient, while lying somewhere between the really adiabatic and the "pseudoadiabatic" curves, probably follows the latter more closely than the former. Presumably, therefore, in practice it would be better, or at least quite as well, to determine the latter gradient (the adiabatic will be considered later, under "Condensation," in chapter XII) and then to add such corrections to it as the circumstances of individual cases suggest. The main curve can be determined as follows:

As before,

$$dQ = C_v\, dT + Ap\, dV.$$

But

$$pV = RT,$$ (R being appropriate to the existing mixture of air and water vapor).

Hence

$$\begin{aligned} dQ &= C_v dT + A(R\, dT - V\, dp) \\ &= (C_v + AR)dT - A\, V\, dp \\ &= C_p dT - A\, V\, dp \end{aligned}$$

[5] *Deutsch. Met. Zeit.*, vol. i, 1884, p. 421.

[6] *Abh. d. K. P. Met. Inst.*, vol. i, No. 6, Berlin, 1900.

But

$$-dp = g\rho dh,$$

where g is gravitational acceleration.

Therefore,

$$dQ = C_p dT + g\,A\,dh.$$

Now the heat, dQ, is added as the result of a quantity of water vapor, dw, being extracted. Hence

$$dQ = -s\,dw,$$

in which s is the heat of vaporization, and therefore,

$$-s\,dw = C_p dT + g\,A\,dh.$$

From this equation it is obvious that to obtain the ratio of dT to dh in terms of measurable quantities it is necessary and sufficient to express dw in similar terms.

But

$$w = \rho\,0.622\,\frac{e}{b},$$

in which w is the total mass of water vapor per cm.[3], ρ the density of the air, 0.622 the ratio of the molecular weight of water vapor to the weighted mean of the molecular weights of the constituents of dry air, e the partial pressure of the water vapor in terms of millimetres, say, of mercury, and b the height, in millimetres, of the barometer.

Hence

$$\frac{dw}{w} = \frac{de}{e} - \frac{db}{b}.$$

But, if D is the density of mercury,

$$-D\,db = \rho\,dh = \frac{p\,dh}{RT} = \frac{D\,b\,g\,dh}{RT}$$

Hence

$$-\frac{db}{b} = g\,\frac{dh}{RT}.$$

and

$$dw = w\,\frac{de}{e} + wg\,\frac{dh}{RT}.$$

Hence, by substitution,

$$C_p\,dT + sw\frac{de}{e} + swg\,\frac{dh}{RT} + gA\,dh = 0$$

or

$$\left(C_p + sw\frac{de}{e\,dT}\right) dT + \left(\frac{sw}{RT} + A\right) g\,dh = 0$$

and

$$\frac{dT}{dh} = -\frac{g\left(A + \frac{sw}{RT}\right)}{C_p + sw\,\frac{de}{e\,dT}} \quad \text{(6)}$$

All the terms on the right-hand side of this equation are known for any definite temperature and assumed value of dT. From this equation, therefore, tables can be written and curves constructed that give the "pseudoadiabatic" gradient under all conditions of temperature and pressure.

Temperature Changes of a Rising (or Falling) Isolated Mass of Air.—The above discussion of the temperature decrease with elevation of dry air applies only to an atmosphere whose potential temperature (temperature any portion would have if carried adiabatically to some given level, usually mean sea level) is the same throughout, such as it would be on thorough adiabatic mixing. As a matter of fact, the actual potential temperature of the atmosphere rarely, if ever, is uniform, and hence it is of some interest to trace the temperature changes with elevation of an isolated mass of air, or other gas, as it rises or falls adiabatically through an atmosphere whose potential temperature is non-uniform.

This subject has recently been discussed by several authors, but most concisely, perhaps, by Exner in his *Dynamische Meteorologie,* and, in substance, as follows:

Let the absolute temperature at a given point within the adiabatically-cooling (or warming) isolated mass of air be T, and that of the surrounding air at the same level, and where the pressure, therefore, is also the same, T'. Then, as already explained, at any point within this mass

$$\frac{dT}{dp} = \frac{ART}{pC_p}$$

whatever the cause of the pressure change, dp.

Now, let dp be due wholly to change of level of the isolated mass in the surrounding air; then

$$-dp = \frac{gp\,dh}{RT'}.$$

Hence the temperature gradient of (not within) the rising mass at the place in question is given by the equation

$$\frac{dT}{dh} = -\frac{gAT}{C_pT'} = -a\frac{T}{T'},$$

in which a is the adiabatic temperature gradient. That is, the rising air will cool at a greater or less rate than the "adiabatic" according as its temperature is higher or lower than that of the adjacent atmosphere, and by roughly 0.4 of one per cent. of the adiabatic rate for each 1° C. difference.

Let the height under consideration be h, and let the temperature of the free air decrease uniformly with elevation. Then, if T'_h is the temperature of the free air at the elevation h, and T'_o its temperature at the surface,

$$T_h' = T_o' - lh,$$

in which l is the uniform lapse-rate, or ratio of temperature change to change of elevation, of the free air. Hence

$$(dT)_h = -a\frac{Tdh}{T_o'-lh}$$

or

$$\frac{dT}{T_h} = -a\frac{dh}{T_o' - lh}$$

and

$$\log T_h = \frac{a}{l}\log(T_o' - lh) + \text{a constant}$$

If T_o is the initial or surface temperature of the rising mass, then

$$\frac{T_h}{T_o} = \left(\frac{T_o' - lh}{T_o'}\right)^{a/l}.$$

Hence, in general, the cooling with elevation is given by the equation

$$\left(\frac{dT}{dh}\right)_h = -a\left(\frac{T_o'-lh}{T_o'}\right)^{\frac{a-l}{l}}\frac{T_o}{T_o'}.$$

When the temperature of the free air has the "adiabatic" distribution, or $l = a$,

$$\frac{dT}{dh} = -a\frac{T_o}{T_o'}.$$

That is, the rising air then also cools at a constant rate, but one more or less different from the "adiabatic," except in the special case when it has the same initial temperature as the adjacent air.

When $l = o$, or the temperature of the free air remains constant with elevation, as it does in the isothermal region, the value of dT/dh clearly can not be determined from the above equation. However, from

$$T_h = T_o \left(1 - \frac{lh}{T_o}\right)^{a/l},$$

it appears that when $l = o$,

$$T_h = T_o e^{-a h/T'_o},$$

and

$$\left(\frac{dT}{dh}\right)_h = -a \frac{T_o}{T'_o} e^{-ah/T'_o}.$$

Hence in an isothermal region the rate of cooling of a rising mass of air decreases with elevation.

Finally, from the equation

$$T_h = T_o \left(\frac{T_o' - lh}{T_o'}\right)^{a/l},$$

it appears that

$$T_h = T_o' - lh,$$

or that the rising air comes to the temperature of the surrounding atmosphere, and thus into a position of rest, at the elevation

$$h = \frac{1}{l}\left[T'_o - \left(\frac{T_o'^a}{T_o^l}\right)^{\frac{1}{a-l}}\right].$$

If, for instance, $l = a/2$, $T'_o = 290°$ A., and $T_o = 300°$ A., then $h = 1.99$ kilometres, approximately; whereas, if the rising air cooled according to the adiabatic rate (1° C. per 103 metres), as usually assumed, the value of h would be 2.06 kilometres.

If the roughly approximate value of the adiabatic rate, 1° C. per 100 metres, is used, the above values of h become 1.93 kilometres and 2 kilometres, respectively.

Chapter III.

OBSERVED VERTICAL TEMPERATURE GRADIENTS.

The temperature of the surface air is well known at many places and at various altitudes, from sea level up to about six kilometres. But the temperature records obtained by the aid of kites and balloons, both manned and free, show that the mountain air temperatures generally differ materially from the temperature of the free air at the same elevation and latitude.

According to Hann,[7] the average temperature of the surface decreases approximately at the rate of 1° C. per each 180 metres, 200 metres, and 250 metres increase of elevation on mountains, hills, and plateaus, respectively. In the free atmosphere, however, the result is quite different. Here the decrease of temperature with increase of altitude, except at very great elevations, is, roughly, the same at different parts of the world.

The records obtained by kites, manned balloons, and sounding balloons all agree, of course, so far as they apply to the same levels, but as the free or sounding balloon, with its automatically registering apparatus, has gone far higher than either manned balloons or kites, and as ascensions by it have been quite numerous, only the records thus obtained will be considered in what follows. Again, and for the sake of still further uniformity, the first part of the discussion will be confined to only those records which were obtained at Munich, Strassburg, Trappes, and Uccle, four European stations of about the same latitude and more or less similar climates. It seems also desirable to divide the records of vertical temperature distribution according to season, winter (December, January, February, and March) and summer (June, July, August, and September), and prevailing type of weather, or, to be more exact, the height of the barometer, high, low, and neutral. Spring and fall observations will not be used in the typical or most general temperature distributions, owing to the transitional nature of these seasons, or the overlapping and confusion at these times of summer and winter conditions.

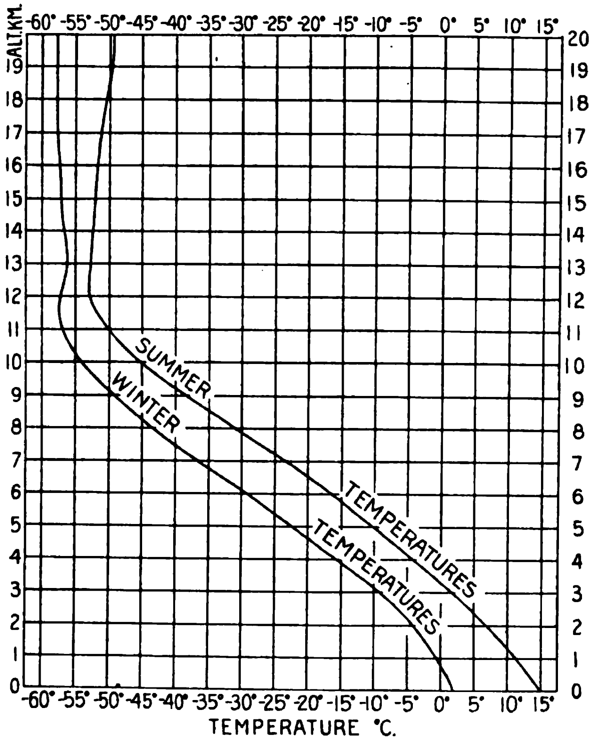
At the time these data were assembled, the middle of 1918, all the available records of the stations mentioned were used.

[7] "Lehrbuch der Meteorologie," 3d ed., p. 126.

there being 185 winter records and 231 summer records. A larger number of flights would, of course, furnish a somewhat more reliable average, but, as the several stations gave substantially the same results, it would seem that no great change would be made in the season averages, however large the number of combined observations.

Fig. 16 gives the average winter and summer vertical tem-

FIG. 16.



Winter and summer vertical distribution of temperature.

perature gradients of the stations in question, or the graphs obtained by plotting the average temperatures (derived from the average of the observed temperature gradients) of the given season against the corresponding altitudes at which they were obtained.

A number of interesting points are brought out by these two curves, each of which calls for an explanation. Among other

things, the two gradients are, roughly, parallel to each other throughout their whole range. This is because the temperature of the atmosphere from top to bottom is determined by the same factors in the winter that determine it in the summer; that is, by radiation, conduction, and convection, all mainly from the surface of the earth and the lower atmosphere. Since all these factors are less in winter than in summer, it follows that their combined result, the temperature of the higher atmosphere, must also be less at every level; hence the substantial parallelism of the two gradients.

Again, it appears, as shown by the figure, that up to about 2½ kilometres the temperature decreases less rapidly with increase of elevation during winter than it does during summer. The reason for this, while not quite obvious, will become apparent from the following considerations:

The surface of the earth, which is a much better radiator than the atmosphere, often cools, especially during clear nights, to a decidedly lower temperature than the air 100 metres or so above it. Hence, late at night, when the sky is clear and the wind is light, the temperature near the surface usually increases with increase of elevation, and even when there is sufficient wind to prevent this "temperature inversion," as it is called, the lower atmosphere still is colder than it otherwise would be. Obviously, too, the amount of this surface cooling, and therefore the magnitude of the temperature inversion, depends jointly upon the rates of radiation to and from the sky and the time involved. Now the rate of the output of surface and lower air radiation is less in winter than in summer, both because of their lower temperatures at that time and because the atmosphere then contains less water vapor, its chief radiating constituent. Nevertheless, even though the radiation loss from the surface of the earth and adjacent air is greater in summer than in winter, the concurrent radiation gain from the upper air may, perhaps, usually render the difference, or net loss, somewhat less.

At any rate, partly for this reason, it may be, but mainly because of the relatively greater length of the nights and greater dryness and consequent diathermacy of the atmosphere, the total surface cooling, and therefore the morning temperature inversions, is much more pronounced in winter than in summer. Hence the average decrease of temperature with increase of elevation

through the first one or two kilometres is decidedly less during the colder than during the warmer season.

Another peculiarity shown by the curves is the fact that between the elevations of approximately four and eight kilometres the temperature decreases rather more rapidly during winter than summer. Throughout this region the temperature of the atmosphere depends in part upon convection from lower levels and in part upon its gain and loss of heat through radiation. But even this midair, or radiational, change in temperature can result only in immediate convection. Consequently, so long as saturation is not reached, the convectional temperature gradient must be very approximately that of a totally dry atmosphere. If, however, condensation takes place, the latent heat of vaporization becomes sensible heat, and the decrease of temperature with increase of altitude is correspondingly less. When a condensation temperature gradient is once established in this mid-region of the atmosphere it tends to persist, even after condensation has ceased and the clouds have evaporated, because, whatever condition—the presence or absence of sunshine, for instance—produces a temperature change in one part of it is likely to produce similar temperature changes in other parts, and therefore approximately the same variation for each level, a variation which would leave the general temperature gradient substantially as before. We should, then, expect to find the average vertical temperature gradient following, roughly, the gradient for saturated air for the given temperature, and such, indeed, are the gradients actually found.

Hence, as the atmosphere between the elevations of four and eight kilometres is quite out of the reach of surface inversions, and as it is also warmer and more humid during summer than during winter, we should expect the summer temperature of this region to decrease less rapidly with increase of altitude than does the winter temperature, precisely as balloon records show to be the case.

Another point brought out by Fig. 16 is the fact that up to eight kilometres, or thereabouts, the ratio of the decrease of temperature to increase of altitude itself increases with altitude. The explanation of this phenomenon is precisely the same as that of the difference between the winter and summer gradients from four to eight kilometres. That is to say, it depends upon the

amount of water vapor necessary to produce saturation at the various levels, since the less this vapor is in proportion to the total gases present the more nearly does the actual temperature gradient follow the adiabatic curve for dry air.

One striking feature of each of the temperature gradients is its gradual change, between the levels of nine and twelve kilometres, from a rapid decrease of temperature with increase of elevation to an approximately isothermal condition. Normally, however, the temperature gradient changes from a rapid to practically a zero decrease, and even, as usually recorded, to a slow increase of temperature with increase of elevation, much more abruptly than one would infer from the given curves. But this more or less abrupt change varies considerably in altitude from day to day. Therefore, when a large number of actual gradients are averaged an *apparent* gradual transition is indicated.

Two other features calling for some attention are shown by the lower portion of the summer gradient; namely, the fact that through the first half kilometre the temperature decreases but slowly, and the further fact that through the second half kilometre it decreases more rapidly than anywhere else, short of very considerable altitudes. Now nearly all the observations from which this summer gradient was constructed were obtained during the early forenoon. Hence the average slow decrease of temperature with increase of elevation is the result only of ordinary morning inversions. On the other hand, the rapid decrease of temperature through the second half kilometre is expressive of the adiabatic gradient of unsaturated air that commonly exists during summer afternoons up to a level of at least one kilometre. In this case it has persisted in its upper portion throughout the night, and been modified, as explained, by temperature inversions only in its lower half.

Why the Temperature of the Atmosphere Decreases with Increase of Elevation.—It is not, perhaps, obvious why the temperature of the atmosphere should rapidly decrease with increase of elevation, as it does through at least the first several kilometres, as shown by Fig. 16. Essentially, however, this phenomenon depends on the following facts:

1. The atmosphere, as is known from observation, transmits directly to the surface of the earth half, roughly, of the effective radiation received from the sun—that is, half of the portion

absorbed and not lost by reflection. Consequently, it is this surface, where the energy absorption is concentrated, and not the atmosphere, through which it is diffused, that is chiefly heated by insolation. The heated surface in turn warms the air above it, partly by contact and partly by the long wave-length radiation it emits, and of which the atmosphere is far more absorptive than it is of the comparatively short wave-length solar radiation.

2. Furthermore, and this is an equally vital part of the explanation, the lower atmosphere (below about 10 kilometres), under all ordinary conditions, emits more radiant energy than it absorbs. It is these two phenomena, (*a*) the net loss of heat by radiation (cooling above), and (*b*) the surface heating (warming below), that together establish and maintain the vertical convections of the atmosphere under which, since the descending portions grow warmer through compression and the ascending colder through expansion, the whole of the convective region is made to decrease in temperature with increase of elevation.

But since the coefficient of absorption of the air, as of other objects, changes but little, if at all, with temperature, while its emissive power decreases rapidly as it grows colder, and since the intensity of the incident terrestrial (including atmospheric) radiation remains roughly constant up to an altitude of many kilometres beyond the first 4 or 5, it follows that the upper limit of the convective region is not, as formerly supposed, the outermost limit of the atmosphere, but at that elevation at which the temperature is so low that the loss of heat by radiation is no longer in excess of, but now equal to, its gain by absorption. Beyond this level temperature does not decrease, or does so but slightly, with increase of elevation; nor would it so decrease, at least at nothing like the present rate, beyond any level, however low, at which absorption and radiation became equal.

In short, then, the air grows colder with elevation because (1) owing to its transparency to solar radiation it is heated mainly at the surface of the earth, and (2) because at ordinary temperatures it emits more radiation than it absorbs. These together so affect the density of the atmosphere as to induce vertical convections, and thereby to establish and maintain, throughout the region in which they are active, a rapid decrease of temperature with increase of elevation.

Chapter IV.

THE ISOTHERMAL REGION, OR STRATOSPHERE.

Of all the conditions indicated by the temperature gradients of Fig. 16, by far the most surprising, and most difficult fully to explain, is the approximately isothermal state of the upper atmosphere. Indeed, the discovery of the fact that the temperature of the upper atmosphere changes but little with altitude, and the supplementary discovery of its physical explanation, constitute one of the most important advances in modern meteorology.

The exploration of the atmosphere by small balloons carrying meteorological instruments was suggested in 1809,[8] but the idea was first carried out by Hermite[9] on March 21, 1893, when an elevation of 16 kilometres was attained. In April, 1898, Teisserenc de Bort,[10] with improved apparatus, began at Trappes, France, a long series of frequent atmospheric soundings. Among other things, he soon found temperature records that indicated something unsuspected: either errors in the thermometers themselves or surprising temperature conditions in the upper atmosphere. However, numerous temperature records subsequently obtained by himself and many others in various countries and with different kinds of apparatus have shown that, in general, the temperature of the upper atmosphere actually does change but little with change of elevation. Indeed, as a rule, the change is so small that the whole region characterized by this approximate constancy of temperature has been called the "isothermal region." At present it is more generally known as the "stratosphere," though the older and less used term certainly is more suggestive of its distinguishing characteristic.

The height at which this region begins and its temperature both depend upon season, upon storm conditions, and upon latitude; but, while all these are important details, they are secondary to the fact that there is an isothermal region at all.

As soon as observations left no doubt of the actual existence of the isothermal region many explanations of it were proposed.

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[8] *Ann. Harvard Obs.*, **68**, pt. 1, p. 1.

[9] *L'Aérophile*, 1, p. 45 (1893).

[10] *C. R.*, 129, p. 417 (1899).

but for a number of years all such suggestions proved unavailing. Finally, however, independently and nearly simultaneously, the generally accepted explanation occurred to Gold [11] of England and Humphreys [12] of America. The same subject has also been discussed at length by Emden.[13] The key to the explanation is this: The temperature of every portion of the atmosphere is determined, in part at least, by counteracting radiation—radiation absorbed and radiation emitted—and wherever these two are equal there is substantial constancy of temperature.

Mr. Gold's method of procedure was to take the best-known data concerning atmospheric absorption and radiation and to obtain, by the aid of suitable mathematics, a general solution of the problem. The chief difficulty in the application of this direct and elegant method, apart from the troublesome equations involved, is that due to our imperfect knowledge of the necessary radiation and absorption constants. Numerical values in these particulars are not accurately known and certainly not easy to determine.

On the other hand, the solution offered by Humphreys, while not so direct, reduces the necessary mathematics to a minimum. In brief, it is as follows: Since the average yearly temperature of the atmosphere at any given place does not greatly change, it follows that the absorption of solar radiation by the earth as a whole is substantially equal to the total outgoing earth radiation, and in amount approximately equal to that which a black or perfectly radiating surface, equal in area to the surface of the earth would emit if at the absolute temperature 259° C.[14] Further, since at ordinary atmospheric temperatures water vapor, in any considerable quantity, absorbs and, presumably, also radiates substantially as does a black body at the same temperature, while dry air is exceedingly diathermanous, it follows that the planetary radiation of the earth is essentially water vapor radiation.

Now the records of sounding balloons show that at some altitude, in general about 11 kilometres above sea level in middle latitudes, the average temperature ceases to decrease with increase of elevation. Individual flights show many peculiarities that call

[11] *Proc. Roy. Soc.*, series A, vol. 82, 1909, p. 43.

[12] *Astrophys. Jour.*, vol. 29, 1909, p. 14.

[13] *Sitz. der K. Bayr. Akad. der Wis.*, Jahr., 1913, p. 55.

[14] Abbot and Fowle, *Annals of the Astrophysical Observatory of the Smithsonian Institution*, vol. 2, p. 174.

for special explanation, but the purpose here is to consider only the general explanation of the main effect, and therefore average conditions are considered.

If, then, as is approximately true, the temperature does not decrease with increase of altitude above 11 kilometres, it follows that this must be the limit of anything like a marked vertical convection. And from this in turn it follows, since conduction is negligible, that the upper atmosphere must be warmed almost wholly by absorption of radiation, in part solar and in part terrestrial; but exactly how much of the final temperature of the upper atmosphere is due to the one source of heat and how much to the other it is not possible to say. However, there are certain facts that seem clearly to indicate the relative importance in this respect of the two sources. Thus the summer and winter gradients as given by Fig. 16 show a difference of temperature in the isothermal region of only about the amount that might be expected on the assumption that the temperature of the upper air is wholly dependent upon the radiation from the lower. That is to say, the seasonal temperatures of the lower atmosphere differ distinctly more than do those of the upper. It should be clearly kept in mind, too, that the particular seasonal gradients given in Fig. 16 were obtained at a latitude of, roughly, 50 degrees, where the number of hours of summer and winter sunshine differ greatly, and therefore where the seasonal temperature of the isothermal region, if essentially determined by absorption of solar radiation, should differ somewhat correspondingly. But, as no such great difference in these temperatures exists it would appear that the temperature of the isothermal region must be due chiefly to absorption of long wave-length radiation given off by the water vapor and other constituents of the atmosphere at lower levels, and to only a very minor degree to the absorption of solar radiation. Hence, as a first approximation, one may consider this radiation alone, and for the lower atmosphere as it actually exists substitute the radiationally equivalent black surface at the absolute temperature of 259° C. Obviously, too, this surface, surrounding as it would the entire earth, could be regarded as horizontal and of infinite length and breadth in comparison to any elevation attainable by sounding balloons, and therefore as giving radiation of equal intensity at all available altitudes.

Now consider two such surfaces, parallel and directly facing each other, at a distance apart small in comparison to their width, and having the absolute temperature T_2, and let an object of any kind whatever be placed at the centre of the practically enclosed space. Obviously, according to the laws of radiation, the final temperature of the object in question will also be approximately T_2. If, now, one of the parallel planes should be removed, the uncovered object would be in substantially the same situation, so far as exposure to radiation is concerned, as is the atmosphere of the isothermal region in its exposure to the radiation from the lower atmosphere. Of course, each particle of the upper air receives some radiation from the adjacent atmosphere, but this is small in comparison to that from the lower water vapor and may, therefore, provisionally be neglected. Hence the problem, as an approximation, is to find the final temperature to which an object, assumed infinitesimally small, to fit the case of a gas, will come when exposed to the radiation of a single black plane of infinite extent.

Now, whether between the parallel planes or facing but one, the object in question is in temperature equilibrium when, and only when, it loses as much energy by radiation as it gains by absorption. Furthermore, so long as its chemical nature remains the same, its coefficient of absorption is but little affected by even considerable changes in temperature. Therefore, whatever the nature of the object, since it is exposed to twice as much radiation when between the two planes as it is when facing but one, it must, in the former case, both absorb and emit twice as much energy as in the latter. Or, using symbols,

$$E_2 = 2E_1$$

in which E_2 and E_1 are the quantities of heat radiated by the object per second, say, when between the two planes and when facing but one, respectively.

Again,

$$E_2 = K_2 T_2^{n_2}$$

and

$$E_1 = K_1 T_1^{n_1}$$

in which T_2 and T_1 are the respective absolute temperatures of the object under the given conditions, and K and n its radiation constants.

For every substance there are definite values of K and n which, so long as the chemical nature of the object remains the same, do not rapidly vary with change of temperature. Hence, assuming $K_2 = K_1$ and $n_2 = n_1$, we have, from the equation

$$E_2 = 2E_1$$
$$T_2 = \sqrt[n]{2}\ T_1$$

From this it appears that there must be some minimum temperature T_1 below which the radiation of the earth and lower atmosphere will not permit the upper atmosphere to fall, though what it is for a given value of T_2 depends upon the value of n.

Presumably the radiation of the upper atmosphere is purely a thermal radiation, and therefore in full agreement, as is the thermal radiation of water vapor, carbon dioxide, and certain other gases, with the Kirchhoff [15] law. In other words, the ratio of emission to absorption for any given wave-length, presumably, is wholly a question of temperature, and is numerically equal to the radiation of a black body at the same temperature and wave-length. In symbols,

$$\left(\frac{He}{h}\right)_{\lambda, t} = (E)_{\lambda, t}$$

in which H is the incident energy, h the energy absorbed, and e the energy emitted by the body or gas in question at the wave-length λ and temperature t, and E the black body emission at the same wave-length and temperature, all per equal area and time.

To fix the ideas, let the body of gas under consideration be a shell one centimetre thick, surrounding the earth at a fixed distance—20 kilometres, say, above sea level—and let the black body be a very thin shell at the same temperature inside and outside, that may, if we wish, take the place of the gas shell. Now, since nearly all the incident radiation under consideration, the radiation of the earth and its atmosphere onto a shell at 20 kilometres elevation, or anywhere else in the isothermal region, comes from below, we may assume it, or its normal equivalent, to be substantially the same for all levels of the upper atmosphere, and assume the emitted radiation to be all the energy

[15] Pringsheim, "Congrès International de Physique," Paris, 1900, vol. 2, p. 127.

sent out by the shell on *either* or on *both* sides; only, whatever the assumption for one shell, the same must be made for the other.

Returning to a consideration of the temperature of the upper atmosphere under the influence of radiation from the lower gases: Since the composition of the upper atmosphere is not appreciably changed by a change of even 50° C., it follows that such a change of temperature will not materially alter its coefficient of absorption. Hence a change in the intensity of the incident radiation H will make substantially the same proportionate change in the rate of absorption h, whatever the alteration in temperature. In short,

$$\frac{H}{h} = K \text{ a constant, presumably.}$$

Hence

$$\frac{e_{\lambda, t_1}}{e_{\lambda, t_2}} = \frac{E_{\lambda, t_1}}{E_{\lambda, t_2}}$$

or

$$\frac{e_{\lambda, t_1}}{E_{\lambda, t_1}} = \frac{e_{\lambda, t_2}}{E_{\lambda, t_2}}$$

Unfortunately, nothing is known of the spectral distribution of the energy radiation of the cold upper atmosphere, though possibly it is of the irregular, but more or less continuous broad band, type. If this is its distribution, and if for each wave-length the increase of black body radiation, for a small increase of temperature, is proportional to the total radiation at that wave length, which it is to a rough first approximation, then to about the same average approximation,

$$\frac{e_{t_1}}{e_{t_2}} = \frac{E_{t_1}}{E_{t_2}},$$

in which the symbols stand for the total radiation of all wave-lengths.

But from the Stefan law in regard to the total radiation of black bodies, we know that

$$\frac{E_{t_1}}{E_{t_2}} = \frac{T_1^4}{T_2^4}$$

in which T_1 and T_2 are the respective absolute temperatures.

Hence, as explained above, if the spectral distribution of the radiation of the upper atmosphere is continuous, or nearly so

(no matter how irregular), and not confined chiefly to lines with zero radiation between them, it follows that in the equation,

$$T_2 = \sqrt[n]{2}\, T_1$$

the numerical value of n must be 4, roughly. But, as already explained, the value of T_2 is substantially 259° C. absolute; hence, on the assumption that $n = 4$, it follows that $T_1 \doteq 218°$ C. absolute. And this is the value, approximately, that observation gives.

Whatever the facts in regard to the radiation constants of the atmosphere, the laws of radiation and absorption demand that the temperature of the upper atmosphere shall change but little with change of elevation. Besides, while the exact value of this temperature—the temperature of the isothermal region—is, of course, best determined by actual observation, it also may be computed approximately from the known intensity of outgoing radiation, together with the thermal properties of the gases of the atmosphere.

Doubtless solar radiation affects the temperature of the isothermal region to some extent, but, presumably, not very much, since the radiation from the lower levels seems competent not only to produce an isothermal condition in the upper levels, but also to maintain them at substantially the observed temperature. Further, the lower atmosphere obviously is slightly warmed and its radiation correspondingly increased by return radiation from the upper, but this presumably does not affect the general validity of the above reasoning, which is based on the action of the *total* outgoing radiation.

Given the isothermal condition of the upper atmosphere, it follows that the heated surface air can, under favorable circumstances, rise till, but only till, by expansion it has cooled down to that temperature (the temperature of the isothermal region) below which the radiation from the lower atmosphere will not allow it to fall.

The existence of an upper isothermal region and the vertical temperature gradient (Fig. 16) suggests rational explanations of a number of otherwise obscure meteorological phenomena—why the clouds of a given region have a fairly well-defined maximum elevation; why this elevation is greater in summer than in winter; why it is a level of maximum cloud formation,

and the like—but all these are special phenomena that will be discussed independently later on.

INEQUALITY OF SEASONAL TEMPERATURE CHANGE OF LOWER AND UPPER ATMOSPHERE.

As just explained, if T_2 is the absolute temperature of the black surface that gives off radiation equivalent to that sent out by the convective portion of the atmosphere and T_1 the absolute temperature of the isothermal region, then

$$T_1 = T_2/\sqrt[4]{2} = 0.84\ T_2, \text{ roughly.}$$

Hence the greater T_2, or the warmer the lower atmosphere, if its composition remains the same, the greater the difference between T_1 and T_2, or the greater the contrast between the temperature of the lower atmosphere and that of the isothermal region. This is in keeping with the observed fact (Fig. 16), that the seasonal difference in the temperature of the isothermal region, while in the same sense as that of the lower atmosphere, is not so great as is the latter. Because of seasonal differences in the composition of the lower atmosphere, especially in the average amount and distribution of water vapor, there can be no constant relation between the above temperature differences—only the qualitative relation as given.

HEIGHT OF THE ISOTHERMAL REGION.

If H_1 is the height and T_1 the temperature of the under surface of the isothermal region above the level H_0, whatever that may be, whose temperature is T_0, then

$$H_1 = H_0 + \int_{T_0}^{T_1} \frac{dH}{dT} dT$$

As above explained, the greater the temperature of the lower atmosphere, the greater the difference between this temperature and that of the isothermal region, or, in symbols, the greater T_0 the greater $T_0 - T_1$, and therefore the greater H_1. Hence the isothermal region should be at a greater elevation during summer than during winter. Another way of showing this same thing is as follows:

Let the difference between the summer and winter temperatures of the lower atmosphere be ΔT_2 throughout, and the cor-

responding difference between the temperatures of the upper atmosphere ΔT_1, then, according to the above theory, $\Delta T_1 = 0.84\Delta T_2$, roughly. This, as already explained, is a radiation result. The inequality or $0.16\Delta T_2$ is produced by convection. Now if h is the change in elevation corresponding to 1° C., we have

$$\Delta H = 0.16\Delta T_2 h.$$

But, ΔT_2, winter to summer, is roughly 12° C., and h about 110 metres. Hence the change in the seasonal elevation of the isothermal region, if there is constancy in atmospheric composition, and other conditions, except temperature, is, roughly,

$$\Delta H = 0.16 \times 12 \times 110 = 211.2 \text{ metres.}$$

It must be distinctly noted, however, that many disturbing elements, such as quantity and distribution of water vapor, frequency and extent of cirrus clouds, and the like, so modify these simple relations that they apply only to average conditions, and to them but approximately.

STORM EFFECTS ON TEMPERATURE GRADIENTS.

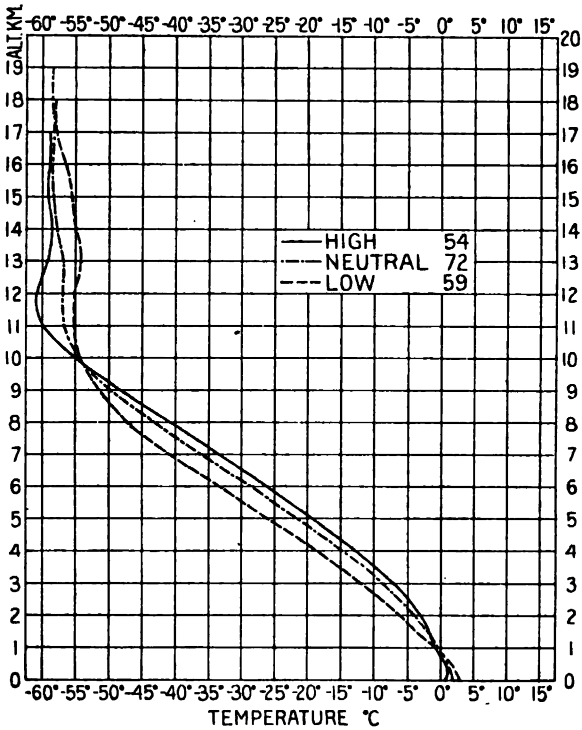
The average of a season's (winter or summer) vertical temperature gradients gives a fairly regular curve, and, of course, the same would be true of the average of these averages, or what might be called the annual gradient for any given locality. However, each particular flight yields its own temperature-altitude curve, which differs more or less from others of the same place and season, especially in the values of the gradients in the first two or three kilometres elevation, in the absolute temperatures at other levels, and in the location of the upper inversion.

With the view of determining the causes of some of these flight-to-flight irregularities, both the summer and the winter records from which the corresponding seasonal gradients were determined were grouped, according to the heights of the barometer at the times and places of observation, into "highs," "neutrals," and "lows." Thus the "highs" belong to barometric readings of 5 mm. or more above, and the "lows" to readings of 5 mm. or more below, the seasonal normal, and the "neutrals" to the various intermediate values, all reduced to sea level.

Fig. 17 shows the winter averages, respectively, of 54 highs, 72 neutrals, and 59 lows. Commonly, as the figure shows, a high barometer in the winter is accompanied by low surface tempera-

tures, a slow decrease of temperature up to an elevation of about three kilometres, relatively warm air, in general, between the levels of two and nine kilometres, a high upper inversion, a cold isothermal region, and a marked minimum temperature in the lower portion of the stratosphere. A winter low, on the contrary, and in comparison with a high of the same season, is accom-

FIG. 17.



Temperature gradients at different pressures, winter.

panied by warm surface temperatures, a more rapid decrease of temperature with increase of elevation through the first three kilometres, relatively cold air from, roughly, two to nine kilometres elevation, a low upper inversion, and a warm isothermal region.

The normal barometer, as one would expect, is accompanied by intermediate values in all particulars.

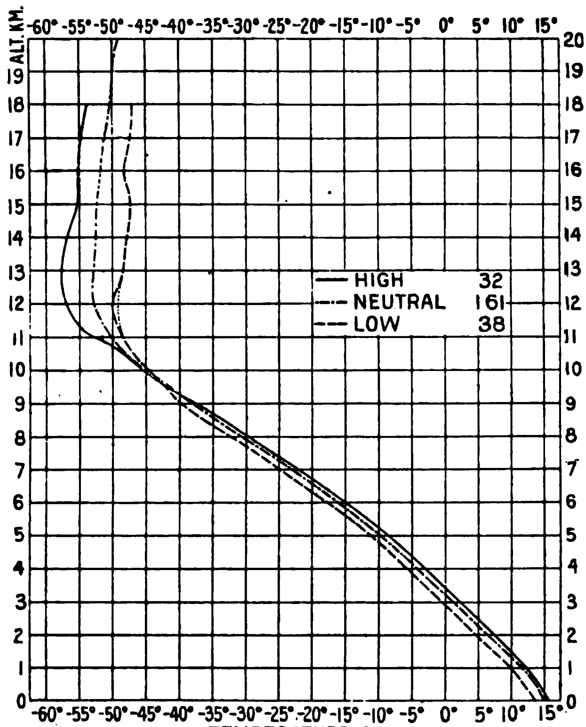
The corresponding summer gradients (averages, respectively,

of 32 highs, 161 neutrals, and 38 lows), given in Fig. 18, show, except near the surface, where the lows remain cold and the highs warm, the same characteristics as do those of winter.

Both the summer and the winter curves follow exactly the averages of the observations, as per the accompanying table.

An obvious contributing cause of these differences in tem-

FIG. 18.



Temperature gradients at different pressures, summer.

perature is the warming of the air by compression and its cooling by expansion incident to barometric changes; an amount which, starting with dry air at 0° C., is given, as explained on page 30, by the equation

$$\frac{dT}{dp} = \frac{77.6}{p},$$

in which p is the pressure, expressed in millimetres of mercury, and dT the change of temperature in degrees Centigrade.

5

TABLE I

Average temperatures, Centigrade (from average temperature gradients) from 416 sounding balloon flights at Trappes, Uccle, Strassburg, and Munich, 1900–1912. Temperature T; number of cases N.

Altitude in km. above sea level	Winter								Summer							
	High		Neutral		Low		Mean		High		Neutral		Low		Mean	
	T	*N*	*T*	*N*	*T*	*N*	*T*	*N*	*T*	*N*	*T*	*N*	*T*	*N*	*T*	*N*
0.0	0.67	43	1.61	55	2.85	49	1.72	147	15.54	29	14.82	121	13.58	28	14.76	178
0.5	0.87	43	1.01	55	1.77	49	1.23	147	13.98	29	13.90	121	11.93	28	13.60	178
1.0	− 0.63	54	− 0.49	72	− 0.80	59	− 0.63	185	12.39	32	12.12	161	9.98	38	11.81	231
1.5	− 1.56	54	− 1.77	72	− 3.50	59	− 2.26	185	9.57	32	9.34	161	7.23	38	9.03	231
2.0	− 2.75	54	− 3.76	72	− 5.92	59	− 4.16	185	7.09	32	6.54	161	4.55	38	6.29	231
2.5	− 4.60	54	− 6.10	72	− 8.66	59	− 6.48	185	4.37	32	3.84	161	2.12	38	3.63	231
3.0	− 6.92	54	− 8.65	72	−11.75	59	− 9.14	185	2.00	32	1.19	161	− 0.49	38	1.02	231
4.0	−12.69	54	−14.69	72	−18.44	59	−15.30	185	− 3.16	32	− 4.16	161	− 5.74	38	− 4.28	231
5.0	−19.22	54	−21.49	72	−25.91	59	−22.24	185	− 8.74	32	− 9.83	161	−11.33	38	− 9.93	231
6.0	−26.18	54	−28.40	72	−33.37	59	−29.33	185	−15.11	32	−16.15	161	−17.86	38	−16.29	231
7.0	−33.27	54	−35.80	71	−40.83	59	−36.63	184	−22.18	32	−23.12	161	−24.80	38	−23.26	231
8.0	−40.83	53	−43.10	70	−47.13	59	−43.63	182	−29.82	32	−30.59	161	−32.34	38	−30.77	231
9.0	−48.05	50	−49.70	67	−51.33	57	−49.63	174	−37.62	31	−38.08	159	−39.77	37	−38.27	227
10.0	−54.85	48	−54.30	64	−54.23	53	−54.33	165	−45.32	31	−44.88	155	−44.57	36	−44.87	222
11.0	−59.65	44	−56.50	56	−55.03	47	−56.83	147	−52.62	31	−50.08	149	−48.37	34	−50.07	214
12.0	−60.75	38	−56.80	45	−54.93	37	−57.23	120	−57.02	30	−52.68	133	−49.97	30	−52.87	193
13.0	−59.05	34	−56.70	34	−54.23	30	−56.33	98	−57.62	27	−52.78	117	−48.27	27	−52.77	171
14.0	−58.35	26	−57.70	22	−54.73	24	−56.53	72	−56.82	22	−52.48	104	−47.87	21	−52.37	147
15.0	−58.85	17	−58.20	16	−55.43	16	−57.13	49	−55.32	18	−52.38	81	−47.37	17	−51.97	116
16.0	−58.55	11	−58.30	12	−56.43	10	−57.33	33	−55.32	13	−51.68	61	−48.17	8	−51.57	82
17.0	−58.65	5	−58.00	9	−57.63	7	−57.63	21	−54.82	9	−51.18	36	−47.37	4	−51.07	49
18.0	....	..	−58.30	6	−57.83	5	−57.63	13	−53.82	8	−50.38	22	−46.97	3	−50.17	33
19.0	....	..	−58.20	5	....	..	−57.63	8	....	..	−49.88	17	....	..	−49.57	21
20.0	....	..	....	..	....	..	−57.63	5	....	..	−48.98	15	....	..	−48.97	18

According to Figs. 17 and 18, the temperature at the altitude of four kilometres is considerably warmer both winter and summer in the regions of high barometric pressure than it is in the regions of low pressure. But to secure a temperature difference of 7° C., say, as a result of pressure change only, would require a rise or fall of the barometer at this level of about 40 mm., or something like 70 mm. at sea level; and, since this is of several-fold the average pressure change, it is obvious that the observed temperature differences, often more than 7° C., cannot in the main be accounted for in this way, though, of course, the pressure effect must be present to some extent.

Another contributing cause of temperature differences, generally associated with the height of the barometer, is the clear and cloudy condition of the sky, or the humid and the dry state of the atmosphere. A barometric high, as we know, commonly is accompanied by clear skies and a dry atmosphere, while in the region of a low the sky ordinarily is overcast, the atmosphere relatively moist, and precipitation abundant—conditions that have much to do with air temperatures. Thus, generally, at the end of any consecutive 24 hours of clear weather the surface of the earth will be warmer in the summer time and colder in the winter because of the unequal lengths during those seasons of the day and night. On the whole, the earth gains heat, especially in clear weather, during summer and loses it in the winter. A cloud covering, however, greatly reduces the rate of this gain or loss, and therefore during winter the surface temperature is lowest when the barometer is high and the earth can radiate most freely to space, while it is warmest when the barometer is low and the sky so overcast as to check radiation loss.

In the summer time, as explained, the conditions of gain and loss of heat are the reverse of those during winter, consequently the highest summer surface temperatures accompany the high barometer, or clear weather, while the lowest accompany cloudy skies.

When the barometer is distinctly above normal the temperature fall with increase of altitude near the stratosphere generally follows approximately the adiabatic curve for dry air. When, however, the barometer is low the temperature gradient usually is far less constant at all elevations, owing to irregularities in the humidity distribution and the consequent varying amounts

and places of precipitation. In many cases the temperature gradient over varying heights is essentially the adiabatic curve for saturated air at the prevailing temperature and pressure; that is, a fall of temperature per given change in altitude is less, other things being equal, the greater the amount of uncondensed moisture present.

Since the curves of Figs. 17 and 18 are the averages of a number of flights, it may be approximately correct, in an effort to account for their differences, to start with something like average conditions and trace the consequences. Let these conditions be a moist atmosphere in the one case and a dry one in the other, each having the same temperature as the other at all levels. The moist atmosphere, because a better radiator than the dry, will, under the same conditions of exposure and at the end of the same interval of time, cool to a lower temperature, but in so doing—that is, in getting rid of its own heat most rapidly—it at the same time supplies heat at greater rate to any neighboring region that receives it by radiation only. Therefore when the lower atmosphere is moist it will, under like conditions, radiate heat most rapidly to and through the always dry air of the isothermal region, and while getting cold itself will at the same time warm this region to a temperature above its average. On the contrary, in the region of a high barometer the lower air, being relatively dry and therefore a poor radiator, will conserve its own temperature, but in so doing will allow the isothermal region to get cold in comparison with its temperature during the prevalence of a low at the same season.

Since the temperature of the isothermal region depends essentially upon the amount of radiation received from the lower atmosphere, it follows that, on the average, the temperatures of the two regions must vary in the same sense, or warm and cool together, and this, indeed, is just what happens, as the winter and summer gradients of Fig. 16 indicate. It is surprising, therefore, when we find the temperatures of these regions varying in the opposite sense, the one getting warm while the other is getting cold. But it must be remembered that the lower atmosphere is warmed convectionally, in large part, while the upper air is warmed almost wholly by radiation. Hence whatever increases radiation from the lower air, increase of humidity or temperature, or both, must tend to increase the temperature of the iso-

thermal region, and whatever decreases this radiation, decrease of humidity or temperature, or both, must decrease its temperature below that which it otherwise would have. Now the average seasonal change of the lower atmosphere is primarily one of temperature, while the average storm difference of a given season appears to be largely one of humidity. The high temperature of summer obviously affords a more abundant radiation than the relatively low temperature of winter and should give, as we have seen, a warmer isothermal region. Similarly, the different intensities of radiation from humid and relatively dry air would lead one to expect the stratosphere to be warmer over a cyclonic than over an anticyclonic area.

Another important factor, presumably the controlling one, in the temperature contrasts between cyclonic and anticyclonic regions is the vertical movement of the atmosphere, upward in the former, downward in the latter. Probably, at least, a large portion of the anticyclonic excess of air that gives the increase of pressure and makes good the loss by outflow is fed in at considerable altitudes, though below the isothermal level. If the air movement is as here supposed, there necessarily must be dynamical heating throughout the lower atmosphere, partly because of the initially increased pressure, but mainly through descent. At the same time, the atmosphere of the isothermal region would be more or less lifted and cooled by the resulting expansion.

Conversely, if, as there is much reason to believe, the chief removal of atmosphere over cyclonic areas is also from considerable altitudes, but below the isothermal region, the lower atmosphere must be dynamically cooled, partly by virtue of the immediately decreased pressure, but chiefly through ascent. The stratosphere would be lowered and its temperature thereby increased. It seems likely, too, that under these conditions there would be a somewhat continuous slow movement of air from the stratosphere down into the region of the troposphere (convection region), with, of course, counterflows elsewhere and subsequent readjustments of the level of the isothermal region as temperature and other conditions changed.

Radiation intensity, barometric pressure, and vertical circulation, therefore, appear all to coöperate in lowering the temperature of the troposphere and in raising that of the stratosphere in cyclonic regions. Conversely, they appear equally to coöperate

in raising the temperature of the troposphere and lowering that of the stratosphere in anticyclonic regions.

The above temperature conditions are averages, respectively, for the whole of cyclonic and anticyclonic areas. Subdivisions of these areas show temperature contrasts between their several quadrants, owing, in part at least, to differences in horizontal wind direction and the distribution of condensation and evaporation. This interesting detail, however, scarcely belongs to a discussion of the isothermal region, but rather to an account of the two types of weather concerned, under which heads it will receive further consideration.

RELATION OF THE ISOTHERMAL REGION TO LATITUDE.

It is well known that both the height and temperature of the stratosphere are functions of latitude. In the northern hemisphere, during summer, the under surface of the stratosphere gradually rises from about 10 kilometres above sea level at latitude 60° to approximately 15 kilometres at the equator, while the temperature correspondingly changes, roughly, from –45° C. to –70° C. Similar altitude and temperature changes of the stratosphere with latitude obtain, so far as observed, during all seasons and in both hemispheres, though the exact cause, or causes, of these variations is not known.

Changes in the amount of radiation from below and changes in the diathermacy of the upper air at once occur as possible explanations of the above latitude effects. The warm surface temperatures of equatorial regions necessarily cause, through vertical convection, abundant cloudiness at high altitudes. These clouds, in turn, intercept much of the radiation from below, either reflecting or absorbing it. The portion reflected obviously does not directly warm the cloud, but somewhat raises the temperature of the troposphere, while a portion of that which is absorbed merely produces evaporation rather than a change of temperature. Hence, presumably, the high clouds of equatorial regions, and also the considerable humidity there at great altitudes, owing to the persistent vertical convection, raise the level and thereby decrease the temperature of the effective radiating surface, thus diminishing the intensity of the radiation that reaches the stratosphere, and permitting its temperature to be correspondingly low.

Some of the heat thus absorbed in the lower atmosphere con-

ceivably may manifest itself in an acceleration of interzonal circulation in equatorial regions and an increase of outgoing radiation at higher latitudes.

There is, of course, abundant cloudiness over the higher latitudes as well as in equatorial regions, but there the atmosphere is more generally descending instead of ascending, the clouds low, the humid layer shallow, and consequently the effective radiating surface and the stratosphere comparatively warm.

Conceivably, too, the composition of the stratosphere may differ with latitude. Because of auroral concentration and because of any poleward drift there may be of the upper atmosphere, perhaps there is more ozone, intensely absorptive of earth radiation, over high than over low latitudes. Doubtless, also, there are local differences in the water vapor content of the stratosphere, but at most the absolute humidity of this region must be exceedingly small, as is obvious from its excessively low temperature, and from the fact that marked temperature changes of the upper atmosphere, amounting at times to 20° C., or more, never, so far as known, produce clouds above the high cirrus; that is, above the upper levels of the troposphere.

As previously stated, the essential cause of the relation of the temperature of the stratosphere to latitude is not known, but it seems probable that this relation may depend, in part at least, upon the distribution of clouds, water vapor, and ozone, and therefore that each deserves further observation and study in this connection.

CHAPTER V.

COMPOSITION OF THE ATMOSPHERE.

IN the previous discussions the actual composition of the atmosphere was of little or no importance. In some that follow, however, barometric hypsometry, for instance, or the determination of altitude from pressure, it is a factor that cannot always be neglected. It will be convenient, therefore, before considering such subjects, to note of what substances the atmosphere consists and in what proportions they occur.

If we disregard such obviously foreign things as dust, fog, and cloud, then whatever remains appears to be ideally homogeneous, under ordinary conditions, and in many discussions, such as most of those of the previous pages, it conveniently may be so treated. The Greek philosophers, indeed, regarded the atmosphere as one of the four elements that singly and combined constituted the whole of the material universe. To them it was an element in the strictest sense—a thing that cannot be divided into dissimilar parts.

In reality it is not even a single substance like water, much less a single element, but a mixture of a number of gases and vapors that radically differ from each other in every particular; nor are even the relative percentages of the several distinct constituents at all constant. The story of the chemical conquest of the atmosphere, from the calcination and combustion experiments of the seventeenth and eighteenth centuries that established its complexity down to the refined analyses of the present day that note and account for even the faintest traces, is full of instruction and inspiration. However, it is practicable to give here only some of the final results.

According to Hann,[16] the chief independent gases that are blended into a dry atmosphere at the surface of the earth, and their respective volume percentages, are as follows:

Element	Nitrogen	Oxygen	Argon	Carbon dioxide	Hydrogen	Neon	Helium
Volume, per cent	78.03	20.99	0.94	0.03	0.01	0.0012	0.0004

In addition to these, krypton and xenon also occur as permanent constituents of the atmosphere. There are also many sub-

[16] "Lehrbuch der Meteorologie," 3d ed., p. 5.

stances, such as radio-active emanations, the oxides of nitrogen, ozone, and, above all, water vapor, that are found in varying amounts, but of these only water vapor commonly forms an appreciable percentage of the total atmosphere, a percentage that depends chiefly upon temperature in the sense that, for any given pressure, the higher the temperature the greater the possible percentage of water vapor. This relation holds up to the boiling-point of water at the given pressure, when, assuming saturation, there will be nothing but water vapor present, as in the spout, for instance, of a vigorously boiling kettle.

Because of this relation of water vapor to temperature its volume percentage decreases in the lower atmosphere from the equator towards the poles, while that of each of the other constituents of the atmosphere correspondingly increases. The annual average values, again quoting from Hann,[17] are:

	Nitrogen	Oxygen	Argon	Water vapor	Carbon dioxide
Equator	75.99	20.44	0.92	2.63	0.02
50° N.	77.32	20.80	0.94	0.92	0.02
70° N.	77.87	20.94	0.94	0.22	0.03

Except for the change in the amount of water vapor, the composition of the surface atmosphere is substantially the same at all parts of the earth. Its composition at different elevations, however, probably differs greatly, as considerations presently to follow will indicate.

But this discussion requires the use of barometric hypsometry. Therefore an equation must be developed for this purpose.

BAROMETRIC HYPSOMETRY.

Let ρ be the density of the atmosphere at the height h, and p its pressure in dynes per square centimetre. Then at the level h the decrease in pressure, $-dp$, due to the increase in height, dh, is given by the equation,

$$-dp = \rho g\, dh \quad \text{(1)}$$

in which g is the acceleration of gravity at the point in question.

To within practically negligible limits the density of the atmosphere is directly proportional to the pressure—*Boyle's law;*

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[17] *Ibid.*, p. 5.

inversely proportional to the absolute temperature—*Charles's law;* and directly proportional to the sum obtained by adding together the molecular weights of the several gases present, each multiplied by the ratio of its partial pressure (the pressure which it alone produces) to the total pressure—*Avogadro's law.*

If, now, ρ_0 is the density of dry air at the temperature 0° C., and under the pressure p, then

$$\rho = \rho_0 \frac{\left(1 - 0.378\frac{w}{p}\right)}{1 + at} \quad \ldots\ldots (2)$$

in which w is the partial pressure of the water vapor present, t the existing temperature in degrees Centigrade, and a the coefficient of gas expansion, or $\frac{1}{273}$, at 0° C.

Substituting the value of ρ, as given by equation (2), in equation (1), we have

$$-dp = \rho_0 g \frac{\left(1 - 0.378\frac{w}{p}\right)}{1 + at} dh \quad \ldots\ldots (3)$$

Further, let H be the height of the "homogeneous" atmosphere, or height it would have if everywhere at the same temperature and pressure, and therefore of constant density. Obviously, according to Boyle's law, the value of H is independent of the actual value of the pressure. If, for instance, the amount of gas is doubled, p becomes doubled and the density doubled, and consequently H remains unchanged. Similarly for any other multiple or submultiple of the pressure.

At any given place the pressure, p, clearly is equal to the continuous product of the gas density, local acceleration of gravity, and homogeneous height; that is,

$$p = \rho_0 g H \quad \ldots\ldots (4)$$

Hence, substituting in (3)

$$-dp = \frac{p}{H} \frac{\left(1 - 0.378\frac{w}{p}\right)}{1 + at} dh \quad \ldots\ldots (5)$$

But, as is seen from equation (4), H is inversely proportional to g. Hence before we can assign a numerical value to H it is necessary to specify the value of g to which it applies. Now the height of the "homogeneous" atmosphere corresponding to tem-

perature t and gravity g is found by multiplying any barometric height b, representing, under gravity g, a pressure p, by the ratio of the density of mercury at the temperature for which b was determined to the density of dry air at the given temperature t, and assumed pressure p. For 0° C. and normal gravity G (that is, the value of gravity acceleration at sea level at latitude 45° N.),

$$H_G = 7991 \text{ metres.}$$

Therefore H, or, specifically,

$$H_g = 7991\frac{G}{g} \text{ metres.}$$

Hence, substituting in (5), and measuring in metres,

$$-7991\frac{dp}{p} = \frac{\left(1-0.378\frac{w}{p}\right)}{1+at}\,\frac{g}{G}dh \quad\text{.............................. (6)}$$

Integrating this from h_0, p_0 to h, p, we have

$$7991 \log_e\left(\frac{p_0}{p}\right) = \int_{h=o}^{h=h} \frac{\left(1-0.378\frac{w}{p}\right)}{1+at}\,\frac{g}{G}dh. \quad\text{.................... (7)}$$

However, the integration is not rigidly possible, since t and the ratio $\frac{w}{p}$ both are more or less irregular and variable functions of h. The value of g is also a function, but, for any given place, a fixed one, of h. Nevertheless, when the coincident values of p, t, and w are closely known, as they may be, it is possible to determine with equal accuracy the corresponding values of h; that is, to obtain an approximate solution of the above nonintegrable equation. This, of course, is most accurately done by dividing the total height into intervals over each of which t and the ratio $\frac{w}{p}$ both change very nearly uniformly.

If at the elevation h_0 we have the ratio $\frac{w_0}{p_0}$ and at h the ratio $\frac{w}{p}$, then, when $h - h_0$ is not too great, we may, with but little error, assume the ratio constant and its value for the interval in question to be

$$\left(\frac{w_0}{p_0}+\frac{w}{p}\right)/2 = W \quad\text{.................................... (8)}$$

Also, if the temperature varies approximately uniformly between the given levels, we can assume, again with but little error,

the mean, t_m, of the two limiting temperatures to be that of the whole layer between h and h_0. Finally, as the value of g changes but little through all attainable levels in the atmosphere, its mean value, g_m, between the two levels, may be used as a very close approximation.

Hence, with all these approximations,

$$h = 7991 \log_e \left(\frac{p_0}{p}\right) \frac{1 + at_m}{1 - 0.378W} \frac{G}{g_m} \quad \text{(9)}$$

It will be noticed that since $1 + at = \frac{T}{273}$, in which T is the absolute temperature, $\frac{T_M}{273}$ should, theoretically, be used in place of $1 + at_m$, where $\frac{1}{T_M}$ is the harmonic mean $\left(\frac{n}{T_M} = \frac{1}{T_1} + \frac{1}{T_2} + \cdots + \frac{1}{T_n}\right)$ of the absolute temperatures (average) of the equally spaced short intervals between the given levels. Probably, however, this refinement is seldom justified by the data, except for elevations greater than 4 or 5 kilometres.

If, instead of natural logarithms with base e, ordinary logarithms with base 10 are used, equation (9) becomes

$$h = 18400 \log_{10} \left(\frac{p_0}{p}\right) \frac{1 + at_m}{1 - 0.378W} \frac{G}{g_m} \quad \text{(10)}$$

Now g_m differs in general from G both because of difference in latitude and because of difference in elevation. Thus the shape of the earth causes the value of gravity so to vary at sea level that at latitude l

$$g_l = G(1 - 0.00264 \cos 2l + 0.000007 \cos^2 2l),$$

while with elevation it varies inversely, nearly, as the square of the distance from the centre of the earth.

Let R be the radius of the earth at the place of observation, and d the elevation at which the value of g is desired. Then

$\frac{g_0}{g_h} = \frac{(R+d)^2}{R^2}$, nearly (increase of g due to mass of air left below is negligible),

and, to the same approximation,

$$g_h = g_0\left(1 - 2\frac{d}{R} + 3\left(\frac{d}{R}\right)^2 - 4\left(\frac{d}{R}\right)^3 + \ldots\right) \quad \text{(11)}$$

But even if d is as great as 10 kilometres, the fraction $\frac{d}{R}$ is still so small, roughly $\frac{1}{637}$, that an error of less than 1 in 400,000 will be made by writing

$$g_h = g_0\left(1 - 2\frac{d}{R}\right)$$

Hence, finally,

$$g_{l,d} = G\,(1 - 0.00264 \cos 2l + 0.000007 \cos^2 2l)\left(1 - 2\frac{d}{R}\right), \text{ nearly,}$$

and

$$h = 18400 \log_{10}\left(\frac{p_0}{p}\right)\frac{1 + at_m}{1 - 0.378\,W}\,\frac{1}{(1 - 0.00264 \cos 2l + 0.000007 \cos^2 2l)}\left(\frac{1}{1 - 2\frac{d}{R}}\right)$$

nearly . . . (12)

But as the amount of water vapor in the atmosphere seldom amounts to more than 2.5 per cent. of the total gases present, it follows that

$\frac{1}{1 - 0.378W} = 1 + 0.378\,W$, to within 1 part in 10,000. Similarly,

$\frac{1}{1 - 0.00264 \cos 2l + 0.000007 \cos^2 2l} = 1 + 0.00264 \cos 2l - 0.000007 \cos^2 2l$, usually to within 1 part in 1,000,000 and

$\left(\frac{1}{1 - 2\frac{d}{R}}\right) = 1 + 2\frac{d}{R}$, when $d = 10$ kilometres, to within 1 part in 100,000.

Hence, for convenience, if $d = \frac{h_0 + h}{2}$, we may write as a close approximation,

$$h = 18400 \log_{10}\left(\frac{p_0}{p}\right)(1 + at_m)(1 + 0.378W)(1 + 0.00264 \cos 2l - 0.000007 \cos^2 2l)\left(1 + \frac{h_0 + h}{R}\right) \ldots (13)$$

If standard gravity g_s, that is, 980.665 c.m./sec.^{2}, is used instead of normal gravity G, the term $1 + 0.00264 \cos 2l - 0.000007 \cos^2 2l$ in equation (13) must be replaced by the term $1 + \frac{g_s - g_l}{g_s}$.

Since the two pressures, p_0 and p, occur in this equation as a ratio, it is correct and customary to substitute for them the corresponding barometric readings—properly corrected, of course, for t and g. But as the value of g, in turn, depends upon h, the evaluation of the latter would appear to require a series of approximations. Rigidly this is true, but, as the value of g varies so little through attainable altitudes, a very rough approximation to the value of h is sufficient for the altitude correction of g.

Obviously, in general, the recorded values of t, W, and the barometric reading b are all in error, and therefore it will be well

to see what effects such errors will have on the computed value of h.

Assuming an error to be made in b only, amounting to db, we have from (13), substituting $\frac{b_0}{b}$ for $\frac{p_0}{p}$ and using natural logarithms, $dh = -7991 \frac{db}{b} (1 + at_m)(1 + 0.378\ W)(1 + 0.0026 \cos 2l)\left(1 + \frac{h_0+h}{R}\right)$ or, very approximately,

$$dh = -7991 \frac{db}{b}(1 + at_m)$$

Hence the greater the altitude, or the smaller the value of b, the more important become the errors in pressure. Under the reasonable conditions that $t_m = 0°$ C., $db = 1$ mm., and $b = 500$ mm., corresponding to an altitude of, roughly, 3350 metres, the error $dh = 16$ metres, nearly. Hence, to avoid serious errors in barometrically ascertained altitudes, the value of b must be determined with great care.

Assume, now, an error in the temperature amounting to dt, then $dh = 18400 \log_{10}\left(\frac{b_0}{b}\right) a\ dt$, approximately.

Hence

$$\frac{dh}{h} = \frac{adt}{1+at_m} = \frac{dt}{t_m+273} = \frac{dt}{T}$$

in which T is the absolute temperature.

Again, let $b = 500$ mm., $t_m = 0°$ C., and $dt = 1°$ C. Then

$dh = 12.25$ metres, approximately.

Clearly, then, to avoid considerable errors in hypsometric altitude determinations the temperature must also be known with considerable accuracy.

Finally, assume an error in the value of W, that is to say, in $\left(\frac{w_0}{p_0} + \frac{w}{p}\right)/2$. As there should be no error of consequence in w_0, assuming it to be the vapor tension at the surface of the earth, it follows that the chief error is likely to be in w, the vapor tension at the elevation where the total pressure is p.

Let w and p both be expressed in terms of barometric height, then

$$dh = 18400 \log_{10}\left(\frac{b_0}{b}\right)(1 + at_m)\ 0.378 \frac{dw}{2b}$$

If, as above, we let $b = 500$ mm., then w will usually be less than 4 mm. Hence, assuming $b_0 = 760$ mm., $t_m = 0°$ C., and a 25 per cent. or 1 mm. error in w,

$$dh = 1.25 \text{ metres, approximately.}$$

Hence an error in the value of the humidity produces only a small effect on the altitude determination in comparison with that due to an error of the same order in either the temperature or the total pressure.

Errors in the force of gravity, whether from latitude or from elevation, have already been shown to be very small.

For all ordinary purposes, therefore, altitudes in metres may be determined by the greatly simplified equation,

$$h = 18400 \log_{10}\left(\frac{b_0}{b}\right)(1 + at_m) \quad \text{(14)}$$

Obviously these hypsometric formulæ apply only to so much of the atmosphere as is of substantially constant composition, since the same "homogeneous" altitude, 7991 metres, is assumed throughout. Clearly, too, this condition of constant composition must apply, very approximately, up to the greatest altitude to which vigorous vertical convection extends, or in middle latitudes, as we shall see later, to an elevation of about 11 kilometres above sea level.

Beyond this level, up at least to the greatest altitude yet reached by sounding balloons and presumably much higher still. the temperature changes comparatively little with change of elevation. Hence in this region there can be relatively little vertical movement of the atmosphere, and therefore a chance, presumably, for the several gases, oxygen, nitrogen, and others, to distribute themselves, each as though it alone were present.

For this more or less isothermal region, then, it is sufficient for most purposes to use the simple equation,

$$-dp = p\frac{dh}{H} \quad \text{(15)}$$

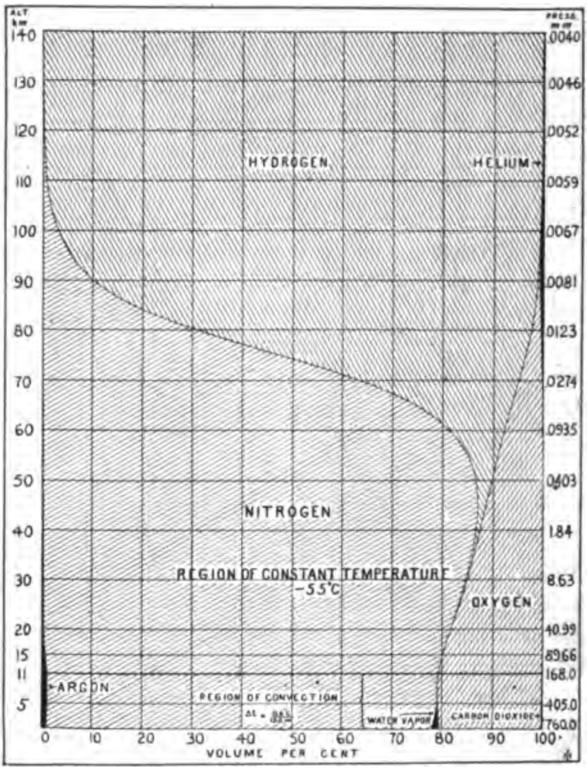
in which p is the partial pressure of the gas under consideration at the place in question, dh the change in elevation, and H the virtual height of the given gas, or its height, assuming its density throughout to be the same as at the initial level, necessary to produce the pressure p. This equation neglects any changes in the force of gravity, but, as already explained, such changes are small, and therefore the equation as it stands gives a close first approxi-

mation. It is not convenient, however, for numerical calculations, but for this purpose can be put into the following form:

$$\log_{10}p = \log_{10}p_0 - \frac{0.434295}{H}(h - h_0) \quad \text{(16)}$$

Equation (14) is applicable as far up as the composition of the atmosphere is essentially constant, or to an elevation of about 11 kilometres, while above that level, where, owing to the practi-

FIG. 19.



Composition of the atmosphere at different levels.

cal absence of vertical convections, each gas presumably is distributed substantially as though it alone were present, equation (16) may be used, with, of course, the proper value of H for each gas considered.

This value is given by the following equation:

$$H = 7991 \frac{D_a}{D} \frac{T}{273}$$

in which T is the absolute temperature, D_a the density of dry air, and D that of the gas in question, both at the same pressure (no matter what) and at 0° C.

Table II, computed by the aid of equations (14) and (16), and Fig. 19, drawn in accordance with this table, give the approximate composition and barometric pressure of the atmosphere at various levels. The assumptions upon which they are based are in close agreement with the average conditions of middle latitudes, and are as follows:

TABLE II.

Percentage Distribution of Gases in the Atmosphere.

Height in kilometres	Gases							
	Argon	Nitrogen	Water vapor	Oxygen	Carbon dioxide	Hydrogen	Helium	Total pressure in millimetres
140	....	0.01	....	.....	....	99.15	0.84	0.0040
130	....	0.04	....	.....	....	99.00	0.96	0.0046
120	....	0.19	....	.....	....	98.74	1.07	0.0052
110	....	0.67	0.02	0.02	....	98.10	1.19	0.0059
100	....	2.95	0.05	0.11	....	95.58	1.31	0.0067
90	....	9.78	0.10	0.49	....	88.28	1.35	0.0081
80	....	32.18	0.17	1.85	....	64.70	1.10	0.0123
70	0.03	61.83	0.20	4.72	....	32.61	0.61	0.0274
60	0.03	81.22	0.15	7.69	....	10.68	0.23	0.0935
50	0.12	86.78	0.10	10.17	....	2.76	0.07	0.403
40	0.22	86.42	0.06	12.61	....	0.67	0.02	1.84
30	0.35	84.26	0.03	15.18	0.01	0.16	0.01	8.63
20	0.59	81.24	0.02	18.10	0.01	0.04	....	40.99
15	0.77	79.52	0.01	19.66	0.02	0.02	....	89.66
11	0.94	78.02	0.01	20.99	0.03	0.01	....	168.00
5	0.94	77.89	0.18	20.95	0.03	0.01	....	405.
0	0.93	77.08	1.20	20.75	0.03	0.01	....	760.

1. That at the surface of the earth the principal gases of the atmosphere and their respective volume percentages in dry air are:

Nitrogen	78.03	Neon	0.0012
Oxygen	20.99	Helium	0.0004
Argon	0.94	Carbon dioxide	0.03
Hydrogen	0.01		

2. That at the surface of the earth water vapor supplies 1.2 per cent. of the total number of gas molecules present.

3. That the absolute humidity rapidly decreases, under the

influence of lower temperatures, with increase of elevation, to a negligible amount at or below the level of 10 kilometres.

4. That the temperature decreases uniformly at the rate of 6° C. per kilometre from 11° C. at sea level to −55° C. at an elevation of 11 kilometres.

5. That beyond 11 kilometres above sea level the temperature remains constant at −55° C.

6. That up to the level of 11 kilometres the relative percentages of the several gases, excepting water vapor, remain constant—a result, of course, of vertical convection.

7. That above 11 kilometres, where the temperature changes but little with elevation, and where vertical convection, therefore, is practically absent, the several gases are distributed according to their respective molecular weights.

A number of atmospheric gases—neon, krypton, xenon, ozone, etc.—are omitted both from Table I and from its accompanying figure. This is because all these occur—in the lower atmosphere, at any rate—in quantities too small for graphical illustration in the same diagram and to the same scale as are the principal gases.

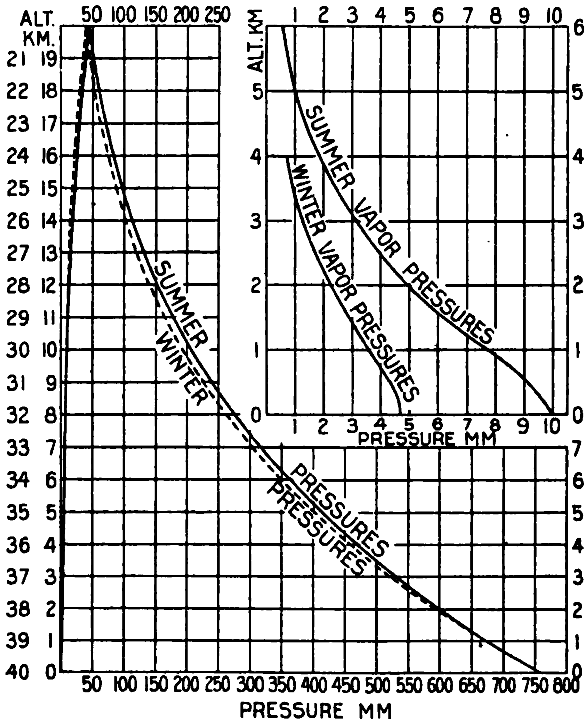
In using this diagram it should be distinctly remembered that it is supported by direct experimental observations only from the surface of the earth up to a level of about 30 kilometres, and that, while the extrapolated values are based upon apparently sound logic and not mere surmises, they necessarily become less and less certain with increase of elevation.

The table and the figure bring out a few points not generally realized. One of these is the fact that the total amount of argon in the atmosphere is much greater than the average total amount of water vapor. Another is the surprisingly small amount of water vapor, especially in view of the wonderful things it does, and of its vital importance to life of every kind. There may also be a little surprise that, according to calculation, the percentage of water vapor reaches a certain maximum at an elevation of 70 to 80 kilometres, where it is, roughly, twenty-fold what it is at, say, 11 kilometres. This, however, does not mean that the total amount of water vapor increases with elevation, but that it decreases less rapidly than do the heavier constituents, and more rapidly than the two lighter ones, hydrogen and helium.

DENSITY OF THE ATMOSPHERE.

Fig. 19, though serving the useful purpose of graphically representing the percentage distribution of the several gases of the atmosphere, nevertheless, is likely to be misleading in respect to their combined pressure and density, especially at great elevations. This latter information is given in the accompanying

FIG. 20.



Summer and winter pressures at various elevations.

table and shown by Figs. 20 and 21, in which the abscissas at every altitude are proportional to the corresponding atmospheric pressure, and density, to as great elevation, about 40 kilometres, as the size of the figures will permit.

In computing the pressure and density values in the accompanying table the complete hyposometric equation was used. That is, the effect of water vapor, and of both the latitude and

TABLE III.

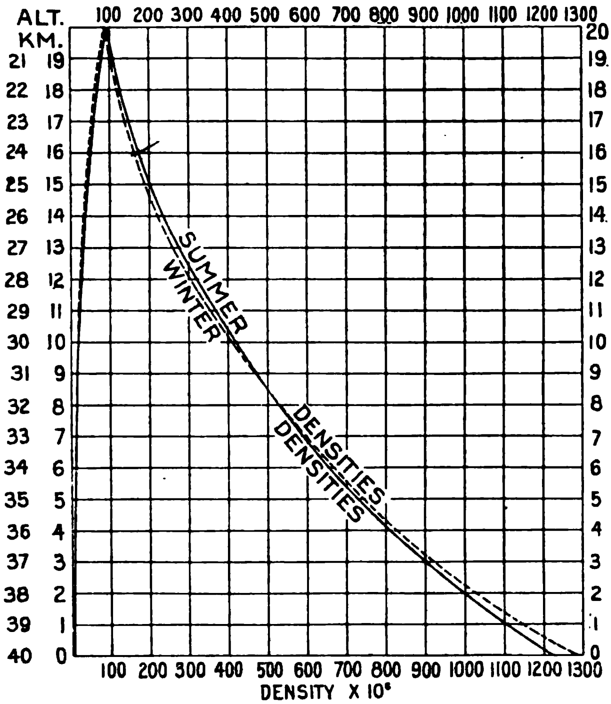
Average Grammes per Cubic Metre ($\rho \times 10^6$, ρ = density) and Millimetres Pressure (total, and Water Vapor) from 231 Summer and 185 Winter Sounding Balloon Flights at Trappes, Uccle, Strassburg, and Munich, 1900–1912.

Altitude Km. above sea level	Summer			Winter		
	Total pressure	Vapor pressure	Grammes per cubic metre	Total pressure	Vapor pressure	Grammes per cubic metre
0.0	762.55*	10.46	1224.42	763.35*	4.69	1287.58
0.5	718.75	9.17	1159.17	717.42	4.35	1212.31
1.0	677.24	7.81	1099.61	674.11	3.56	1147.23
1.5	637.81	6.21	1046.50	633.12	2.93	1084.23
2.0	600.31	4.97	995.19	594.37	2.27	1025.03
2.5	564.67	3.97	945.56	557.71	1.71	970.08
3.0	530.82	3.12	897.73	522.99	1.30	919.87
4.0	468.23	1.87	808.07	458.91	0.72	826.62
5.0	411.93	1.06	726.57	401.32	......	743.33
6.0	361.32	0.57	653.35	349.62	......	666.41
7.0	315.84	......	587.39	303.34	......	596.05
8.0	274.98	......	527.26	261.94	......	530.41
9.0	238.39	......	471.70	225.37	......	468.61
10.0	205.77	......	418.94	193.19	......	410.34
11.0	176.95	......	368.66	165.19	......	355.20
12.0	151.80	......	319.03	141.11	......	303.43
13.0	130.14	......	273.51	120.55	......	259.22
14.0	111.58	......	234.50	102.99	......	221.46
15.0	95.67	......	201.06	87.99	......	189.20
16.0	82.03	......	172.40	75.18	......	161.66
17.0	70.34	......	147.83	64.24	......	138.13
18.0	60.32	......	126.77	54.89	......	118.03
19.0	51.73	......	108.72	46.91	......	100.87
20.0	44.37	......	93.25	40.09	......	86.20
21.0	38.05	......	79.97	34.26	......	73.67
22.0	32.64	......	68.60	29.28	......	62.96
23.0	27.99	......	58.82	25.02	......	53.80
24.0	24.01	......	50.46	21.39	......	45.99
25.0	20.60	......	43.29	18.28	......	39.31
26.0	17.67	......	37.14	15.63	......	33.61
27.0	15.16	......	31.86	13.36	......	28.73
28.0	13.01	......	27.34	11.42	......	24.56
29.0	11.16	......	23.45	9.77	......	21.01
30.0	9.58	......	20.13	8.35	......	17.95
31.0	8.22	......	17.28	7.14	......	15.35
32.0	7.05	......	14.82	6.10	......	13.12
33.0	6.05	......	12.72	5.22	......	11.24
34.0	5.19	......	10.91	4.46	......	9.59
35.0	4.46	......	9.37	3.82	......	8.21
36.0	3.83	......	8.05	3.27	......	7.03
37.0	3.28	......	6.89	2.79	......	6.00
38.0	2.82	......	5.93	2.39	......	5.14
39.0	2.42	......	5.09	2.04	......	4.39
40.0	2.08	......	4.37	1.75	......	3.76

*Normal for the season.

altitude changes of gravity were all allowed for. No allowance, however, was made for the probable change with altitude in composition of the upper atmosphere because *a*, the exact amount of

FIG. 21.



Summer and winter densities at various elevations.

this change, is not certain, and *b*, at most, could not alter the values at the level of even 40 kilometres by more than about 1 per cent.

CHAPTER VI.

INSOLATION.

THE temperature variations of the atmosphere, both as to time and to place, and also the actual average temperature for any given locality, all obviously are of the utmost importance within themselves, and of equal importance indirectly through such meteorological elements as humidity, precipitation, wind direction, wind velocity, and nearly everything else that contributes to the sum-total of both weather and climate. Hence it is imperative that in any general discussion of meteorology some consideration be given to the question of the source, or sources, of the heat energy necessary to these conditions, where it is delivered, and how distributed.

A little heat is given to the surface of the earth and the atmosphere surrounding it by conduction from the heated interior, a little as a result of certain chemical changes, some from tidal action, and another small amount by the absorption of stellar and lunar radiation. But the sum-total of all these several amounts is so small in comparison to that which results from the absorption of solar radiation that for even a close approximation to the total amount of thermal energy given to the atmosphere it is sufficient to consider the sun as its only source.

The rate at which heat is delivered to any place on the earth from the sun depends upon:

a. The solar output of radiation.
b. Distance from the sun.
c. Inclination of the rays to the plane of the horizon, or solar elevation.
d. Transmission and absorption of the atmosphere.

These factors, then, determine the earth's heat income, and will be considered *seriatim*. How this heat is conserved, distributed, and expended also are important problems, which will be taken up later.

SOLAR OUTPUT OF RADIATION.

There is no *a priori* reason for assuming that the total output of radiation from the sun must remain strictly constant from

age to age, from year to year, or even day to day. Neither is there any known *a priori* reason for supposing that the solar radiation must greatly vary, either periodically or irregularly. Hence it is distinctly a subject for continuous and careful observation, a sort that, fortunately, is already well under way. Beginning with the summer of 1905, Messrs. Abbot and Fowle, of the Astrophysical Observatory of the Smithsonian Institution, and others working with them, have made numerous determinations of the solar constant, or intensity of the solar radiation as received at the outer surface of the atmosphere. Many of these observations were made on consecutive days, especially those obtained on Mount Wilson, California, and show irregular changes both as to time and quantity, changes often amounting to several per cent. in from five to ten days. In general, the observed values of the solar constant varied from one extreme to the other gradually and not by jumps. Besides, observations taken simultaneously, or at least on the same day, at stations so widely separated as Mount Wilson, California, and Bassour, Algeria, gave values that varied substantially together. "Hence," they say, "the most probable conclusion is that the sun actually varies from day to day in its output of radiation within limits of from five to ten per cent. in quantity and in irregular periods of from five to ten days." But further observations are much needed.

Some hold that the variations of even these short periods produce noticeable weather effects;[18] while a change of, say, five per cent. in the solar constant, if long continued, obviously would be a matter of very great climatic importance. A five per cent. increase in the amount of radiation received means, of course, a correspondingly higher temperature equilibrium and, when that is established, a like increase in the amount of radiation that must somehow be sent out to space. An increase in temperature would, among other things, increase the cloudiness to some extent and therefore the amount of solar radiation directly reflected, but probably the chief increase of the outgoing radiation would be due to the greater temperature of the earth and the atmosphere. Suppose the terrestrial radiation to be increased by four per cent., roughly the amount that might be expected from a long-continued five per cent. increase of solar radiation. Then, since the effective absolute temperature of the earth as a full radiator is approxi-

[18] Clayton, Smithsonian Miscellaneous Collections, 68, No. 3, 1917; 71, No. 3, 1920.

mately 260° C.,[19] it follows, from the fourth power law, that the effective temperature increase, essential to this four per cent. greater terrestrial output of radiation, would be 2.6° C., an amount that, if continued for a number of years, would be very important. But, as already stated, the observed periods of solar variations, while irregular, appear always to be short, in fact so short as to produce only much smaller variations from the average temperature.

There is, however, some evidence that, in addition to all short period changes, there are also other changes of the solar constant coincident in period and time with the sunspot variation, and which therefore go through a complete cycle in about 11 years. The evidence in favor of this 11-year cyclic change is not conclusive, but it is found both in the relative pyrheliometric values and in the absolute determinations of the solar constant. Both indicate that the solar constant may be, roughly, two per cent. greater at the times of sunspot maxima than at the times of sunspot minima. This indicated change probably is in the same sense that most persons would anticipate from the fact that the solar surface is most agitated at the times of spot maxima. Nevertheless, it apparently is in direct conflict with the conclusion, supported by much statistical evidence, that the atmosphere as a whole is slightly warmer at the times of spot minima than at spot maxima. There is, however, a logical way out of the paradox which will be given later.

DISTANCE FROM THE SUN.

If the apparent disk of the sun radiated equally all over, it would be strictly accurate to say that the intensity of its energy received at any particular point is directly proportional to the solid angle subtended by it at that point. But it does not radiate equally from all parts, either in amount or kind. The quantity of radiation per unit area of the apparent solar disk decreases from centre to circumference, while at the same time the spectral region of maximum intensity gradually shifts to longer and longer wave-lengths. Hence the total insolation at a given point cannot be rigidly proportional to the solid angle indicated. However, this departure from exact proportionality must in most cases

[19] Abbot and Fowle, *Annals Astrophys. Obs. Smithsonian Institution*, 2, p. 175 (1908).

be wholly negligible. Besides, the size of the solid angle subtended by the sun at the surface of the earth is so small that but little error would be made in problems of total radiation by regarding this angle as strictly zero and therefore applying without correction the law of inverse squares.

At aphelion the distance of the earth from the sun is, roughly, 3.3 per cent. greater than at perihelion. Hence the solar constant at perihelion, other things being equal, must be approximately 6.6 per cent. greater than at aphelion. And, since the earth may be regarded as a full radiator at about 260° C. absolute, it follows that its effective temperature must be greater when a maximum than when a minimum by about 1.6 per cent. of the average value, or, roughly, by 4° C.

All these calculations assume complete equilibrium between radiation and absorption, and, while there necessarily is some lag, an approach to equilibrium, as a little calculation will show, probably comes much sooner than one might suppose to be the case. Hence northern winters are not only shorter but also warmer than they would be if they occurred at times of aphelion instead of, as they do, at times of perihelion; while the winters of the southern hemisphere, during which the earth is farthest removed from the sun, have now a maximum both of duration and severity.

Notwithstanding—really, because of—the marked difference between the perihelion and aphelion intensities of the solar radiation at the limit of the atmosphere, it is easy to show that the total amount of insolation on the earth as a whole is constant per constant angular travel along its orbit; and that each hemisphere, regardless of the perihelion phase, or exact date on which perihelion occurs, receives during the course of a whole year exactly the same amount of solar radiation as does the other. This is shown as follows:

Let R be the solar distance, $d\theta$ the angle at the sun swept over by the earth in the time dt. Then, from the law of equal areas,

$$R^2 d\theta = C dt,$$

in which C is a constant.

Also, if dQ is the amount of solar energy incident upon the earth in the time dt,

$$dQ = \frac{I}{R^2} dt$$

in which I is the solar constant at unit solar distance. Hence

$$dQ = \frac{I}{C} d\theta,$$

or the energy received by the earth from the sun, assuming the solar output to be constant, is directly proportional to the angular distance between the initial and final radii vectors. Since the direction of the earth's axis is practically fixed in space, it follows that, to the same degree of approximation, each hemisphere must be inclined toward or from the sun over exactly one-half the angular orbit, and hence that the total yearly amount of heat received by one hemisphere is the same as that received by the other, and also that the earth as a whole gets precisely the same amount of radiant energy during the aphelion half of its orbit that it does during the perihelion half—what it loses in distance it exactly makes up in time.

It must not be supposed that this equality of heat supply means equality of world temperatures. Indeed, it means quite the reverse, for the equal quantities are delivered in unequal times; the time being longest and therefore the world temperature the lowest, except in so far as there may be a lag, or, perhaps, counter land and water effects, during aphelion, and shortest with highest temperature, as previously explained, during perihelion.

SOLAR ALTITUDE.

Leaving out, for the present, all questions of atmospheric absorption, it is obvious that the intensity of insolation is directly proportional to the sine of the angle of solar altitude, or to the cosine of the sun's zenith distance. But neither of these angles is directly known, and therefore can be expressed only in terms of those that are known. Each, however, is a function of latitude, of the time of day, expressible as an hour angle, and solar declination, as will be explained by the aid of Fig. 22.

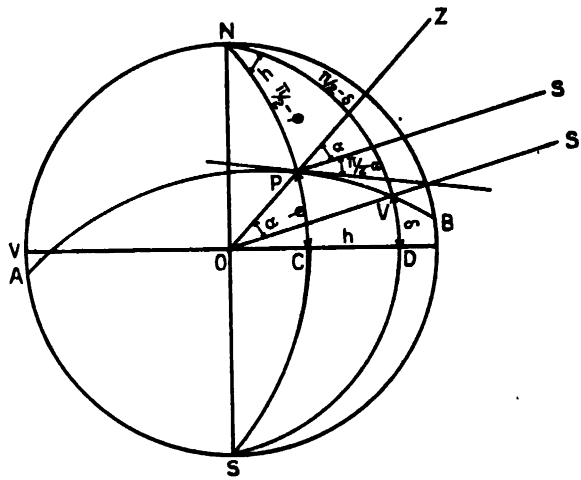
Let P be the point of observation, with its zenith at Z, and let the sun be off in the direction OS or PS. Clearly, then, the plane of OZ and OS intersects the surface of the earth in a great circle, and the angle ZOS, measured by the arc PV of this circle, is equal to the sun's zenith distance. But in the spherical triangle NPV it is obvious that the arc NV, V being directly under the sun, is the codeclination, NP the colatitude, and the angle h at N, measured by the arc CD on the equator, the hour angle, or

angle through which the earth must turn to bring the meridian of P directly under the sun. Hence

$$I_n = I \cos \alpha = I \ (\sin \ \varphi \ \sin \ \delta + \cos \varphi \cos \ \delta \ \cos \ h),$$

in which I_n is the normal component of the full solar intensity I, φ the latitude of the point P, δ the solar declination, and h the hour angle, as explained.

FIG. 22.



Relation of insolation to latitude, declination, and hour angle.

To find the amount of solar energy delivered at a given point we have the equation,

$$dQ = I_n dt = I \ (\sin \ \varphi \ \sin \ \delta + \cos \ \varphi \ \cos \ \delta \ \cos \ h) \ dh/\omega$$

in which dh is the change in the angle h in the time dt, and ω the angular velocity of the earth with reference to the sun or $\omega = \frac{\pi}{12}$ radians per hour. To obtain the total energy delivered per unit area in the course of a day we may consider I and δ constant for that time, and therefore write,

$$Q = 2I \ (\sin \ \varphi \ \sin \ \delta \ H + \cos \ \varphi \ \cos \ \delta \ \sin \ H)/\omega$$

in which H is the hour angle between noon and sunrise or sunset.

H obviously is a function of φ and δ. Thus at sunrise, say, $\alpha = \frac{\pi}{2}$. Hence,

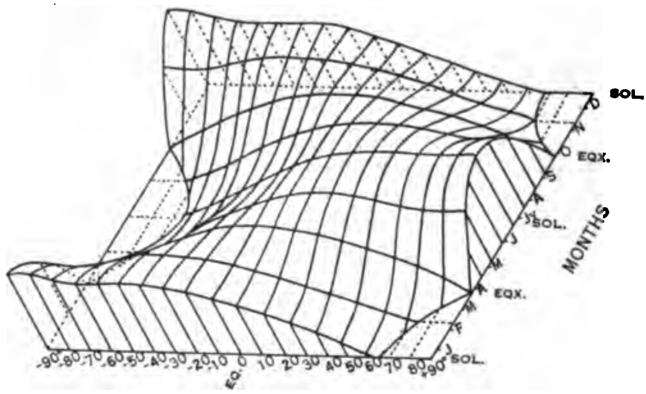
$$\cos \alpha = 0 = \sin \ \varphi \ \sin \ \delta + \cos \ \varphi \cos \ \delta \ \cos \ H$$

and

$$\cos H = - \tan \ \varphi \ \tan \ \delta.$$

Now φ, the latitude of the place in question, is known, and δ, the solar declination for any given day, is obtainable from an ephemeris. Hence, assuming a constant output of solar energy and allowing for variations in solar distance, the relative per diem amounts of insolation delivered to the earth (outer atmosphere) at different latitudes and different times may readily be computed. A few of these values, in terms of the total insolation received at the equator on the day of the vernal equinox, are given in the

FIG. 23.



Relation of insolation to season and latitude.

following table, and the complete data for all seasons and all latitudes in Fig. 23, both of which are copied from Davis's "Elementary Meteorology," Ginn & Co.:

Latitude..........	0°	+20°	+40°	+60°	+90°	−90°
March 20.........	1.000	0.934	0.763	0.499	0.000	0.000
June 21............	0.881	1.040	1.103	1.090	1.202	0.000
September 22.....	0.984	0.938	0.760	0.499	0.000	0.000
December 21......	0.942	0.679	0.352	0.000	0.000	1.284
Annual total....	347	329	274	197	143	143

TRANSMISSION AND ABSORPTION.

The rate of solar output of energy, distance from the sun, length of day, and solar altitude are all, as above explained, of vital importance in determining the amount of radiant energy

delivered to unit horizontal area during any interval of time—any consecutive 24 hours, say. But they give only the quantity of insolation delivered, and not the amount of energy actually absorbed and thereby rendered effective in maintaining temperature. This smaller quantity, the absorbed energy, depends upon not only the total insolation and therefore upon each of the above factors, but also upon at least two others; namely, reflection and scattering. Thus it is obvious that all the solar radiation which is reflected back to space, whether by clouds or by the surface of the earth, is immediately and completely lost, so far as heating the atmosphere is concerned, and that the same is also true of that smaller portion lost through the process of scattering, whether by dust particles or by air molecules.

The total loss of insolation by these two processes, reflection and scattering, amounts, according to Abbot and Fowle,[20] to about 37 per cent. for the whole earth. In other words, more than one-third of all the energy delivered to the earth is unused—merely scattered to space.

When solar radiation is minutely analyzed spectrally it exhibits thousands of irregularities in the wave-length distribution of energy that were present even before the outer atmosphere was reached. The minima, indicating strong absorptions in the solar atmosphere, constitute the well-known Fraunhofer lines.

In addition to the vast number of intensity deficiencies, absorption or Fraunhofer lines, as they are called, inherent in solar radiation, there are also many similar deficiencies resulting from its passage through the earth's atmosphere. These are caused by oxygen, carbon dioxide, water vapor, and ozone, and possibly even by other substances. Three of these, carbon dioxide, water vapor, and ozone, are also strongly absorptive of the long wave-length earth radiation. Oxygen and water vapor have many exceedingly restricted regions of absorption—so restricted, indeed, that in mere appearance they are indistinguishable from the narrow Fraunhofer lines. But in addition to these numerous narrow lines there are also a number of broad absorption bands, certainly of water vapor, ozone, and carbon dioxide. Oxygen, too, seems to have a broad absorption band in the region of exceedingly short wave-lengths. It is possible that many, if not all, of the bands are simply aggregates of large numbers of

[20] *Annals Astrophys. Obs. Smithsonian Institution*, 2, p. 163.

individual lines, but this is not known to be true of any of them.

Whatever the actual process of absorption, it is certain that to within observational errors the amount of energy absorbed increases arithmetically with the intensity of the incident radiation and, for monochromatic radiation, geometrically with the quantity of the absorbing material passed through, provided it is all under the same physical condition. Thus, if I_0 is the initial intensity of a parallel beam of monochromatic radiation, and aI_0 its intensity after passing normally through a homogeneous layer of absorbing material of unit thickness, then its intensity after traversing a distance of m units in the same material is given by the equation,

$$I = I_0 a^m.$$

In the case, also, of scattering of monochromatic radiation the extinction progresses according to the same laws that apply to absorption. That is to say, it is always a constant fraction of the remaining radiation that gets through a unit quantity of the scattering material.

The coefficients both of direct absorption and of extinction by scattering are radically different for radiations of different wave-lengths. But if I_0 is the initial intensity of the radiation of a given wave-length and aI_0 its intensity after it has passed normally through a layer of absorbing material of unit thickness, then after normal transmission through layers m and n units thick respectively,

$$I_m = I_0 a^m, \text{ and } I_n = I_0 a^n$$

Hence,

$$a = \left(\frac{I_m}{I_n}\right)^{\frac{1}{m-n}}, \text{ and } I_0 = I_m \left(\frac{I_m}{I_n}\right)^{\frac{m}{n-m}}$$

Now, in the case of the atmosphere, while in a vertical direction there is neither homogeneity in dust content nor in density, there is approximate horizontal homogeneity in respect to both conditions. Hence by observing the sun at different angles of elevation a considerable range in the ratio between m and n can be obtained for the atmosphere as a whole.

For simplicity it is desirable to take $n = 2m$, from which,

$$I_0 = \frac{I^2_m}{I_{2m}}, \text{ (equation of Bouguer).}$$

As both I_m and I_n are measurable, anywhere at the surface of the earth, I_0, or the intensity of the radiation outside the atmosphere, is at once determined to approximately the same degree of accuracy. The equation, however, is accurate only for monochromatic radiation. If applied to the whole, or even any considerable part, of the solar radiation as a unit the value thus obtained for the intensity of the initial radiation will be too small, as Langley showed long ago. This can be seen as follows:

Let the intensities of the several monochromatic radiations be A_0, B_0, C_0, etc., and their respective coefficients of transmission a, b, c, etc. Then their combined residual intensities after passing through the thicknesses m and $2m$ of the absorbing medium will be, respectively,

$$A_0a^m+B_0b^m+C_0c^m+, \text{ etc.}=R_m$$

and

$$A_0a^{2m}+B_0b^{2m}+C_0c^{2m}+, \text{ etc.}=R_{2m}$$

Hence the initial intensity, as computed by the Bouguer equation, is

$$R_0=\frac{R^2{}_m}{R_{2m}}$$

But the difference between the actual and the computed initial intensity is

$$A_0+B_0+C_0+, \text{ etc.}-R_0=$$

$$A_0+B_0+C_0+ \text{ etc.}-\frac{(A_0a^m+B_0b^m+C_0c^m+\text{etc.})^2}{A_0a^{2m}+B_0b^{2m}+C_0c^{2m}+\text{etc.}}=$$

$$\frac{A_0B_0(a^m-b^m)^2+A_0C_0(a^m-c^m)^2+\ldots\ldots+B_0C_0(b^m-c^m)^2+\text{etc.}}{A_0a^{2m}+B_0b^{2m}+C_0c^{2m}+\text{ etc.}}$$

An occasional term in the numerator of this final fraction may reduce to zero, since possibly $a=k$, $c=l$, etc., but in general no two of the coefficients, a, b, c, etc., are equal to each other. Hence every term in the numerator, except the few zero ones, if such exist, and consequently the fraction as a whole, is both real and positive. The Bouguer equation, therefore, when applied to complex radiation, always gives too small a value for the initial intensity.

Clearly, then, to determine to the highest degree of accuracy the intensity of the radiation reaching the outer atmosphere (that is, the amount per unit normal surface per unit time) it is necessary first to analyze it into its spectroscopic components and

then either to determine the initial intensity of each or else to adopt some equivalent process. The direct method of measuring the energy in each small spectral range would be very tedious, and, besides, would involve a difficult instrumental standardization, hence the following method has been found more convenient:

1. Analyze the insolation and obtain, with the bolometer, the relative distribution of energy through the spectrum for different solar altitudes but, as nearly as possible, constant sky conditions.

In each case the value of m, the air mass as it is called, is proportional to the secant of the solar zenith distance. Hence when the solar altitudes at which the bolograms were taken are known, the ratios of the corresponding m's, being the ratio of the respective zenith distance secants, are also known.

2. Measure with a pyrheliometer the rate at which solar energy, exclusive of sky radiation, is delivered per unit normal area during the same time that one of the bolograms was being obtained.

3. Extrapolate, according to the Bouguer equation, each portion of the bolograms to zero atmosphere and thus obtain the initial bologram, or energy distribution through the solar spectrum outside the atmosphere.

4. Measure the areas between the base line, corresponding to zero insolation, and the two bolograms, the extrapolated and the one corresponding to the pyrheliometric reading.

5. From these areas, A_0 and A, respectively. and the observed solar intensity I, compute I_0, the intensity of solar radiation outside the atmosphere, by the equation,

$$I_0 = I\frac{A_0}{A}$$

When expressed in terms of gram-calories per square centimetre normal surface per minute the average value of I_0, known in this form as the solar constant, is about 1.93.[21]

As stated above, careful estimates show that about 37 per cent. of this radiant energy is wholly lost to the earth, leaving only some 63 per cent. directly absorbed in roughly equal amounts by the earth and the atmosphere. And since the air usually is nearly opaque to terrestrial radiation, it follows that approxi-

[21] Abbot and Fowle, *Annals Astrophys. Obs. Smithsonian Institution*, 3, p. 134 (1913).

mately 60 per cent. of the incident solar energy ultimately heats the atmosphere.

The more conspicuous notches in the bolometric curve coincide with water vapor absorption bands, from which it is inferred that most of the direct absorption of solar energy in the atmosphere is due to water vapor. All these bands, however, are of longer wave-length than the region of maximum intensity in the solar spectrum, as are also the absorption bands of carbon dioxide and the stronger bands of ozone. Nitrogen and argon have no known absorption bands, while oxygen, the only other important constituent of the atmosphere, has only one, and that in the extreme ultraviolet or Schumann region, except some fine lines in the red. Further, the general absorption of all three is so feeble that, to a first approximation, it may be regarded as wholly negligible. Hence, atmospheric absorption of radiation, whether solar or terrestrial, obviously is due almost wholly to water vapor, carbon dioxide, and ozone; and, since the approximate amount of carbon dioxide in the atmosphere is always known and that of water vapor at least often determinable, it frequently is possible, by the aid of laboratory data, to know roughly the actual absorption in any portion of the spectrum due to these two substances, either singly or jointly.

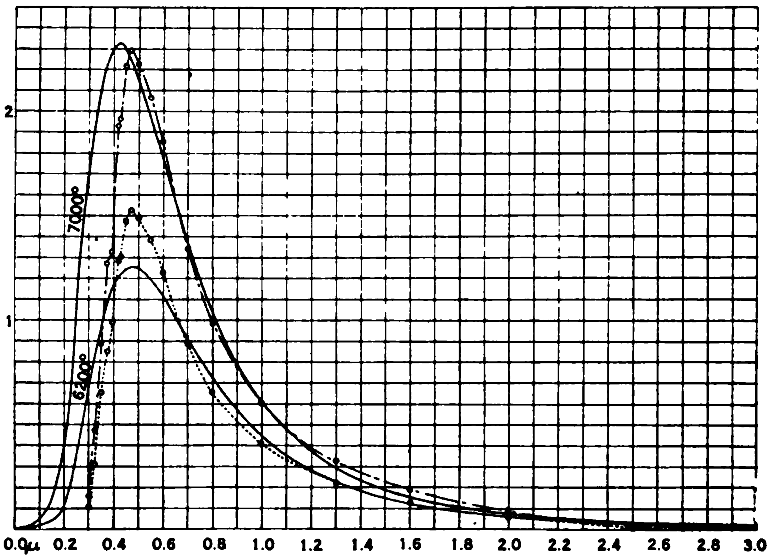
The magnitude of the ozone effect, however, is always uncertain because the quantity of this gas in the atmosphere is not known. In the presence of moisture and at ordinary temperatures it soon reverts to ordinary oxygen—a sufficient reason, perhaps, why only traces of it are found in the lower atmosphere. In the high atmosphere, on the other hand, where there must be very little moisture and where the temperature is about −55° C. in mid-latitudes, and even lower in the tropics, it obviously is far more stable. Hence, since extreme ultraviolet radiation, such as there is every reason to believe is emitted by the sun, on passing through cold dry oxygen converts much of it into ozone, it appears exceedingly probable that this substance must exist to appreciable amounts in the higher portions of the atmosphere. Indeed, the presence of ozone in the upper atmosphere has been fully demonstrated spectroscopically by Ångström,[22] Fabry and Buisson,[23]

[22] *Arkiv. för Matematik, Astronomi och Fysik.*, 1, p. 395 (1914).
[23] *Journal de physique* (5), 3, p. 196 (1913).

Fowler and Strutt,[23a] and Abbot.[23b] Strutt [23c] also proves, spectroscopically, that it is not nearly so concentrated, if present at all, in the lower atmosphere as in the upper.

The form of the solar energy spectrum curve outside the atmosphere as determined by Abbot and Fowle and its comparison with "black-body" curves at 6200° and 7000° absolute C. are given by Fig. 24 (a copy of Fig, 29, vol. iii, *Annals of the Astrophysical Observatory of the Smithsonian Institution*). The

FIG. 24.



Comparison of solar and "black-body" energy distribution.

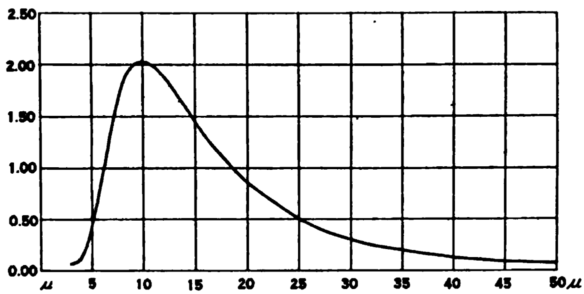
curve of terrestrial radiation intensities, on the other hand, is not known, but it obviously must be within that of a full radiator at the earth's temperature, but close to it, because of the universal presence of the highly absorptive substances, water vapor, especially, carbon dioxide, and ozone. That is, it must be within, but close to, the black-body curve for 287.2° C. absolute, as shown in Fig. 25, copied from Plate XX, vol. ii, *Annals of the Astro-*

[23a] *Proc. Roy. Soc.* v. 93 A, p. 77, 1917.
[23b] *Proc. National Acad.*, vol. 4, p. 104, 1918.
[23c] *Nature*, vol. 100, p. 144, 1917; *Proc. Roy. Soc. A.* 94, p. 260, 1918.

physical Observatory of the Smithsonian Institution (Abbot and Fowle).

The absorptions of earth radiation by water vapor, carbon dioxide, and ozone are shown in Fig. 26. The outer or enveloping curve, a copy of Fig. 25, gives the intensity distribution of radiation from a black body at the temperature 287.2° C. absolute. The area of this curve below the irregular full line near its top, at least up to 20μ, and, presumably, through much, if not all, the region of longer wave-lengths, shows the absorption of earth radiation by a column of 1.13 grammes of water vapor per square centimetre cross-section, as computed by Bouguer's equation from

FIG. 25.



"Black-body" radiation at earth temperature.

the laboratory data of Rubens and Aschkinass.[24] Similar data by Fowle,[24a] however, give smaller coefficients of absorption. The amount of water vapor assumed, 1.13 grammes per square centimetre cross-section of the column, is that which Abbot and Fowle [25] have computed to be the average amount in the atmosphere as a whole above the 1780-metre level. The areas below the two broken curves show the absorption by carbon dioxide above the same level, as computed from the experimental data of Schaefer,[26] and Rubens and Aschkinass.[27] Finally, the area below the dotted line gives some idea of the absorption of earth radia-

[24] *Ann. der Phys.*, 64, p. 584 (1898).

[24a] Smithsonian Misc. Col., v. 68, No. 8, 1917.

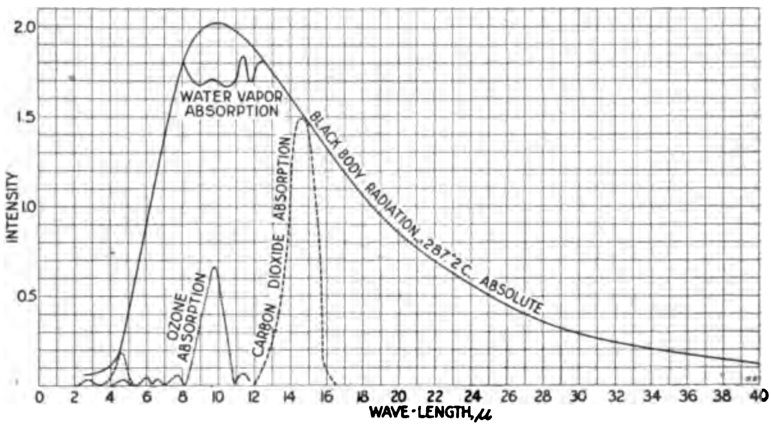
[25] *Annals of the Astrophys. Obs. Smithsonian Institution*, vol. ii, p. 168 (1908).

[26] *Ann. der Phys.*, 16, p. 93 (1905).

[27] *Loc. cit.*

tion by ozone, computed, after making certain assumptions explained below, from the observations of Ladenburg and Lehmann.[28] The amount of the ozone absorption, however, is uncertain, as implied, for several reasons: (*a*) Laboratory measurements have not extended beyond 12μ. (*b*) While Ladenburg and Lehmann give the length of the absorbing column used, one metre, and describe minutely their ingenious manometer, they do not state the actual pressures at which their data were obtained. But as their ozone was nearly pure, and as they refer to the color of the gas at 50 mm. and 200 to 300 mm., it would seem that they used the equivalent, roughly, of a column 30 centimetres

FIG. 26.



Water vapor, carbon dioxide, and ozone absorption.

long at, say, 15° C. and 760 mm. (*c*) The amount of ozone in the atmosphere is not known. Pring,[29] who appears to have done the most careful work on this subject, estimates it to be the equivalent of a layer of the pure gas 4.2 centimetres thick at normal temperature and pressure. Hence the assumptions referred to above were (1) that the ozone in the absorption tube was the equivalent of a column 30 centimetres long, and (2) that the ozone in the atmosphere is equivalent to a layer whose thickness is only 4.2 centimetres, each at atmospheric pressure and room temperature.

It appears, then, as shown by Fig. 26, that the absorption of

[28] *Ann. der Phys.*, 21, p. 305 (1906).

[29] *Proc. Roy. Soc.*, *A*, vol. 90, p. 204 (1914).

earth radiation by water vapor is nearly perfect, even beyond the level of one to two kilometres; that neither at this level nor, perhaps, at any other (consult Fig. 19 and its accompanying table for probable distribution of carbon dioxide and water vapor) does the carbon dioxide appreciably affect the amount of absorption—already complete (through the presence of water vapor) in the regions of its bands; and that, since ozone absorption is very strong where that of water vapor is weakest and imperfect and earth radiation at its maximum, the presence of this gas in the atmosphere has a slight heat-conserving or warming effect.

It is also obvious that, although each of these substances is an effective absorber in the region of appreciable to strong solar radiation, their joint effect on earth radiation is far greater, so much so, indeed, that for low levels absorption is practically complete.

SURFACE TEMPERATURES AND ABSORBING GASES.

The radiation from a particle of water vapor, or any other substance in or of the atmosphere, clearly, is equal in all directions. Hence the amount of radiant energy incident on the surface of the earth and the resulting temperature are generally different from what they would be in the absence of any such absorbing and radiating medium. During the night, when there is no incoming radiation to consider, the process of absorption and radiation by the constituents of the atmosphere manifestly checks the rate of surface cooling, and thereby insures higher night temperatures than otherwise would obtain. Whether this process also increases day temperatures is not so obvious. It does, as the following argument will show, when, and only when, the substance involved, whether the ozone shell, presumably in the isothermal region, the water vapor shell of the lower atmosphere, or any other such shell, is more absorptive of the outgoing terrestrial radiation than of the incoming solar radiation.

The numerical solution of the problem as applied to any one of these shells is complicated by the alternation between night and day; by constant changes in the solar inclination; by reflection; and probably by many other conditions of importance. However, since the coefficient of absorption is independent of intensity of radiation, it is possible to obtain an approximate evaluation of the temperature effect due to any given absorbing layer or shell.

Let I be the average intensity of the normal component of the

absorbed portion of solar radiation. Then, assuming, as already explained, that 37 per cent. of the incident solar energy is wholly lost by reflection, representing the solar intensity outside the atmosphere by I_0, and dividing the cross-section of the beam, multiplied by its effectiveness by the area over which it is distributed,

$$I = \frac{0.63 I_0 \pi R^2}{4\pi R^2} = 0.16\, I_0, \text{ about,}$$

in which R is the effective radius of the earth as an intercepter of incoming radiation.

Also let aI be the average intensity of solar energy, including both the direct and the reflected, absorbed by the outermost shell, say, or by the ozone of the upper atmosphere. Half this absorbed energy is radiated to space and half to the earth, including the lower atmosphere.

The earth, which for convenience may be considered initially cold, now receives radiation of the average intensity $(1-a)\ I$ from the sun, and of the intensity $\frac{a}{2}\ I$ from the absorbing layer under consideration. Together these amount to $I\ (1-\frac{a}{2})$.

After a time the earth will return an equal average radiation, but of very different spectral distribution. Let $b\,I\ (1-\frac{a}{2})$ be the part of this long-wave earth radiation absorbed by the layer in question. As before, one half, or $\frac{b}{2}\ I\ (1-\frac{a}{2})$, will be radiated back to the earth, there absorbed, because of its long wavelength, and again sent out. The next amount returned by the shell is $\left(\frac{b}{2}\right)^2 I\ (1-\frac{a}{2})$, and so on infinitely.

Hence the total average intensity, I_e, of the radiation reaching the earth and lower atmosphere is given by the equation,

$$I_e = I\left\{1-\frac{a}{2}\right\}\left\{1+\frac{b}{2}+\left(\frac{b}{2}\right)^2+\ldots\ldots+\left(\frac{b}{2}\right)^\infty\right\}$$

$$= I\left\{1+K(b-a)\right\}-\frac{a}{2}\left(\frac{b}{2}\right)^\infty.$$

$$= I\left\{1+K(b-a)\right\}, \text{ since } b \text{ cannot exceed unity,}$$

in which

$$K=\frac{1}{2}\left\{1+\frac{b}{2}+\left(\frac{b}{2}\right)^2+\ldots\ldots+\left(\frac{b}{2}\right)^\infty\right\}$$

Now, b is positive and therefore K is also positive. Hence

$$I\left\{1+K(b-a)\right\} \gtreqless I$$

according as

$$b \gtreqless a$$

That is, the total amount of radiation reaching the earth is increased, unchanged, or decreased by the presence of an absorbing shell according as its coefficient of absorption of terrestrial radiation is greater than, equal to, or less than its coefficient of absorption of solar radiation. But water vapor, carbon dioxide, and ozone are all more absorptive of earth radiation than of the comparatively short wave-length solar energy, and therefore each, but water vapor especially, keeps the average temperature higher than it otherwise would be.

Suppose $b = 20$ per cent. and $a = 2$ per cent., then,

$$I\left\{1+K(b-a)\right\} = 1.1I, \text{ about.}$$

That is, the incoming radiation, and hence also the outgoing radiation, is increased 10 per cent. over what it would be if such an absorbing layer did not exist. But as the earth, largely because of its water vapor, radiates substantially as a black body, or in proportion to the fourth power of the absolute temperature, it follows that a 10 per cent. increase of radiation implies approximately a 2.5 per cent. increase of the absolute temperature. In other words, an absorbing shell with the properties assumed, properties that possibly are of the same order of magnitude as those of the existing ozone in the upper atmosphere, would maintain the average temperature of the earth about 7° C. higher than it otherwise would be.

Any increase, then, in the amount of ozone, or other similar absorbing material, in the outer atmosphere must more or less increase the average temperature of the earth. Hence variations in the output from the sun of the ozonizing or very short wave-length radiation presumably would alter the ozone content of the upper air and through it, as above explained, the average temperature of the earth. Other things being equal, it would seem that there should be a maximum of this extreme ultraviolet radiation and, consequently, maximum average temperature at the

time of a sunspot minimum when the solar atmosphere is comparatively clear, as indicated by a minimum corona; and a minimum, with minimum average temperature, at the time of a spot maximum when the "dustiness" of the solar atmosphere, as shown by a maximum corona, is very great. Even if the solar constant should be a little greater at the time of a spot maximum than at a spot minimum, the above variation of ozone, if it occurs, might lead to the paradoxical concurrence of maximum average temperature with minimum average insolation and minimum average temperature with maximum average insolation.

A greater general prevalence of cirrus and cirrus haze during spot maxima than during spot minima would also account for this paradox; because such clouds, owing to the size of their particles, shut out the short wave-length solar radiation more effectively than they shut in the long wave-length earth radiation. And perhaps these clouds really are generally most prevalent during spot maxima, and therefore at least a contributing factor in the cause of the observed temperature changes. At any rate the auroras are then most frequent, and they obviously generate nitrous oxide and other hygroscopic compounds which, because of their density, slowly fall to the cirrus level where they may produce cloud particles in an atmosphere whose humidity is much below that which otherwise would be essential to cloud formation.

Chapter VII.

ATMOSPHERIC CIRCULATION.

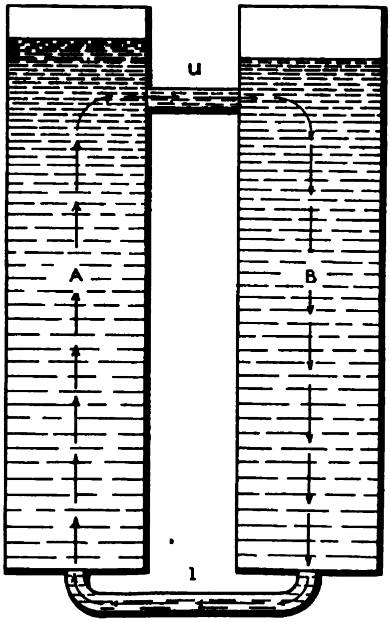
INTRODUCTION.

Atmospheric circulation, whether manifesting itself in a monsoon, for instance, or in only a gentle lake breeze, is a gravitational phenomenon induced and maintained by temperature differences. This can be well illustrated by the flow of water between two adjacent tanks when connected by an upper and a lower pipe and kept at different temperatures.

Let the two tanks, *A* and *B*, Fig. 27, be filled to the same level slightly above the upper pipe *u*, and let them have the same temperature. Under these conditions there will be no flow of water from either tank to the other. Now let the pipes be closed and let the water in tank *A* be equally warmed throughout. It will expand, providing its original temperature was not below 4° C., and the amount of water above each level in *A*, at and below the initial surface, be increased in proportion to its distance from the bottom. Hence the pressure due to gravity is everywhere throughout the original volume correspondingly increased—the maximum increase being at the level of the initial surface. If the lower pipe *l* be now opened, there still will be no flow of water from either tank to the other. But if the upper pipe be opened, water will flow from *A* to *B*, and in so doing will decrease the pressure on all parts of *A* and increase it on all parts of *B*. If *l* is also open, water will flow from *B* to *A*. If both pipes are left open and the water in *A* kept constantly warmer than the water in *B*, there will be continuous circulation of the water from *A* to *B* through the upper pipe and from *B* to *A* through the lower. Obviously the same results could be obtained by applying a cooling process to *B* instead of a warming one to *A*. That is, since the circulation in question is a gravitational phenomenon induced by a temperature difference between the water in the two tanks, it clearly is immaterial how this temperature difference is established, whether by heating the one tank or by cooling the other; similarly in the case of the atmosphere. If two adjacent columns of air, or the masses of air over two adjoining regions, whether large or small, are kept at

different temperatures, there will exist, through the action of gravity, a continuous overflow from the warmer to the colder, and an underflow from the colder to the warmer. Neither does it make any difference in this case how the inequality of temperature is established and maintained, whether by heating the one section or by cooling the other.

FIG. 27.



Circulation between warm and cold tanks.

VERTICAL CONVECTION OF THE ATMOSPHERE.

General Considerations.—Vertical convection of the atmosphere may be divided into two classes: (*a*) mechanically forced convection, as the rise of air on the windward side of a mountain or other obstruction and its fall on the leeward side; (*b*) thermal convection. The latter, involving both warming and cooling, is by far the more important; in fact, it either constitutes or is associated with all natural air movements. It commonly is said to consist of the rising of warm air and the sinking or flowing in of cold air to take its place; but, while this describes the phenomenon of thermal convection, it seems to imply the false

concept that warm air has some inherent ascensional power, whereas, in reality, thermal convection is only a gravitational phenomenon, consisting in the sinking of relatively heavy air and the consequent forcing up of air which, volume for volume and under the same pressure, is relatively light.

The terms "heavy" and "light" are used here advisedly instead of "dense" and "rare," because it is the relative *weights* of two adjacent masses of air of equal volume under the same pressure and not their *densities* that determine which shall fall and which shall be raised.

Three factors enter into the question of weight per unit volume when pressure is constant: (*a*) temperature, (*b*) composition, and (*c*) horizontal velocity, including speed and direction. The first of these weight factors varies widely and is very effective. A change in temperature by any given amount $\pm t$, say, changes the original weight per unit volume, W_1, to the new weight, $W_1 \pm w$ in the ratio,

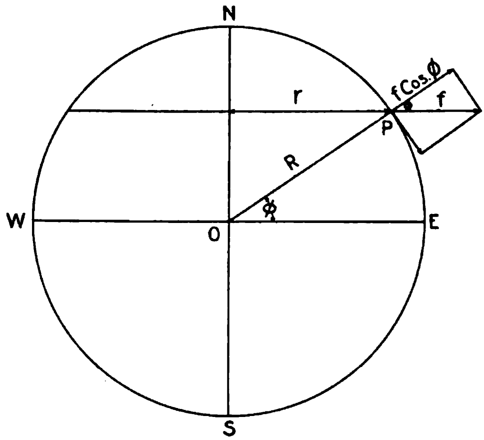
$$\frac{W_1}{W_1 \pm w} = \frac{T \mp t}{T} \quad \text{(Charles's or Gay-Lussac's law),}$$

in which T is the original absolute temperature. Thus if the original temperature is that of melting ice, and it is increased or decreased by 1° C., the weight per unit volume will be decreased or increased, respectively, 1 part in 273.

The effect of the second of the above weight factors, the composition of the atmosphere, is obvious from the following consideration: Since the number of gas molecules per unit volume under a fixed temperature and pressure is independent of the nature of the gas—Avogadro's law—it follows that under these conditions an increase or decrease of water vapor, say, in the atmosphere implies a corresponding decrease or increase of the other molecules present, mainly nitrogen and oxygen. Now the equivalent molecular weight of dry air is approximately 28.94 and the molecular weight of water 18, hence a change in the water vapor, the only constituent of the atmosphere that appreciably varies, amounting to 1 per cent. of the total number of gas molecules present, alters the weight per unit volume by $\frac{1094}{2894} \frac{1}{100} W$, in which W is the weight of the unit volume of dry air under the same conditions of temperature, pressure, and gravity. On very warm days water vapor may amount to 5 per cent. or more of the total gas molecules present, and the air, therefore, be, roughly,

2 per cent. lighter than it would be if perfectly dry. Of course, changes from saturation to utter dryness, or the reverse, do not occur in nature, but a variation of as much as 50 per cent. in the absolute humidity at a given place does occur through evaporation, condensation, and air movement. Hence on very hot days a change of 1 per cent. in the weight per unit volume of the lower air as a result of altered composition alone is quite possible, and indeed often occurs. This produces a difference in buoyancy

FIG. 28



Decrease of weight due to rotation of the earth.

of the same order as that caused by a 3° C. change in temperature and therefore may be decidedly important.

The third factor that affects weight and convection, namely, horizontal velocity, while comparatively small, occasionally may be of some importance. Its numerical value can easily be computed. Let *NS*, Fig. 28, be the axis of the earth's rotation, and let *P* at latitude ϕ be the point under consideration. The centrifugal force f acting on the mass m at the point P is given by the equation,

$$f = m\, r\, \omega^2,$$

in which ω is the angular velocity of the earth's rotation, and r the distance of P from the axis. Numerically,

$$\omega = \frac{2\pi}{86{,}164}.$$

Since the mean radius of the earth is about 6368 kilometres, and since weight equals mass times gravitational acceleration, or $981m$, approximately, in the C.G.S. system, it follows that at latitude 40°

$$f = \frac{w}{378}, \text{ roughly,}$$

in which w is the weight of the object considered, while the decrease, dw, in the weight, or the component of f at right angles to the surface, is given by the equation,

$$d\,w = -f \cos 40° = -\frac{w}{493}, \text{ about}$$

At latitude 40° a velocity of 22.4 metres per second (50 miles per hour) from east to west is equivalent to decreasing ω^2 by 1 part in 8, approximately, and an equal velocity from west to east to increasing it a like amount. That is, at latitude 40° a given mass of air in a west wind of 22.4 metres per second (50 miles per hour) weighs less than an equal amount in an east wind of the same velocity by about 1 part in 1972: similarly for other latitudes in proportion to their cosines. Hence, other things being equal, an east wind tends slightly to underrun an adjacent west wind.

The above special solution of this problem may suffice for most purposes, but the following outline and conclusions of a more general solution probably will be of interest.

Referring to Fig. 29, let $V \cos \phi$ be the horizontal velocity of the surface of the earth at latitude ϕ, v the horizontal velocity of the air with reference to the surface, α the angle between the east direction and the path, and assume the earth to be spherical and concentrically homogeneous. Further, let mg_0 be the gravitational pull on the mass m, or the weight the mass m would have when at rest if the earth were nonrotating, and let mg be its actual weight. Then,

$$mg = mg_0 - m\frac{(V \cos \phi + v \cos \alpha)^2}{R} - \frac{m(v \sin \alpha)^2}{R}$$
$$= mg_0 - \frac{m(V \cos \phi)^2}{R} - \frac{mv(2\,V \cos \phi \cos \alpha + v)}{R}.$$

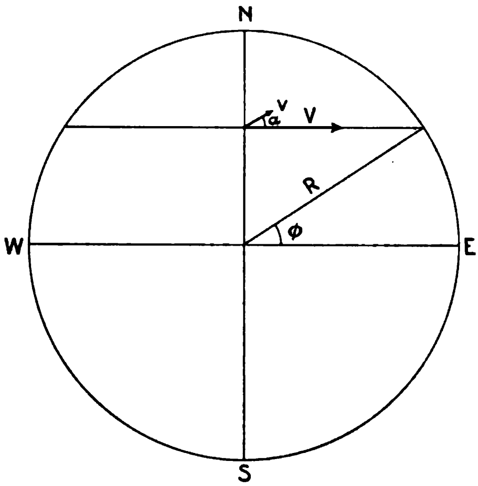
But, $mg_0 - \frac{m(V \cos \phi)^2}{R} =$ the weight of the mass m when at rest on the surface of the rotating earth. Hence, when the mass m

is given a horizontal velocity v, its still-weight, W, is changed by an amount, ΔW, given by the equation,

$$\Delta W = -mv\frac{(2V\cos\phi\cos\alpha + v)}{R}.$$

Since the latitude is limited by 0° and 90°, it follows that the sign of the change (that is, whether the change consists of a decrease or increase from the still-weight) depends upon the value of α, or the direction of motion. Obviously, whenever $\cos\alpha$ is positive, or the velocity has an *easterly* component, the

FIG 29



Change of weight due to horizontal velocity.

change of weight is *negative*—maximum when ϕ and α are both zero, or when the motion is east on the equator. Similarly, since $2V\cos\phi$ is nearly always large in comparison with v, the change of weight ordinarily is *positive* whenever the velocity has an appreciable westward component. This increase of weight clearly is a maximum when $\phi = 0°$ and $\alpha = 180°$, or when the motion is west on the equator.

Further, the direction of the wind in order that there be no change of weight is conditioned by the equation,

$$\cos\alpha = \frac{v}{2V\cos\phi}.$$

But, as above stated, $2V\cos\phi$ nearly always is large in comparison with v, and therefore the direction conditioned by

zero change in weight commonly has only a small westerly component. For example, let $v = 44.7$ metres per second (100 miles per hour) and let $\phi = 60°$. Then, under these somewhat excessive conditions, for zero effect, $\cos \alpha = -\frac{1}{10}$, about, or the direction along which there is no change of weight is less than 6° west of the meridian.

From the above it appears that of the three factors that alter the weight of a given volume of air and thereby determine whether it shall rise to higher levels, sink to lower, or remain where it is, temperature is by far the most important, and horizontal velocity the least important. As a rule, the former alone need be considered.

Local Convection.—There are two distinct ways of thermally inducing convection: (*a*) by *heating below;* (*b*) by *cooling above.* Each is of great importance, both in general atmospheric movements and also in those restricted or local winds to which special names have been given. Where heating alone occurs the rising air does not return, but remains in equilibrium at its new level, where its final temperature is the same as that of the adjacent atmosphere. Similarly, when cooling alone occurs the sinking air does not again immediately rise. In other words, neither heating nor cooling, acting alone, can produce closed circulation in which the same mass passes through a complete cycle of positions, though, of course, a compensating movement must occur somewhere. A strong local uprush, for instance, except in the case of the thunderstorm, to be explained below, is nearly always compensated by a wide settling, so gentle that it cannot be measured. Similarly, restricted down-rushes of air, again except in connection with the thunderstorm, are compensated by wide and gentle upward movements.

Whatever the type of atmospheric disturbance under consideration, the most important general facts to remember are: that all vertical movements of the air are accompanied by dynamical heating or cooling; that rising air cools, roughly, at the adiabatic rate of about 1° C. for each 103 metres increase of elevation; that descending air warms at substantially the same rate; that dynamic cooling limits the height of upward convection; and that in many cases dynamic heating, and not the surface of the ground, limits downward convection.

CLASSIFICATION OF WINDS.

All the above is entirely general and of universal application. Gravity and temperature differences enter directly or indirectly into all atmospheric circulation, both the fundamental and continuous circulation that exists between the warm equatorial and cold polar regions, and those secondary circulations that occur only locally and occasionally. Nevertheless, clearness in any detailed discussion of all winds requires their grouping according to some basis of classification suitable to the purpose in view—in the present case a discussion of their initiating causes and modifying influences. Unfortunately, none of the current classifications of winds (*e.g.*, those listed in Milham's "Meteorology," p. 164 *et seq.*) is adapted to this end, and, therefore, provisionally a somewhat different grouping will be used.

Obviously no subdivision of air movements is possible on the basis either of gravitation or of temperature difference, since, as just explained, each is involved in every such movement.

But there also are modifying factors—friction, viscosity, turbulence, local heating, local cooling, deflection by mountain barriers, deflection due to the earth's rotation, and many others that make one circulation different from another in time of occurrence, extent, duration, direction, and intensity. Hence in considering the origin and nature of winds, it will be convenient to classify them as follows, according to the conditions that initiate or materially modify them, and to discuss separately, under its appropriate head, each distinct and well-recognized subdivision.

A.—Winds due chiefly to local heating—whirlwinds, cumulus convection, valley breezes, sea breezes.

B.—Winds due to cooling—land breezes, mountain breezes, glacier winds, bora, mistral, Norwegian fallwinds, continental fallwinds.

C.—Winds due to simultaneous adjacent local heating and local cooling—thunderstorm winds.

D.—Winds due to widespread heating and cooling—gradient winds, monsoons, trades, antitrades, tropical cyclonic winds, extratropical cyclonic winds, anticyclonic winds.

E.—Forced winds, or winds caused by other winds—eddies, foehns (Chinooks), tornadoes.

WINDS DUE TO LOCAL HEATING.

Whirlwinds.

During clear, calm summer afternoons, particularly during a dry spell when vegetation is parched and the ground strongly heated, dust whirls often develop, and occasionally travel considerable distances before losing their identity. The flatter the region, the more barren, the hotter the surface, and the quieter the air, the more violent these whirls become. Hence, level deserts are especially frequented by such winds, amounting at times to violent storms, though never more than a few metres in diameter. The development of these storms in which convection is strong is not simple, but an understanding of them will materially help to an understanding of convection due to heating in less obvious cases.

It is well known that those regions in which violent dust or sand whirls occur are also the places where inferior mirages are most frequent. The reason for this coincidence is the fact that the density gradient of the atmosphere essential to the production of a mirage simulating a lake, namely, an increase of density with elevation, is most favorable to strong vertical convection. Under these conditions the air is in that same unstable equilibrium that applies to a column of liquid whose under layer is lighter than the upper—whose under layer is oil, say, and upper layer water.

At first sight it might seem that no such condition of considerable extent can occur in nature; that as soon as the under layer became specifically lighter than the one next above, they would change places. Whenever a cork, for instance, is let go under water it bobs up. Similarly, a balloon rises, without exception, whenever the combined weight of gas, envelope, etc., is less than that of the atmosphere displaced. Why then should not surface air, whenever it becomes specifically lighter than the air above it, also rise immediately? This undoubtedly is just what a limited volume of light air would do if actually surrounded on all sides by heavier air. But surface air is not completely surrounded by other air; its condition is somewhat analogous to that of a cork whose flat surface is pressed against the bottom of a vessel of water. The cork in this case does not rise, simply because it is pressed down by water above and not pushed up by water beneath. Similarly, warm air covering an extensive flat surface is pressed down by the superincumbent atmosphere

and not pushed up by denser air below—there is no denser air below to push it up.

Obviously, though, even surface air under the given conditions is in an unstable condition. Hence it is important to determine that vertical temperature gradient which reduces superadjacent layers of air to the same density.

Auto-convection Gradient.—Suppose the atmosphere is perfectly quiet, what temperature gradient must any layer of it have in order that it may just initiate its own convection?

Clearly this gradient must be such that density shall just increase with elevation. That is, the ratio, $\frac{d\rho}{dh}$, must be just positive.

From

$$pV=\frac{p}{\rho}=RT$$

we get

$$\rho=\frac{p}{RT}.$$

Therefore,

$$d\rho=\frac{p}{RT}\left(\frac{dp}{p}-\frac{dT}{T}\right).$$

But

$$dp=-981\,\rho dh=-\frac{981\,dh}{V}=-\frac{981\,pdh}{RT}.$$

Therefore,

$$d\rho=-\frac{p}{RT}\left(\frac{981\,dh}{RT}+\frac{dT}{T}\right).$$

and

$$\frac{d\rho}{dh}=\frac{p}{RT^2}\left(\frac{981}{R}+\frac{dT}{dh}\right).$$

Hence, since $\frac{dT}{dh}$ is negative in the case under consideration,

$$\frac{d\rho}{dh}=0,$$

when

$$\frac{dT}{dh}=\frac{981}{R}.$$

But for dry air,

$$R=2.871\times10^6$$

and

$$\frac{dT}{dh}=-0.0003417.$$

Hence when $dT = -1°$ C., $dh = 29.27$ metres.

That is, in order that an under layer shall have the same density as the next above it the temperature must decrease 1° C. with each 29.27 metres increase of altitude, or 3.52 times faster than the adiabatic rate of 1° C. per 102.93 metres. In order that the lower layer shall be distinctly lighter than the upper, the temperature decrease with increase of altitude must be four or five times the adiabatic rate.

Obviously an extensive layer of warm, light air cannot all rise at the same time. It must rise locally and in streams, if at all. Similarly, the upper air must settle locally, if at all. But during the time mirage conditions are maintained the surface is strongly heated, so that any air that might reach it is not only warmed dynamically by the compression to which it is subjected, in settling down, but also by the heat acquired from contact with the surface. Hence, any settling of colder air from above amounts only to a partial removal of the hot and therefore light surface air.

It might seem, however, that the streams of rising air, above referred to, would ascend with great rapidity and in such volume as quickly to exhaust the supply of warm air. But, as already stated, the heat is being constantly supplied, and therefore the surface layer of hot air continuously renewed. Besides, the difference in weight between a given volume of a rising filament of warm air and an equal volume of adjacent air is at first only an exceedingly small fraction of the weight of either. Hence the acceleration with which it moves is correspondingly small. In nearly all cases the feeble streams of rising hot air presumably are pinched off by the adjacent denser air, and in this manner the breaks, as it were, in the integrity of the hot surface layer mended and the entire convection divided into innumerable discontinuous filaments—mere fitful leaks from the constantly renewed reservoir of hot air.

It occasionally happens, however, that because of some disturbance an unusually large volume of warm air breaks through and rises in a columnar form. Such a column necessarily produces a chimney-like draft, since the air composing it is warmer and therefore lighter than the adjacent air on the outside. Hence, however established, such a column of warm air will maintain its integrity, or, rather, perpetuate itself, so long as the air that

is forced into it from the base is warm and light. In this connection it apparently cannot be too strongly emphasized that the ascending air is not " drawn " up any more than air is " drawn " up a chimney. In each case the weight of the column of warm air is less than the weight of an adjacent equal column of cooler air, and the static unbalance is compensated kinetically; that is, air is forced up the column in question, as up a chimney, because of, and in proportion to, the difference between its density and that of the cooler descending air outside. Even a *vena contracta,* or restricted section, is formed in the column a short distance—one to five metres often—above the surface, as, and for the same reason that, such a restriction occurs in a jet of any fluid shortly after its issuance from an ordinary orifice.

The incoming air is almost certain to be directed to one side of the centre of the rising column, and, as the angular momentum thus established tends to remain constant, a correspondingly vigorous whirl is developed as the place of ascent is approached that gathers up such loose materials as dust, straws, leaves, etc. Furthermore, this rotation, whether clockwise or the reverse, perpetuates itself, though the details of how it does so are, perhaps, not fully understood. Pictet,[80] for instance, reports observing a dust whirl near Cairo, Egypt, that began on a small sand mound, remained stationary for nearly two hours, then, in response to a gentle breeze, wandered away, but maintained its sharply defined outlines and great altitude until lost in the distance, more than three hours later, or about five and a half hours after its inception.

The diameters of these whirls (seldom more than a few metres near the ground) are too small for the direction of their gyration to be greatly influenced by the rotation of the earth. Hence nearly as many turn in one sense as in the other. They have even been reported to reverse. but it is probable that the apparent changes were only optical illusions similar to that which causes the cup anemometer to seem to reverse its rotation.

The height to which the whirling column rises (that is, the distance between the base of the column and its mushroom capital), the violence of the whirl, and, in some measure, even its duration. all depend upon the amount of surface heating and the

[80] Hildebrandsson et Teisserenc de Bort. " Les Bases de la Météorologie dynamique," vol. 2, pp. 286–288.

extent to which the lower temperature gradient has been made greater than the adiabatic. When this heating is slight, only those small and gentle dust whirls with which all are familiar can be generated and sustained. When, however, the heating is pronounced, as it often is over level, desert regions, the whirl may assume almost tornadic violence. But, however violent, this sort of storm is never a tornado; it originates near the surface and is sustained by the supply of warm air from below, while the true tornado is generated and developed by conditions that occur at the cloud level.

When dust whirls pass on to regions where the surface air is not so strongly heated—over bodies of water, for instance, or green vegetation—they no longer are fed with air relatively so light and, as a rule, quickly come to rest. Naturally, too, their frequency varies with topography, ground covering, latitude, season, and time of day. Thus they are most frequent of afternoons and least of early mornings, most likely to occur during summer and fall and least during winter and spring; most generally found in tropical and semitropical countries and least in regions of high latitude; more numerous over barren surfaces than over water and succulent vegetation; and, finally, more favored by level regions than by irregular and broken ground.

Cumulus Convection.

An interesting and important case of rapid vertical convection resulting from the local application of heat and consequent establishment of strong horizontal temperature contrasts is that displayed by the turbulence of the cumulus cloud. That strong vertical and irregular movements of the air often occur in large cumulus clouds is known from the rapid boiling and rolling motions of their upper portions, from the descriptions of aeronauts who have been caught up in the heart of a thunderstorm, and from the formation of hail within them. This latter phenomenon, the formation of hail, implies very definitely an uprush of at least 8 to 10 metres per second (20 miles per hour). This, in turn, on the theory of the chimney-like action of a warm central column of air, would demand, even neglecting viscosity, the equivalent of a column 1500 metres high and 1° C. warmer throughout than the surrounding atmosphere at the same level.

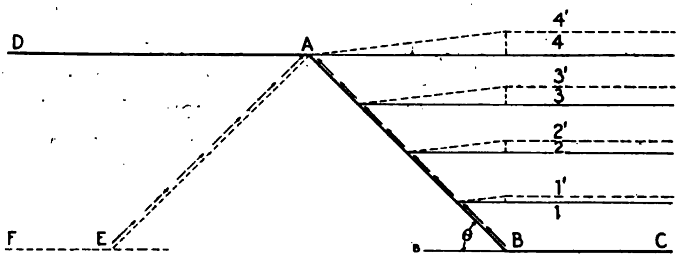
The chief cause of the horizontal temperature contrasts necessary to this rapid uprush obviously is the difference between the current temperature gradient of the surrounding atmosphere and the adiabatic gradient of the saturated air within the cloud itself, full details of which will be given in the chapter on the thunderstorm.

The common level, therefore, of a considerable number of detached but neighboring cumuli is one of rather vigorous convection—up within the clouds and down between them—however free from turmoil the atmosphere may be at other altitudes.

Valley Breeze.

During warm, clear days, when there is but little or no general wind, a gentle breeze, known as the valley breeze, commonly

FIG. 30.



Effect of mountain on levels of isobars.

blows up the sides of mountains. The strength of this breeze varies greatly, owing to the size of the mountain, the material of its covering, and the conditions of its surroundings. The cause of some of these variations can be understood by reference to Fig. 30.

Case 1.—Mountain Slope Connecting Wide Plateaus of Different Level.—Let *AD* be the upper plateau, *BC* the lower, and *AB* the slope connecting them. Further, let the insolation be vertical, or the sun directly overhead. If, now, the slope *AB* is barren, or nearly so, it will become strongly heated and the adjacent air correspondingly expanded. As this lighter air is buoyed up by the adjacent denser atmosphere there results a draft in toward the side of the mountain. But this draft is all along the mountain slope from top to bottom, and thus in a measure the warm air is held in against the mountain side. Hence, the updraft along the side of the mountain is analogous to that in a chimney.

In addition to this obvious chimney effect of the mountain, there is, in the case under consideration, another source of upward winds that causes them to blow up along even shaded and cool ravines; unless snow-filled and very cold, in which circumstance local density is the controlling factor and air drainage, or a downward flow of the air, usually prevails, as will be more fully explained later. The early morning isobaric levels, 1, 2, 3, etc., are raised by heating during the day to higher levels, 1′, 2′, 3′, etc. From any point directly above the foot of the mountain the amount of this expansion obviously drops off, as indicated in the figure, as the side of the mountain is approached. Hence a pressure gradient is established toward the mountain side and plateau beyond. If both plateaus are broad, a perceptible breeze of several hours' duration up the mountain slope and onto the higher plateau may be induced in this manner. Such winds grow stronger as the top of the mountain is approached. At night, after sufficient cooling has taken place, the winds reverse.

To form some idea of the possible magnitude of these effects, let the difference in level between *AD* and *BC* be 1.6 kilometres (one mile), and let the air between these levels be warmed on the average 5.5° C. (10° F.), then the increase of barometric pressure at *A* will be about 2.5 mm. (0.1 inch), with proportionate increases along the side of the mountain—quantities quite sufficient to produce a decided breeze.

Similar overflow winds occur also on the slopes of the isolated mountain, *BAE*, whenever the air on the opposite sides is unequally heated. Thus the landward side of a coast mountain, for instance, on a still warm day should have a breeze blowing up it and out to sea.

Case 2.—Isolated Mountain in the Midst of a Uniformly Heated Plain.—Here, too, the sides of the mountain are heated and corresponding upward currents induced. There also is expansion of the air over the adjacent plains and a tendency to establish pressure gradients towards the mountain, as in the case just discussed. But, however great this expansion, the gradients thus produced never cause, in the present case, more than a negligible wind. To make this statement obvious: Let the ridge *A* of the mountain be 1.6 kilometres (one mile) above the plains *EF* and *BC*; let the width of the base, *EB*, be 3.2 kilometres (two miles), and let the air be heated the same over one plain as over

the other. Let the temperature increase of the air during the day be 5.5° C. (10° F.), or, suppose, 1 part in 50 of the absolute temperature. Under these conditions the compensating flow of air from the two sides must amount jointly to 1 part in 50 of the volume of the mountain; that is, the horizontal flow of the air from either side of the given mountain must average, from top to bottom, 1 part in 50 of 0.8 kilometre (0.5 mile). If, further, this is extended over a period of ten hours, as it may well be, the average velocity will amount to only about 1.5 metres (5 feet) per hour—certainly an imperceptible breeze.

Clearly, then, the breezes that ascend mountain sides on still clear days have two causes: (*a*) a chimney or draft effect due to surface heating—always present—and (*b*) a pressure gradient effect due to expansion of the air over an adjacent plain or valley—present only when this expansion is unequal on the opposite sides of the mountain, or when the base levels are decidedly unequal on the opposite sides.

Sea Breeze.

Whenever a strongly heated region adjoins one whose surface is less heated, a local circulation from the one to the other obtains, unless prevented by winds of a general circulation. Thus along the seashore, beside lakes and even at the edge of favorably situated forests, a breeze (sea breeze, lake breeze, and forest breeze, respectively) of greater or less strength sets in during dry summer forenoons, after the land surface has become sufficiently warmed to establish decided convection.

Since the sea breeze obviously ceases at that level where the barometric pressure is the same above the land that it is above the water, and since the change of pressure with change of altitude is a function of temperature, it follows that its depth, never great, may easily be computed by replacing certain general terms of a suitable equation by observed temperature and pressure data. To develop such an equation:

Let ρ be the density of the air, then

$$-dp = \rho\, g\, dh,$$

in which $-dp$ is the small decrease in pressure corresponding to the small increase, dh, in height, and g the gravitational acceleration.

From the general equation,

$$pV = RT = \frac{p}{\rho},$$

in which p is the pressure, V the specific volume, R the gas constant, and T the absolute temperature, it follows that

$$\rho = \frac{p}{RT}.$$

Therefore,

$$-\frac{dp}{p} = \frac{g\,dh}{RT}.$$

Hence, integrating from p_0, corresponding to $h = 0$, to p, corresponding to $h = h$, if T is independent of h, which, as a first approximation, its average value may be assumed to be,

$$\log_e \frac{p}{p_0} = -\frac{gh}{RT}.$$

The value of the first half of this equation obviously remains the same when the corresponding, but more convenient, barometric readings b and B are substituted for p and p_0, respectively. Hence, also,

$$\log_e \frac{b}{B} = -\frac{gh}{RT}.$$

But the top of the sea breeze clearly is where there is no horizontal difference of pressure, or where $db = 0$, when h is constant. Hence, on differentiating this equation, keeping h constant, it is seen that

$$db = dB\left(\frac{b}{B}\right) + \frac{bgh}{RT^2}dT,$$

and that the depth of the breeze, h, is given by the equation,

$$h = -\frac{dB}{dT}\,\frac{RT^2}{gB}.$$

Consider a typical case: Let the sea-level reading of the barometer on land, less that at sea, or $dB = 0.5$ mm.; let the temperature over the land exceed that over the sea by $dT = 5°$ C.; let $T = 300°$ A.; let B, the sea-level barometer reading at sea, be 760 mm. R for dry air $= 2.871 \times 10^6$. Then the depth or thickness, h, of the sea breeze is given by the equation,

$$h = \frac{0.5}{5}\,\frac{2.871 \times 10^{10} \times 9}{981 \times 760} = 34{,}657 \text{ cm.} = 347 \text{ metres, approximately.}$$

The sea breeze, usually, as just explained, not more than 500 metres deep, starts on the water, seldom attains a greater velocity than 4.5 metres per second (10 miles per hour: the greater the temperature contrast the stronger the breeze), and extends inland, growing feebler and warmer, to a distance of only 16 to 40 kilometres (10 to 25 miles).

In a very important sense the circulation involved in this and all other local air convections is incomplete, since in such cases the path along which a given particle of the atmosphere flows is open and not closed. That is, the air that goes up over

the heated land, in the case of the sea breeze, for instance, does not itself return by way of the water; it simply spreads out at the top of its ascent where its new temperature (above assumption of constancy of temperature not strictly correct), acquired by adiabatic expansion, is the same as that of the adjacent atmosphere, while the return branch, or down-flowing portion of the circulation, is broad and gentle. Hence the surface air always flows from the cooler toward the warmer mass. By day the sea or lake breeze is on shore, because the soil gets warmer than the evaporating water, and the similar forest breeze, always feeble, away from the woods.

WINDS DUE TO COOLING.

Land Breeze.

By night, when the direction of the horizontal temperature gradient is the reverse of that during the day (that is, when the water surface is relatively warm and the soil cool, because of its rapid radiation), the direction of the surface wind is also reversed or offshore. This is the well-known land breeze.

Besides being reversed in direction and occurring at night instead of by day, the land breeze further differs from the sea breeze in usually being very much the weaker of the two, even though aided by the gravity flow of the cooler surface stratum of air. This is because *a*, the temperature contrast between land and water, is less by night than by day, and *b*, the surface friction over land which retards the land breeze, is greater than the water friction that affects the sea breeze. Hence, while the latter, as above stated, reaches 16 to 40 kilometres (10 to 25 miles) inland, the former seldom extends more than 8 to 10 kilometres (5 to 6 miles) to sea.

The depth of the land breeze, usually less than that of the sea breeze, obviously may be computed in precisely the same manner as the latter.

Mountain Breeze, or Gravity Wind.

During clear nights when there is but little or no general wind, there usually is a flow of the surface air, commonly most pronounced in ravines, down the sides and along the basin of every valley. At most places this movement is gentle to very slow, but in those exceptional cases where the valley is long and rather steep, especially if covered with snow and free from forest, and still better if fed by a gently-sloping plateau, the down-flowing

air current may attain the velocity of a gale and become a veritable aerial torrent. This drainage flow is known indifferently as the mountain breeze, or mountain wind; also canyon wind, katabatic wind, and gravity wind.

For simplicity let there be no general wind; let the cross-valley profile be the arc of a circle, and let the covering of the walls be everywhere the same. But even thus simplified the problem of air drainage still requires the consideration of temperature changes of the free air, of the surface air, and of the surface itself.

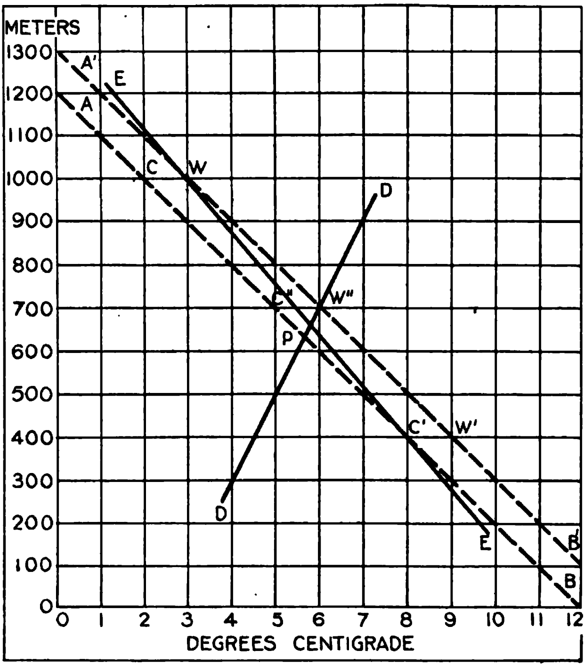
Free Air.—Largely, perhaps almost wholly, because of the dust and vapor always present, the lower atmosphere emits and absorbs radiation abundantly through much the greater portion of the spectrum. But during clear nights the loss, in the lower air at least, usually, if not always, is greater than the gain. However, even on such nights when radiation losses are greatest, the lower air, neglecting surface influences, cools too slowly and, on any given level, too nearly uniformly to produce more than very scattering and very feeble convection currents.

Suppose, though, that a limited mass of free air is cooled to a temperature below that of the adjacent atmosphere at the same level, as must happen at night within a small isolated cloud. What will be the result?

This problem, interesting within itself and essential to the present discussion, can easily be solved graphically. To this end let *AB* and *A'B'*, Fig. 31, be two adiabatic gradients of the free air, indicating a temperature decrease of 1° C. for each 100 metres increase in elevation (the customary approximate value), and let *EE* be any actual temperature gradient different from the adiabatic—in this case, for simplicity, assumed to be 1° C. per 120 metres change of elevation. If, under the given conditions, a limited mass of air at an elevation of 1000 metres, say, be cooled 1° C., or its position in Fig. 31 be shifted from *W* to *C*, it will immediately become denser than the neighboring air of the same level and therefore sink. As it sinks, if there is no interchange of heat by conduction, it will warm up adiabatically and finally come to equilibrium where the adiabatic gradient, *AB*, intersects the actual gradient *EE*, or at *C'*, where the falling air will have reached, through compression, the same temperature as the then adjacent atmosphere. That is, under the above gradient, a limited mass of air at any sufficient elevation cooled 1° C. will

drop 600 metres, and in so doing increase its temperature by 6° C., or become 5° C. warmer than it originally was before the initial cooling. Similarly, if the original limited mass of air, with elevation and temperature indicated by *C′*, say, be warmed 1° C., it will be forced to assume a new equilibrium level and temperature indicated by *W*. When the vertical temperature gradient is "reversed"—temperature increasing with elevation as indicated by *DD* of the figure—the final gain in temperature is

FIG. 31.



Assumed temperature gradients.

less than the initial loss. If, for example, the initial cooling is from *W″* to *C″*, *P* will be the point of equilibrium and the final temperature will be less than the initial. In short, when air whose temperature decreases with elevation is warmed, it proceeds at once to get colder than it was at first, as is evidenced by every cumulus cloud; and when cooled it quickly gets warmer than it originally was. If this gradient is reversed, that is, if the temperature increases with elevation, there still will be dynamical heating and cooling as before, but to an extent less than the initial

cooling and heating, respectively. An initial temperature change different from the one just assumed, 1° C., would, of course, produce, under the same temperature gradient, correspondingly different alterations in level and final warming or cooling, provided always that the process is wholly adiabatic and that it takes place well above the surface of the earth. As a matter of fact, there necessarily is some interchange of heat between the moving limited mass of air and the surrounding stationary atmosphere. In so far, however, as the falling mass of air *gains* heat by conduction or radiation its equilibrium is reached at a correspondingly higher *level* and *colder* temperature. Similarly, so far as the rising mass of air loses heat by conduction or radiation, its equilibrium is reached at a correspondingly *lower* level and *warmer* temperature.

It must be clearly understood that all the above reasoning applies only to free air. When the surface air of a level region or valley basin of negligible slope loses heat it gets *colder* and not warmer, simply because it cannot acquire dynamical heating by falling to a lower level—it is already at the bottom. It must also be noted that the paradoxical results under discussion, "cooling by warming," as Shaw[31] has called it, and its counterpart, warming by cooling, apply to isolated masses of air. When the whole of each layer of air of the same level undergoes the same temperature change, and when this change is but slightly different from that of the next higher or lower level, as obviously is the case over flat regions on still, clear nights, there can be but little local convection, and, therefore, but little dynamical heating. There still will be night-cooling, however, of the free air through at least the lower kilometre or more, but it will be distributed approximately uniformly and nowhere localized in that manner which, as just explained, is essential to marked convection.

Valley Surface.—Since all portions of the valley surface are equally exposed, or nearly so, to the sky, and since the covering is uniform, it follows that on clear nights each portion must lose heat by radiation at a rate that varies closely only with its temperature. At the same time the surface also acquires heat partly by absorption and partly by conduction. But during still, clear nights the net loss of heat by the valley surface, whatever its nature, is more rapid than is the net loss of heat by the slowly-radiating free air. Indeed, it may even be assumed, as a rough approximation, that the atmosphere neither emits nor absorbs

[31] Forecasting the Weather, p. 149, 1911.

radiation; that only the surface covering is effective in these respects, and hence, that all temperature changes of the valley air are results of heat conduction to or from the valley surface and of dynamical heating or cooling.

Surface Air.—Any change in the temperature of the surface is communicated in greater or less measure by conduction, radiation, diffusion, and convection to all the neighboring atmosphere. But as chilled air tends to fall vertically, appreciable cooling in this case extends through only a relatively thin surface layer, as often is obvious to one on crossing a ravine.

Consider then a thin layer of air close to the surface of one of the valley walls, and follow its movements and temperature changes on a still, clear night. As the surface cools, which it does everywhere, the temperature of the adjacent air is also reduced and its density thereby correspondingly increased. Hence as soon as this cooling has proceeded to a lower temperature than that of the free atmosphere at the same elevation the surface air on the valley walls begins to flow to lower levels; overrunning, of course, any pockets of colder air that may be in its path. The turbulence resulting from this flow continuously, and at all places along its course, causes more or less of the initially-chilled air to be separated from the surface, there abandoned, temporarily or permanently, and underrun by other air. Clearly, too, the amount of turbulence and consequent depth of the affected layer, or amount of interchange between free air and surface air, vary owing to velocity of flow, slope of surface, nature of covering, etc.

If the weight of a unit volume of this cooled air is w grams more than that of an equal volume of free air at the same level its contribution, f, to the total force producing, or tending to produce, drainage, or flow down the sides of the valley, is given by the equation,

$$f = g\,w \sin \theta,$$

in which g is the local acceleration of gravity and θ the angle of slope at the particular place along the valley wall where the small quantity of air under consideration happens to be. Sin θ varies, of course, from a minimum at the bottom of the valley, where it is o if the valley happens to be level, to a maximum where the sides are steepest. The value of the other variable, w, depends on the difference between the temperature of the cooled air in question and that of the free atmosphere at the same level. A

steady state, always more or less closely approached, obviously would give

$$g w \sin \theta = K V^n$$

in which K is the coefficient of whatever opposition (friction, etc.) is encountered per unit volume due to the drainage velocity V down the slope, and n a numerical exponent.

Since w is proportional to δT, or difference in temperature between the surface air and free air at the same level, it follows that if δT, and therefore w, remains constant—that is, if the rate of temperature loss by the descending air through conduction to the surface is equal to the rate of its temperature gain over the free air of identical levels, owing to compression during descent, then

$$\frac{1}{mS_p} \frac{de}{dt} = (A - \frac{dT}{dh}) V \sin \theta$$

where de/dt is the rate of loss of heat by the mass m of air to the surface at any given place, S_p the specific heat of air at constant pressure, A the adiabatic gradient, dT/dh the actual vertical temperature gradient in the free air at the given level, and V the velocity in metres per second of the flow down the slope at the same level.

Clearly, then, V must increase with increase of de/dt. Also V must vary with dT/dh, but always be positive (down hill) so long as dT/dh is less than A, which it nearly always is except near the surface under strong insolation. If the temperature gradient of the free air should be super-adiabatic (dT/dh greater than A) V would become negative, or the surface air would need to flow uphill to maintain δT constant. Actually, however, the air would flow down and not up the cooling surface. That is, if the free air were in this unstable condition δT, as applied to any given mass of surface air, instead of tending to remain constant, would rapidly increase.

The more rapid the loss of heat by the surface the more rapid also the loss of heat by the adjacent air, and the swifter its flow, if the slope is sufficient; but, on the other hand, the swifter the flow the more rapid the dynamical gain of heat, and also the greater the retarding effect of surface friction. Hence an automatic adjustment between free-air gradient, velocity of flow, and rate of loss of heat to the radiating surface, is always in operation. Furthermore, as this automatic adjustment prevents the air next the surface from becoming greatly colder than the free air at the same level, and as the latter cools only slowly,

it follows that the temperature along the valley walls cannot decrease at all rapidly except below the inversion level (the nearer the bottom the more pronounced), as presently explained.

Initially, when the temperature of the free air everywhere over the valley decreases with elevation, the speed of the surface air down at least the steepest portions of the cooling walls is quite certain to be sufficient to make its dynamical gain of heat exceed its conduction loss and therefore to cause its temperature to increase with descent. As the bottom of the valley is approached, however, the rate of vertical descent and the consequent dynamical heating become less and less, and finally cease altogether, except in so far as there is drainage along the valley. At and near the bottom, then, where the dynamical heating is absent, or small, the temperature of the surface and the adjacent air necessarily decrease more or less rapidly. In a short while, therefore, the valley basin begins to fill with a river of cold air. The first incoming air doubtless overflows the original bottom layer, but in so doing gets separated from the cooling surface, and in position itself to be underrun by such other air as has cooled to a lower temperature, and this in turn by still colder air, and so on. In this way a temperature inversion is established in the valley. Below the inversion level (level of maximum temperature) the surface flow, though still active, is so decreased in rate of descent that the contact cooling exceeds the dynamical heating. The surface air could not otherwise here underrun the free air since the temperature of the latter decreases, as explained, with decrease of altitude below the inversion level. Above the inversion level the temperature of the down-flowing air, since it cannot anywhere greatly differ from that of the free air at the same elevation, necessarily increases, in general, with descent.

A valley wall produces local cooling up in the atmosphere, guides the resulting drainage of cold air and more or less controls its velocity, while the temperature gradient of the free air (modified over the valley bottom by the inflow from the sides) limits and largely determines the ratio of velocity of flow to rate of loss of heat (not cooling) by the surface air. In the case, therefore, of well-defined valleys it appears that there must be on either side a belt at substantially the same elevation as the then inversion level along which during still, clear nights the dynamical heating and the contact cooling of the descending surface air are exactly equal. Above this level descending air grows contin-

uously warmer, below it continuously cooler. Later in the night the inversion level attains a practically stationary elevation, and then intersects the valley walls in what are known as the "thermal belts."

In this connection it may be interesting to note that there are many practices in recognition of the above facts of air drainage. Thus, for instance, the mountain camper tents above and not below his night fire, so as to avoid the smoke; the Swiss peasant builds his cottage on a knoll to keep above the valley flood of cold air; and the orchardist seeks the "thermal belt" to escape killing frosts.

Mountain Convections.

Vertical convections on the sides of mountains due to temperature contrasts of different origin from those already mentioned are also well known, and, though rarely if ever producing more than a gentle breeze, are worth mentioning. Thus, a considerable shower during the afternoon of a warm summer day, for instance, may leave the atmosphere, level for level, distinctly cooler than the side of a neighboring mountain. Consequently the air will then flow up the adjacent slopes and occasionally carry with it masses of detached fog, or "steam," that gradually merge into a long billow-like crest cloud. Similarly, as the cooler air on the clearing side of a cyclone invades a mountainous region, the relatively warm slopes often, when the general winds are light, induce rising currents. And as these currents also frequently are laden with patches of "steam" cloud the familiar mountain saying: "When the fog rises the rain is over," is well justified.

On the other hand, the air on the stormy side of a cyclone is nearly always warmer during winter, and also frequently warmer during the other seasons, than are the mountains; and therefore on these occasions, as it passes over them, downward currents are induced along their slopes, a circumstance that equally justifies this other common saying of the mountain dweller: "While the fog descends it will continue to rain."

Glacier Winds.

It is well known that a draft of cold air often is found blowing out from a cave-like opening in the lower end of a glacier, hence called glacier wind. Similar winds, blowing out during summer and in during winter, occasionally are found at the mouths of caves, called blowing caverns, on the sides of hills or mountains.

In the volcanic mountains of Japan such places are numerous and extensively used for cold storage.[32] In each case the explanation of the phenomenon is the same and obvious. The cavity extends quite through the glacier, or earth, as the case may be, from its lower to its higher openings. Let this difference in elevation be 250 metres; let the average temperature of the air inside the cavity be 0° C., and of that outside at the same level 15° C. The density of the inner air will be to that of the outer approximately as 19 to 18, and the pressure head, producing an inverse chimney effect, about 14 metres. Neglecting friction—usually, however, an extremely important factor—this would give the exit air a computed velocity of about 16.6 metres per second (37.1 miles per hour)—a very appreciable gale. The velocity or strength of this wind, other things being equal, varies as the square root of the difference in level between the lower and upper openings.

In the case of a glacier, drainage obviously obtains in substantially the same manner, whether the air passes in a concentrated stream through a cavity or along a crevice within the ice, or merely flows in a broad sheet over the top surface. Clearly, too, the same sort of aerial cascades (exaggerated mountain winds) must occur, especially during summer nights, over any banks of snow that may exist in the upper and steeper reaches of canyons and mountain valleys. Such winds necessarily are shallow and therefore, when swift, a treacherous source of danger to the landing aviator.

The Bora.

From the above explanation and examples of aerial drainage it is obvious that similar winds must often blow down steep slopes that separate high, snow-covered plateaus or mountain ranges from adjacent bodies of relatively warm water. Thus when an anticyclone covers such a region during winter the surface air becomes very cold and correspondingly dense until, unless otherwise dissipated, it overflows restraining ridges or drains away through passes and gaps. Clearly, too, this flow must be most frequent and strongest during the early morning, since that is the coldest time of the day, and least frequent of mid-afternoons. In many instances during anticyclonic weather the air as it leaves the snow fields is so cold that in spite of dynamical heating it even reaches the sea at freezing temperatures and very dry. When, however, the drainage is amplified, if not even started, by

[32] S. Suzuki and T. Sone, *Tohoku Univ. Sci. Reports*, 3, pp. 101–111.

the pressure gradients (to which the final velocity bears no special relation) due to a properly situated "low," it usually is associated with a counter cyclonic current above and therefore accompanied by rain, sleet, or snow.

Probably the best known of all these violent fallwinds is the *bora* of the northeast Adriatic, especially at Trieste, Fiume, and Zengg. The boras of these places, however, are not from the north, as the name implies, but rather from the northeast and east-northeast.

Another excellent example of this kind of wind occurs at Novorossisk, a Russian port on the northeast coast of the Black Sea, where it blows down from a nearby pass in the mountains, occasionally with destructive violence. Probably, also, the brief but sudden and violent williwaws of steep, high latitude coasts have a similar origin.

Mistral.

Another instance of convection due essentially to cooling is the well-known mistral, or dry, cold northerly wind of the Rhône Valley. Here the more or less persistent winter low over the warm waters of the Gulf of Lyons to the south and the frequent highs over the snow-covered plateaus of southeastern France to the north often coöperate in such manner as to produce extensive air drainage down the lower Rhône Valley. In general the cause of the mistral and its actions are the same as those of the bora. It is less violent, however—its path less steep, and therefore itself not so distinctly an aerial cataract. Similar winds occur, of course, under like circumstances in other parts of the world. but the mistral is the best known of its class.

Norwegian Fallwinds.

An extensive fallwind, which, because of its importance and the fact that it is more or less unique, deserves especial mention, frequently occurs during winter along the coast of Norway. Sandström,[32a] in one of his interesting atmospheric studies, describes it as follows: "In winter, as one steams along the northwest coast of Norway, there is frequent opportunity to observe a peculiar meteorological phenomenon. Fine weather prevails over a narrow strip along the coast, while a heavy bank of cloud is visible out to seaward. Of course, coastwise traffic is greatly favored by this fine-weather strip and takes full advantage of it.

[32a] *Mount Weather Bulletin*, 5, p. 129, 1912.

Throughout this zone of fine weather prevails a cuttingly cold wind so strong that one can scarce stand against it when on deck. The maximum velocity of this wind is attained near shore, where the water is whipped up into whirls and miniature waterspouts. Evidently the wind here plunges down upon the water from above, and with great force.

"Upon leaving the steamer and travelling inland up the mountain slopes on skis, strong head winds oppose progress. This easterly wind is still very strong on the great divide of the Scandinavian Peninsula. But observations of the cloud caps on the highest peaks of the range show that a westerly wind is blowing at those great altitudes. It is clear that a lively interchange of air between the North Atlantic Ocean and the continent is taking place above the Scandinavian highlands. This exchange takes place along either side of a glide surface whose altitude above the ground at the divide may be estimated at about 1000 metres. In fact, at the kite station Vassijaur it proved almost impossible to raise the kites above that level, evidently because they there encountered a glide surface through which they cannot pass, since the wind has opposite directions on the two sides of this surface, and therefore calm must prevail at the glide surface itself. The altitude of this glide surface decreases to the Atlantic Ocean. The air below this surface flows toward the west, and above the surface it flows toward the east."

Continental Fallwinds.

From the above discussion of winds that result from surface cooling it is obvious that they are of very general occurrence, especially during cold weather and down the valleys and slopes of high, snow-covered regions. Hence one would expect drainage winds to obtain to a greater or less extent during winter over the middle and high latitude regions of every continent. Where the elevation and slope are slight, however, as they are, with but minor exceptions, over all North America east of the Rocky Mountains, over Russia, and over Siberia, except the eastern portion, this drainage necessarily must be comparatively sluggish.

On the other hand, there are two regions of continental extent—Greenland, with an area of about 827,000 square miles, and Antarctica, with an area of, roughly, 4,600,000 square miles—that are ideally located for, and perfectly adapted to, the production of strong and almost continuous fallwinds.

Greenland, as is well known, is continuously covered with an enormous ice cap that rises to a gently rounded plateau of, roughly, from 2000 to 3000 metres (7000 to 10,000 feet) elevation. This plateau, whose crest runs approximately north and south, has been crossed several times—six in all—at as many different places, and in each case nearly constant down-slope or drainage winds of greater or less strength were experienced.

Throughout its great area, therefore, Greenland is a region of almost perpetual aerial cascades and cataracts. The continuous refrigerative influence of its enormous ice cap, covering an area 18 times that of the state of Pennsylvania and rising at places to an elevation of over three kilometres (two miles), not only controls the direction and velocity of nearly all local winds, but obviously must be of decided influence on the general circulation of the middle and higher latitudes of the whole northern hemisphere—an important circumstance that will be taken up later.

Antarctica, according to the reports of all its explorers, is quite as completely covered with ice as is Greenland, and it also rises, more or less dome-like, to fully as great altitudes. Hence it would seem that its general effect on the movement of the air must be very similar to that of its great counterpart in the northern hemisphere—an inference now fully borne out by the many accounts and records of those who have skirted its coasts, crossed its plateaus, or wintered on its borders. Sir Douglas Mawson, for instance, who spent many months during 1912–13 at Adelie Land, latitude 67° S., on the edge of the continent almost directly south of Tasmania, reports an average wind velocity for an entire year, from the interior toward the sea, of more than 22.4 metres per second (50 miles per hour). "Day after day," he says, "the wind fluctuated between a gale and a hurricane." Velocities of 100 miles and over per hour occurred, and gusts of even much greater velocity occasionally were recorded. These measurements were made at the main station on the declivitous border that connects the inner ice plateau with the ocean. Back some distance inland, where the slope is gentle, the winds were less severe. At sea, also, these continental drainage winds decreased in intensity with increase of distance from shore, and ceased altogether at a distance of about 300 kilometres (187 miles), where the westerlies became effective. Obviously, therefore, this particular station was located in one of the windiest places in the world—in an aerial cataract where the cold

drainage air of the ice plateau rushes down a steep coastal slope to the sea.

Similar winds of varying intensity and irregular duration are reported all along the Antarctic border, from every inland station and from end to end of every exploring trail. Clearly, then, the winds of Antarctica, though due essentially to cooling, nevertheless, because of the extensive area they cover, belong also to those great circulations that are strongly influenced by earth rotation, and therefore constitute an important part of the general circulation of the atmosphere, under which head they will again be considered.

WINDS DUE TO SIMULTANEOUS ADJACENT LOCAL HEATING AND LOCAL COOLING.

Thunderstorm Winds.

Shortly, say twenty minutes or so, before the rain of a thunderstorm reaches a given locality the wind at that place, generally light, begins to die down to an approximate calm and to change its direction. At first it usually is from the south or southwest in the extra-tropical portion of the northern hemisphere; from the north or northwest in the corresponding portion of the southern, and in both more or less directly across the path of the storm itself. After the change, it blows for a few minutes rather gently, directly toward the nearest portion of the storm front, and finally, as the rain is almost at hand, abruptly and in rather violent gusts away from the storm and in the same direction, roughly, that it is travelling, a direction that usually differs appreciably from that of the original surface wind. Generally this violent gusty wind lasts through only the earlier portion of the disturbance, and then is gradually but rather quickly succeeded by a comparatively gentle wind, which, though following the storm at first, frequently, after an hour or so, blows in the same general direction as the original surface wind.

The chief cause of these and all other winds peculiar to the thunderstorm, except those within the cumulus cloud itself, is the juxtaposition of warm air immediately in front of the rain and a column or sheet of cold air through which the rain is falling. How this temperature distribution is established and what the results are will be explained later in the chapter on the thunderstorm.

CHAPTER VIII.

ATMOSPHERIC CIRCULATION (*continued*).

Winds Due to Widespread Heating and Cooling.

GENERAL REMARKS.

SINCE the atmosphere is a fluid whose viscosity is not only small but also well known, one might suppose, with respect to any given portion, that it would be quite as easy, through the equations of thermodynamics and hydrodynamics, to foretell its every movement and future position as by the equations of celestial mechanics to predict an eclipse or an occultation. But this is far from being the case, and for many reasons. Thus the irregularities of surface heating and surface friction, and the action of mountains, themselves irregular and broken, in deflecting winds, both horizontally and vertically, complicate the problem beyond exact solution. Besides, there are even discontinuities in the amount of atmosphere involved. Water vapor is added in large amounts by evaporation to the volume of circulating gases, mainly in the regions of "highs," while equal average quantities are withdrawn (not simultaneously) by precipitation, chiefly in the regions of "lows." Hence an exact mathematical solution of the problem of world-wide circulation does not seem possible. Nevertheless, many details of this circulation are clearly understood from physical considerations and admit of at least approximate analyses. Some of these details pertain equally to all the more general winds, and therefore a discussion of them will be given independently as a common introduction to the more extended accounts of certain types of atmospheric circulation that follow under the captions monsoons, hurricanes, trade winds, cyclones, etc.

Irregularities, Gusts or Puffs.—Though the wind at an altitude of 200 metres or more is comparatively steady, except in very rough or mountainous regions, near the surface of the earth any appreciable wind that may exist is always in a turmoil, owing to surface friction that checks the lower layer while the layers above tumble forward and down, and to numerous obstructing objects that block its course and introduce cross-currents. Hence

its direction constantly changes through many degrees, while the velocity irregularly but persistently fluctuates from one extreme to another. At one instant the actual velocity may be anything up to 50 per cent. or more greater than the average velocity, while a second or two later it may be fully 50 per cent. less than the average. The greater the average velocity the greater, roughly, in the same proportion, the absolute change, as illustrated by Fig. 32. So great, indeed, are these irregularities that places near the surface of the earth not more than 15 metres (50 feet) apart, though having the same average wind direction and velocity, commonly have different, often very different, simultaneous directions and velocities, and therefore, for the instant, radically different winds.

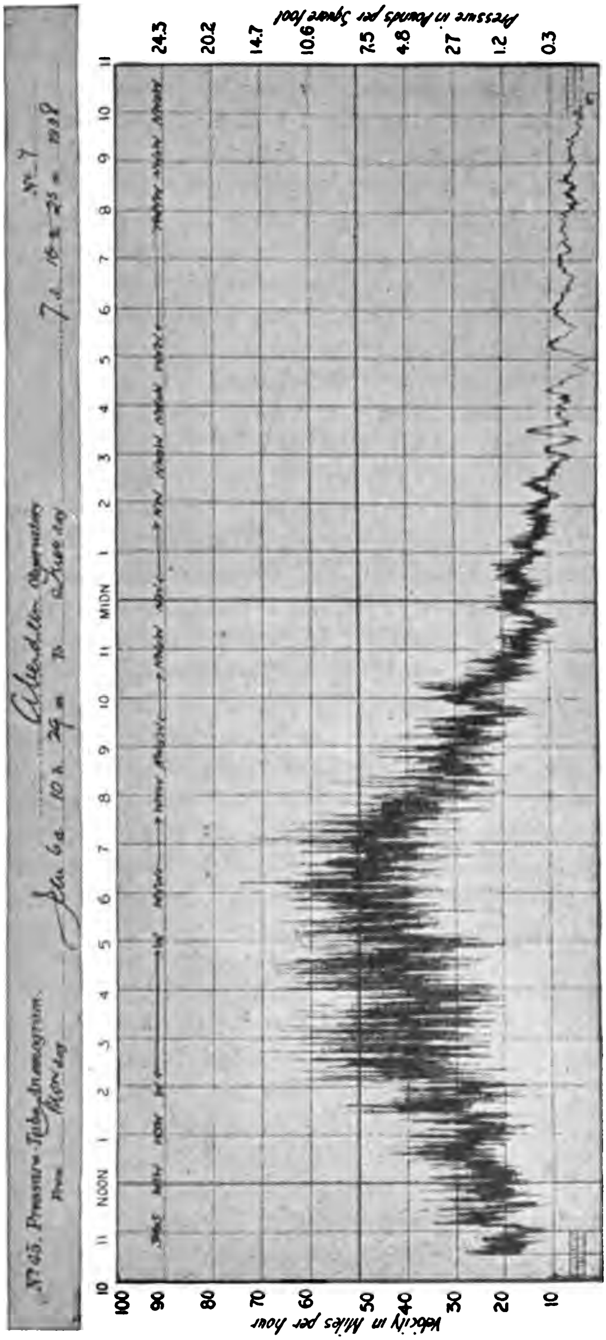
These facts are of great importance with reference to the wracking effect winds have on houses, bridges, and other structures. They also deeply concern the aviator when starting and landing. But, however valuable for many reasons an exact knowledge of surface air movements would be, it is obvious that they defy all formulæ, and that mathematical equations can no more predict the course and speed of a given portion of turbulent surface air than they can mark the path and fix the velocity of water in the swirls and eddies of a mountain torrent. Nevertheless, the average effect of a group of turbulent wind eddies, like the average effect of a large number of gas molecules, does profitably yield to mathematical discussion.[33]

Interzonal Drift.—Although the atmosphere moves mainly from west to east in middle latitudes and from east to west in equatorial regions, it nearly always has a north-south component that produces an interzonal drift. This is simply because the major temperature contrasts of the surface of the earth are between the equatorial and polar regions. Hence, whatever the local or secondary temperature contrasts, of which there are many, and whatever the deflecting barriers and other obstacles that prevent the latitudinal circulation from being free and rapid, it nevertheless must and does obtain to a greater or less extent.

Change of Velocity with Change of Latitude.—The velocity of the earth's surface at and near the equator, as a little calculation shows, is about 1675 kilometres (1040 miles) per hour from west to east, while, with reference to this surface, the veloc-

[33] Taylor, *Phil. Trans. Roy. Soc.*, A, 215, pp. 1–26, 1915, and Richardson, *Proc. Roy. Soc.*, A, 97, p. 354, 1920.

FIG. 32.



Irregularities in the surface wind velocity as indicated by a pressure-tube anemometer.

ity of the atmosphere from east to west in the same region is only a small fraction of this value. In reality, therefore, the atmosphere of equatorial regions is also moving from west to east with a great velocity, though not so great as that of the surface of the earth itself.

When this air moves to higher latitudes, as it does under the influence of the great temperature contrasts between equatorial and polar regions, it is obvious that its velocity with reference to the surface of the earth must change. It might seem, as many have assumed, that the linear velocity of the air about the earth's axis would remain constant, and that the wind would have a west to east component as soon as it had crossed that latitude on which the surface velocity is the same as the original west to east wind movement. This method, however, of considering the problem is theoretically incorrect, as has often been explained. A particle of the atmosphere, or any portion of it moving as a unit, is mainly, at times wholly, rotating around the axis of the earth. In considering latitude effect, therefore, or effect of distance from this axis on wind velocity, it is necessary, as Ferrel first insisted, to remember that the angular momentum $mr^2 \omega$ (in which m is the mass, r its distance from the axis of rotation, and ω the west-east angular velocity), and not the linear momentum mv (in which v is the west-east linear velocity), is a constant. In other words, the law of conservation of areas applies. Hence as a given mass of air reaches higher latitudes the absolute value of its east-west component tends, not to remain the same as is often supposed, but actually to become greater, in proportion to the secant of the latitude. If the trend of the air is to lower latitudes, its west-east velocity obviously tends to get slower at the same rate that, when going to higher latitudes, it tends to become faster.

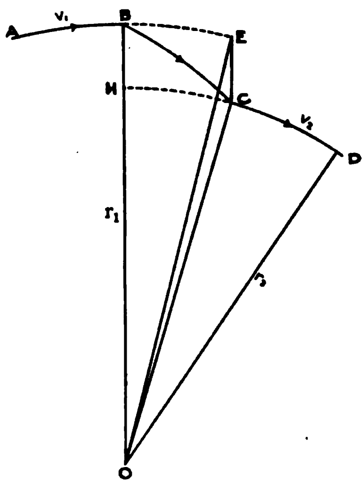
Law of Conservation of Areas.—The law of the conservation of areas as applied to the atmosphere is of sufficient importance to justify its brief demonstration.

Let O, Fig. 33, be the centre about which a mass m is moving with the linear velocity v_1 along the circular arc AB whose radius is r_1. Let m be constrained to follow its path by a central force; that is, a force directed towards O. For instance, let it move over a frictionless horizontal plane, and be kept in its orbit by the tension on a weightless string connecting it with

the centre O. At B let the tension on the string be so increased that the mass m will be drawn in the distance BH in the same time that, undisturbed, it would have reached E. During this time the path will be something like BC. If the tension again becomes constant with the value appropriate to the point C, the new orbital velocity will be v_2 along the arc CD of radius r_2.

By taking the time interval smaller and smaller the velocity along BH, however irregular, approaches uniformity as its limit, while BE and BC both approach straight lines. With the two component velocities along BE and BH respectively uniform it is obvious that the resultant velocity along BC must also be

FIG. 33.



Velocities along arcs of different radii

uniform. The problem of vectorial areas reduces, therefore, to finding the general relation of a radius vector to the component of the corresponding orbital velocity normal thereto. This relation may be shown as follows:

Let BC, Fig. 34, be a rectilinear section of the orbit, usually of infinitesimal length, along which the velocity is uniform, and let OB and OC be two radii. From B draw BE perpendicular to OC extended, and from C draw CA perpendicular to OB. Hence

$$\frac{AC}{BE} = \frac{v_1}{v_2}$$

in which v_1 and v_2 are the components of the uniform velocity along BC at right angles to OB and OC, respectively. But

from the similarity of the triangles OAC and OEB it follows that

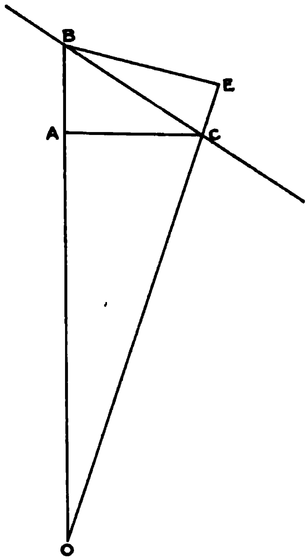
$$\frac{AC}{BE}=\frac{OC}{OB}$$

and, therefore, that

$$\frac{v_1}{v_2}=\frac{OC}{OB}\ .$$

That is, $rv=k$, a constant, in which r is the radius vector and v the component of the orbital velocity at right angles thereto.

FIG. 34.



Conservation of areas in a plane.

But, by elementary geometry,

$$rv=2a,$$

in which a is the rate at which area is covered by the movement of r. Hence whatever the portion of the orbit, the radius vector sweeps over equal areas in equal intervals of time, or whatever the points selected,

$$\frac{v_1}{v_2}=\frac{r_2}{r_1}$$

The kinetic energy E_1 of the mass m while on the arc AB, Fig. 33, is given by the equation,

$$E_1=\tfrac{1}{2}mv_1^2.$$

Similarly, its kinetic energy along CD is given by

$$E_2 = \tfrac{1}{2} m v_2^2.$$

These are unequal, the difference being

$$\frac{m}{2}(v_2^2 - v_1^2),$$

and it will be interesting to find the source of inequality.

At all parts of the orbit the tension f on the string is given by the equation,

$$f = \frac{mv^2}{r}$$

and the work, dw, done on shortening the radius by the small amount dr is expressed by the equation,

$$dw = -\frac{mv^2}{r} dr.$$

But since $rv = k$, $v = \frac{k}{r}$, and

$$dw = -\frac{m\,k^2}{r^3} dr.$$

Hence on shortening the string from r_1 to r_2 the work becomes,

$$W = m\,k^2 \int_{r_2}^{r_1} \frac{dr}{r^3} = \frac{m}{2}\left(\frac{k^2}{r_2^2} - \frac{k^2}{r_1^2}\right).$$

But $k^2 = v^2 r^2 = v_1^2 r_1^2 = v_2^2 r_2^2 =$ etc.

Hence $W = \frac{m}{2}(v_2^2 - v_1^2)$.

But this is identical with the value already found for the difference between the kinetic energies of m at the distances r_1 and r_2 from O. That is, this difference is equal and due to the work done on m by the tension on the string while decreasing the radius from r_1 to r_2.

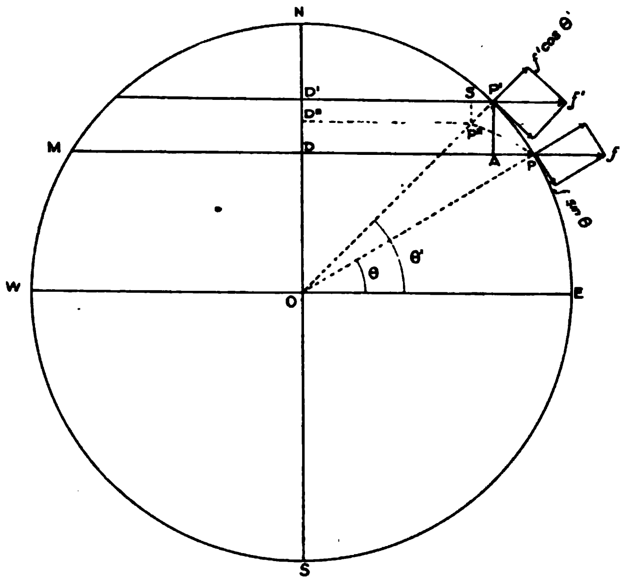
Consider now the velocity of a quantity of air or other mass moving as a unit frictionlessly over the surface of the earth. Let the mass m, regarded as a point (in the case of an extended body it is $\Sigma\ mvr$ that remains constant), be at P, Fig. 35, rotating around the axis NS on the small circle MP and held to the surface by gravity directed towards the centre O. If v is the linear velocity of m, it follows that the radially directed force f is expressed by the equation,

$$f = \frac{mv^2}{r} = \frac{mv^2}{DP}.$$

On forcing this mass to a higher latitude, to P', say, work is done against the horizontal component of f. But from the similarity of triangles it is obvious that at every point along the

arc PP', $f \sin \theta$, the force to overcome, is to f, the centrifugal force, as the rate of approach to the axis is to the rate of progress along the meridian. Thus the work done in going from P to P' is the same as that which would be done in shortening the radius DP to DA. But, as above explained, this work on m, or transfer of energy to it, must appear as so much additional kinetic energy, friction being excluded. Hence the orbital velocity v' at P' is to the orbital velocity v at P as DP is to $D'P'$. That is,

FIG. 35.



Conservation of areas on a sphere.

the conservation of areas holds likewise in this case, where the radius vector is the normal from the moving mass to the axis of rotation.

This same law holds also on the slightly flattened earth. To make this clear, let PP'' be a portion of a greatly flattened meridian, and let the mass m be taken from P to P' as before, and thence to P''. From P' to P'' the work is against the force f' $\cos \theta'$, and obviously equal to the work against f' over the distance $P'S$. Hence the orbital velocity of m at P'' is to its orbital velocity at P as DP is to $D''P''$.

Suppose a quantity of quiet air, air moving strictly with the surface of the earth, at, say, latitude 30 degrees, is forced to higher latitudes, as it actually is by pressure gradients due to temperature differences, what, according to the law of the conservation of areas, will be its final surface velocity at, say, latitude 60 degrees?

At latitude 30 degrees its orbital velocity, being the same as that of the surface, is, approximately,

$$v = \frac{2\pi 3957 \cos 30^\circ}{24} = 897.2 \frac{\text{miles}}{\text{hour}} = 401.1 \frac{\text{metres}}{\text{second}}.$$

At latitude 60 degrees its orbital velocity is, from the principle stated,

$$v' = v\frac{\cos 30^\circ}{\cos 60^\circ} = 1554 \frac{\text{miles}}{\text{hour}} = 694.7 \frac{\text{metres}}{\text{second}},$$

while at latitude 60 degrees the orbital velocity of the surface is

$$s = \frac{2\pi 3957 \cos 60^\circ}{24} = 518 \frac{\text{miles}}{\text{hour}} = 231.6 \frac{\text{metres}}{\text{second}}.$$

Hence the velocity of the transferred air in question with reference to the surface is

$$v' - s = 1036 \frac{\text{miles}}{\text{hour}} = 463.1 \frac{\text{metres}}{\text{second}}.$$

As a matter of fact, no such enormous velocities of the wind as the principle of the conservation of areas would lead one to expect in the higher latitudes are ever found, either at the surface or at other levels. This, however, does not argue against the applicability of the principle itself, but only shows that in the case of atmospheric circulation there are very effective damping or retarding influences in operation.

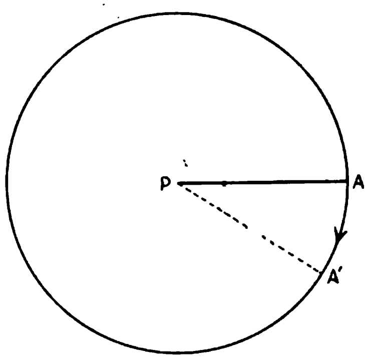
The resistance due to the viscosity of the atmosphere is one of these retarding influences, but its effect probably is very small. A larger effect doubtless comes from surface turbulence induced by trees, hills, and other irregularities. A still greater velocity control, probably so great that all others are nearly negligible in comparison, except near the surface, is vertical convection. This phenomenon leads to extensive interchange between lower and upper layers of the atmosphere, thus indirectly increasing the effect of surface friction probably several fold and tending to bring all the lower, vigorously convective, portion of the atmosphere to a common velocity. Because of these several means of control the actual wind velocity everywhere is different from that which, and at high latitudes much less than, it otherwise would be.

Not only is the velocity of the wind changed through change of latitude, but also the rate at which its direction with reference to the surface of the earth varies or tends to vary, as will appear from what follows.

Deflection Due to the Earth's Rotation.—The effects of the rotation of the earth on the direction of the wind are of extreme importance to the science of meteorology. It will be convenient, therefore, before going further, to consider how these important results are produced and to form some idea of their approximate magnitudes.

An exact discussion of this problem is somewhat tedious, but a very approximate solution is readily obtained. To this end, let

FIG. 36.



Deflection, at pole, due to the earth's rotation.

P, Fig. 36, be one pole of the earth—the south pole, say—assume the surface to be flat, which it very approximately is, at and near this point, and let a particle of air cross it in the direction *PA* with the uniform velocity v; let the earth rotate in the direction AA' with the angular velocity ω, and let the distance which the air particle under consideration has gone from *P* in the brief time dt be such that

$$PA = dr = v dt.$$

Let the meridian along which the particle started as it left *P* have the position PA' at the end of the time dt, or when the particle, keeping a constant direction in space, has arrived at *A*. Obviously the velocity with which the earth moves under the particle increases directly with the distance from *P*. But as this latter is directly proportional to the time dt since the particle left

P, v being a constant, it is clear that the distance, ds, travelled normally to the instantaneous meridian, may be expressed in terms of a constant acceleration, a, in the direction opposite to that of rotation. That is,

$$ds = \tfrac{1}{2} a(dt)^2.$$

Also

$$ds = dr\, \omega\, dt = v\, dt\, \omega\, dt = v\, \omega\, (dt)^2.$$

Hence,

$$\tfrac{1}{2} a\ (dt)^2 = v\, \omega\, (dt)^2,$$

or

$$a = 2\, \omega\, v.$$

Since a force is measured in terms of mass times acceleration, it follows that the west-east deflective force, f, that would keep a mass m, of atmosphere or anything else, next the pole in the same meridian, or the east-west force that, if the earth were still, would produce the given motion with reference to its surface, is very approximately given by the equation,

$$f = ma = 2\, m\, \omega\, v,$$

where v and ω have the values above assigned.

Let the moving particle under consideration be not at one of the poles, but at some other point, such as P, Fig. 37, at latitude ϕ. Resolve the angular velocity ω, about ON, into its components about the right-angled axes OP and OP'. These components, as is well known, are $\omega \sin \phi$ about OP and $\omega \cos \phi$ about OP'.

Now all points on and exceedingly close to the equator of a rotating sphere have sensibly the same velocity, hence the direction and velocity of horizontally moving particles at P are affected by the component of rotation about OP only, and not at all by the component about OP'.

That is, at N, as already explained,

$$a = 2\, \omega\, v, \text{ and } f = 2\, m\, \omega\, v,$$

while at latitude ϕ, where the angular rotation is $\omega \sin \phi$,

$$a = 2\, \omega\, v \sin \phi, \text{ and } f = 2\, m \omega v \sin \phi,$$

to the right (going forward with the particle) in the northern hemisphere, to the left in the southern.

From this simple equation, assuming it to be exact, which it very nearly is, it follows that the deflective force due to the rota-

10

tion of the earth acting on a quantity of moving air (that is, the force that, assuming the earth to be still, measures the existing tendency of wind to change its direction with reference to any fixed line on the adjacent surface) is:

a. Directly proportional to its mass.

b. Directly proportional to its horizontal velocity.

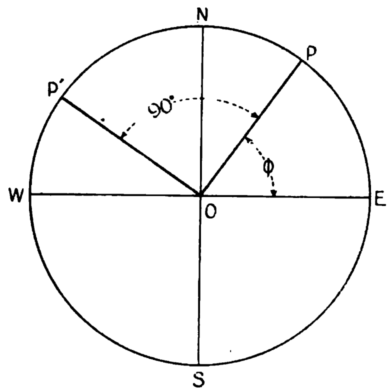
c. Directly proportional to the angular velocity of the earth's rotation.

d. Directly proportional to the sine of the latitude of its location.

e. Exactly the same whatever its horizontal direction.

f. Always at right angles to its instantaneous direction and

FIG. 37.



Deflection, away from pole, due to earth's rotation.

therefore wholly without influence on the velocity with reference to the surface.

g. Opposite to the direction of the earth's rotation. Hence to the right, or clockwise in the northern hemisphere; to the left, or counter-clockwise in the southern.

Since the deflection acceleration is all the time at right angles to the path, it follows that an object moving freely over the surface of the earth would describe an endless series of curves. At each point on this path the acceleration at right angles to it, measured with reference to the surface of the earth, is given, as explained, by the equation,

$$a = 2 \omega v \sin \phi,$$

in which the terms have the values assigned above. It is also

given by the well-known equation for acceleration along a radius of curvature. That is,

$$a = \frac{v^2}{r},$$

in which, since the force is tangent to the earth, $r = R \tan \alpha$ where R is the radius of the earth, and α the angle subtended at the centre of the earth by the radius of the path at the point under consideration. Hence, when the only deflective influence is that due to the earth's rotation,

$$r = \frac{v}{2\omega \sin\phi}, \text{ and } \tan\alpha = \frac{v}{2R\omega \sin\phi}.$$

That is, the greater the linear velocity of the moving object over the surface of the earth, and the nearer it is to the equator, the greater the radius of curvature. On the equator the radius of curvature is infinite, or the path a straight line. However, unless moving along the equator the object soon crosses to the other hemisphere and its direction of curvature changes, as is well shown by the summer wind tracks over the Indian Ocean.

Rate of Change of Wind Direction.—The theoretical rate of change of wind direction at any point on the surface of the earth obviously is the angular velocity ω' of the earth about an axis passing through its centre and the point in question. This change of direction, therefore, clockwise in the northern hemisphere, counter-clockwise in the southern, occurs at a rate *wholly independent* of wind velocity, and is given by the equation,

$$\omega' = \omega \sin \phi,$$

in which ϕ is the latitude of the place under consideration and ω the angular velocity about the axis through the north and south poles.

But, as the earth turns completely around during a sidereal day,

$\omega = \frac{2\pi}{86{,}164}$ per second $= 15^\circ\ 2'\ 26''$ per hour, or 15° per hour, roughly. Hence,

$$\omega' = 15^\circ \sin \phi \text{ per hour, roughly.}$$

The approximate values of ω' corresponding to different latitudes may conveniently be tabulated as follows:

Earth Rotation in Degrees per Hour at Different Latitudes.

φ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
ω'	0.00°	1.31°	261°·	3.89°	5.14°	6.36°	7.52°	8.63°	9.68°	10.64°
φ	50°	55°	60°	65°	70°	75°	80°	85°	90°	
ω'	11.52°	12.32°	13.02°	13.63°	14.13°	14.53°	14.81°	14.98°	15.04°	

It must be clearly understood that the above values of rate of change of direction are based on the assumption that there is no friction and no disturbing horizontal pressure gradient, neither of which is true with reference to actual winds. Nevertheless, the values obtained are important, since they indicate the natural torque of the atmosphere, or its tendency to rotate when set in motion by pressure gradients.

It is interesting and quite practicable, as Whipple[34] has shown, to compute the path of a frictionless mass over the rotating earth when its direction and velocity are given for any definite point, but this will be omitted, since in the case of the actual atmosphere, because of viscosity, turbulence, surface obstacles, horizontal pressure gradients, etc., the departures from theoretical values are so great that it seems hardly necessary or even safe to go beyond the above simple and general relations.

Centrifugal Deflecting Force of Winds.—Usually the paths of winds are more or less curved, and therefore the moving air exhibits a "centrifugal force," or inertia force away from the centre of curvature in opposition to any applied "centripetal force." The value of this force, f, in the plane of the curve, not the plane of the horizon, is given by the equation,

$$f = \frac{mv^2}{r}.$$

in which m is the mass concerned, v its linear velocity, and r the radius of curvature of the path at the place and time under consideration. or radius of the "small circle" in which the air is then moving.

Relative Values of Centrifugal and Rotational Components.—The ratio between the two deflective forces, rotational and cen-

[34] *Phil. Mag.*, 33, p. 457, 1917.

trifugal (centrifugal action of the earth's rotation and centrifugal action due to curvature of path), varies greatly with the velocity of the wind, radius of curvature of its path, and latitude of its location. Within 20 degrees, at least, of the equator cyclonic storm winds commonly move on curves whose radii are comparatively small, 150 kilometres (93.2 miles) or less, in which case the centrifugal deflective force generally is greater than the rotational. In middle and higher latitudes, however, the average radii of cyclonic wind-paths usually are much larger, say 600 kilometres (373 miles), and the rotational deflective force greater than the centrifugal.

A few numerical examples will be interesting. No effort has been made to get average values, but only such as presumably often occur.

Ratios of Deflective Forces Under Given Conditions.

Latitude	Radius of curvature	Gradient velocity	$\frac{\text{Rotational force}}{\text{Centrifugal force}}$
	Miles	Miles per hour	
10°	20	80	1/44
20°	30	70	1/13
30°	100	50	10/19
40°	400	40	10/3
50°	400	35	14/3
60°	400	35	26/5

In tropical cyclones, therefore, the pressure gradient is balanced mainly by the centrifugal force, while in those of middle latitudes it is balanced chiefly by the rotational deflective force.

Ordinarily, except in the neighborhood of a well-marked low, the radius of curvature is much larger than any of the values above assumed, and consequently the ratio of rotation to centrifugal force correspondingly greater.

Total Horizontal Deflecting Force.—If the path of the air is at all curved, as it usually is, the total *horizontal* deflecting force, F, due to its velocity is given by the equation,

$$F = 2m\omega v \sin\phi \pm \frac{mv^2}{r},$$

in which $r = r' \sec\alpha$, r' and α being the linear and angular (as seen from the centre of the earth) radii respectively of the "small circle" in which the air is moving, or $r = R \tan\alpha$, R being the

radius of the earth, and the other symbols have the meanings given above. The positive sign is used, or the deflective forces are additive, in the northern hemisphere when the course of the wind is counter-clockwise; in the southern hemisphere when the course is clockwise. The negative sign is used in each case when the sense of rotation is reversed. In cyclones, therefore, the total deflecting force is equal to the sum of the centrifugal and rotational deflective forces; in anticyclones to their difference.

When the winds become approximately steady the deflective force obviously is balanced against the gravitational pressure gradient. In symbols,

$$\frac{1}{\rho}\frac{dp}{dn} = 2\,\omega\, v \sin\phi \pm \frac{v^2}{r},$$

in which ρ is the density of the air, dp the slight difference between the pressures at the ends of the short horizontal distance dn at right angles to the path at the place considered. The meanings of the other symbols are given above. If the gradient is zero (that is, if the air moves without lateral restraint), $r = \frac{v}{2\omega \sin\phi}$, as previously shown, or $R \tan\alpha = \frac{v}{2\omega \sin\varphi}$; also, from the equation just given, $2\,\omega\, v \sin\phi \pm \frac{v^2}{r} = 0$. Hence, under the assumed conditions, $r = R \tan\alpha = \infty$; that is, $\alpha = 90°$, or the path is a great circle. However, these latter equations have but little more than a theoretical interest, since without the driving force of a pressure gradient wind velocity can neither be acquired nor (because of friction) maintained.

GRADIENT WIND.

Gradient Velocity.—That velocity of the air at which the deflective force due to the rotation of the earth and the centrifugal force jointly balance the horizontal pressure gradient is called the gradient velocity. It does not occur near the surface of the earth, owing to surface friction, but "from kite observations, it appears that at 1500 feet above the surface the agreement [between the observed and 'gradient' velocities] is generally very close,"[35] especially in the absence of thunderstorms and other local disturbances. This does not mean that the wind has the same velocity at all levels beyond ½ kilometre, but only that above this height the velocity of an approximately steady wind

[35] Shaw, "Forecasting the Weather," p. 45.

is very nearly the gradient velocity appropriate to the atmospheric density, horizontal pressure gradient, and latitude at the place in question.

If the sense of rotation is that which exists in a cyclone, the two deflective forces are additive, as stated above, and the gradient velocity is given by the general equation,

$$v = \pm\sqrt{\frac{r}{\rho}\frac{dp}{dn} + (r\,\omega \sin\phi)^2} - r\,\omega\sin\phi, \quad . \quad . \quad . \quad . \quad . \quad (A)$$

in which the sign of the radical remains to be determined.

But obviously $v = 0$ when $\frac{dp}{dn} = 0$, as there can be no wind without a pressure gradient, from which it follows that the sign of the radical is positive, and that actually

$$v = \sqrt{\frac{r}{\rho}\frac{dp}{dn} + (r\,\omega\sin\phi)^2} - r\,\omega\sin\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

Theoretically this wind at any place is *along* the corresponding isobar, parallel, roughly, through the first one or two kilometres of elevation, to the surface isobar, and always in such direction that one moving with it will have the *lower* pressure to his *left*. It must be distinctly noted, however, that the above equations presuppose absence of friction and the attainment of a steady state. They therefore give the approximate wind velocity and direction only for levels above the appreciable reach of surface turbulence, and even there in the cases only of smooth and regular isobars. Near the surface where the velocity is checked by friction the wind direction is correspondingly deflected toward the region of lower pressure.

In the C. G. S. system of units:

v = centimetres per second.

$\frac{dp}{dn}$ = difference in dynes pressure per square centimetre, per centimetre horizontal distance at right angles to isobars. $r = r'$ sec α; r' = radius of curvature, in centimetres, of wind-path at time and place of observation, *not distance to the centre of the low;* α = angular radius (measured from the centre of the earth) of the "small circle" along which the wind is moving. Ordinarily r differs from r' by less than 1 part per 100, and therefore, in practice, they may be assumed to have equal values.

ρ = grammes of air per cubic centimetre.

ω = angle through which the earth turns per second, $\frac{2\pi}{86{,}164}$.

sin ϕ = natural sine of the angle of latitude.

If the path of the wind is straight (that is, if there is no centrifugal force), the equation for gradient velocity is

$$v = \frac{\frac{dp}{dn}}{2\omega\rho \sin \phi}.$$

In anticyclonic regions gradient winds, as explained, obey the equation,

$$\frac{1}{\rho}\frac{dp}{dn} = 2\omega v \sin\phi - \frac{v^2}{r}$$

or,

$$v = r\omega \sin \phi \pm \sqrt{(r\omega \sin \phi)^2 - \frac{r}{\rho}\frac{dp}{dn}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (C)$$

As above,

$$v = 0 \text{ when } \frac{dp}{dn} = 0.$$

Hence, in this case, the sign of the radical is negative and

$$v = r\omega \sin \phi - \sqrt{(r\omega \sin \phi)^2 - \frac{r}{\rho}\frac{dp}{dn}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (D)$$

Obviously, then, as pointed out by Gold,[30] steady anticyclonic winds cannot become intense, since the maximum possible velocity is given by the equation,

$$v \max = r\,\omega \sin \phi.$$

For example, if r = 500 kilometres, and $\phi = 40°$,

$$v \max = 23.4 \text{ metres per second} = 52.3 \text{ miles per hour.}$$

The gradient that produces this velocity is given by the equation,

$$\frac{dp}{dn} = \rho\, r\,(\omega \sin \phi)^2.$$

On substituting this gradient in the equation above for straight winds, it appears that it would give the velocity,

$$v = \frac{\rho\, r\,(\omega \sin \phi)^2}{2\,\rho\omega \sin \phi} = \tfrac{1}{2}\, r\,\omega \sin \phi.$$

That is, the limiting velocity of anticyclonic winds,

$$v \max = r\,\omega \sin \phi,$$

[30] M. Q., No. 190, "Barometric Gradient and Wind Force," London, 1908.

is just twice that which the corresponding pressure gradient would give to straight winds.

Since " for the time being, we may regard the gradient winds as the best estimate we can give of the actual winds at, say, 1500 feet above the surface," [87] it seemed advisable to construct tables (see Appendix I), one for cyclonic, the other for anticyclonic conditions, that give the theoretical wind velocities in metres per second, kilometres per hour, and miles per hour for various latitudes, radii of curvature, and pressure gradients, as indicated, each to be used in conjunction, of course, with the current weather map, corrected, if need be, by estimation or by special reports, for the hours that have elapsed since the observations were made from which it was constructed.

As the equations demand and the tables indicate, the winds of an anticyclone, gradient for gradient, latitude for latitude, and curvature for curvature, are stronger, often much stronger, than those of a cyclone. This may seem to be flatly contradicted by the fact that anticyclones are characterized by relatively light winds, but the contradiction is only apparent, for, as the equations show, steep gradients cannot obtain in anticyclonic regions, nor, therefore, heavy winds except near their borders, or when r is large. However, in general, strong anticyclonic winds cover only a narrow strip of territory, and their duration, therefore, is comparatively brief.

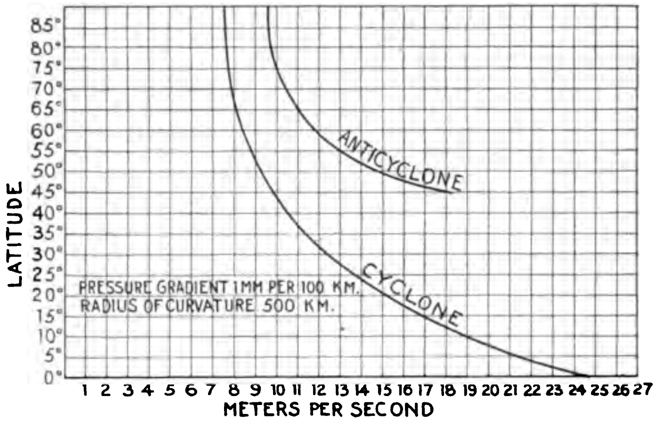
Figs. 38, 39, and 40 represent respectively the effect of latitude, pressure gradient, and radius of curvature on the " gradient " velocity, other things in each case being constant.

Surface gradients and surface isobars, when well defined and in the absence of local disturbances, may be used for approximate values up to elevations of, roughly, two kilometres, except wherever the horizontal temperature gradient is steep and opposite in direction to the horizontal pressure gradient. In such cases the temperature tends to weaken and finally reverse the pressure gradient with increase of elevation. Hence, since the temperature gradient is nearly always more or less poleward, in extra-tropical regions the east wind generally is the shallowest and the least likely to have, at an elevation of two kilometres, say, the direction and velocity computed from the surface system of isobars; and

[87] Shaw, " Forecasting the Weather," p. 49.

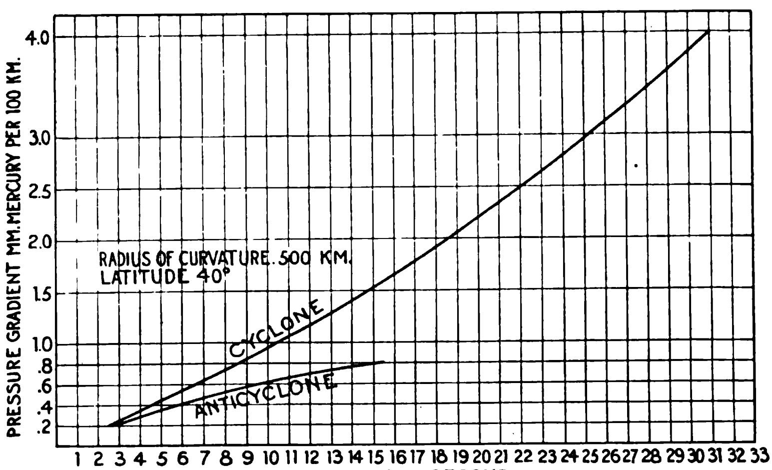
the west wind the deepest, with direction and velocity closest in agreement with the values thus computed. In short, with increase of elevation the isobars, usually closed on the surface, tend to

FIG. 38.



Relation of gradient velocity to latitude.

FIG. 39.



Relation of gradient velocity to pressure gradient.

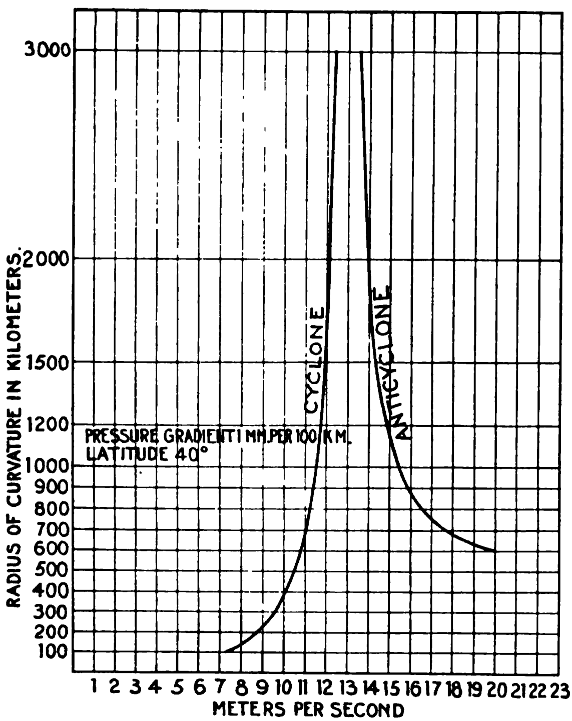
open out and roughly to follow the parallels of latitude with the decrease of pressure directed poleward, at least to well above the troposphere.

Gradient Velocity Nomogram.—The general gradient velocity equation, $F = 2m\omega v \sin\phi \pm \frac{mv^2}{r}$, reduces, on dividing by *m*, to

$$2\,\omega\, v \sin\phi - \frac{1}{\rho}\frac{dp}{dn} = \pm\frac{v^2}{r},$$

the upper sign being used for anticyclones and the lower for cyclones.

FIG. 40.



Relation of gradient velocity to radius of curvature.

A straight line nomogram that solves this equation has been constructed by Mr. Herbert Bell, of the University of Chicago, after the method developed by Professor d'Ocagne in his "Traité de Nomographie." The solution is as follows:

Writing

$$u = -\frac{20}{\rho}\frac{dp}{dn} \text{ and } w = 10^5\,\omega \sin\phi,$$

u and w being scales along the lines $x = -10$ and $x = 10$ respectively, the velocity equation becomes

$$2500\,u + v\,w = \pm\,50{,}000\,\frac{v^2}{r}$$

which is linear in u and w.

If, then, a network is constructed of the two families

$$x = 10\,\frac{v - 2500}{v + 2500} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

and

$$y = \pm\,\frac{50{,}000\,\dfrac{v^2}{r}}{v + 2500}$$

$$= \pm\,\frac{6.25 \times 10^6}{r}\,\frac{(10 + x)^2}{10 - x} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

the point c, say, determined by (1) and (2) from values of v and r, will be collinear with the point A on the u scale, fixed by the given value of $\frac{dp}{dn}$, and the point B on the w scale indicated by the latitude.

The resulting diagram, with gradients in terms of millimetres difference of barometer reading per 100 kilometres, and velocity in metres per second, is given in Fig. 41.*

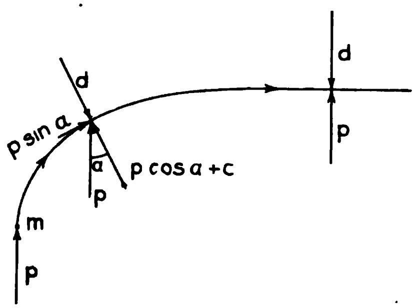
To find the gradient wind velocity, connect the known pressure gradient (marked on lower left border of the diagram) and the latitude of the place in question (given on the upper right border) with a straight edge or stretched string and note where it cuts the curve representing the radius of curvature of the local isobar. For *cyclones* the vertical through this point gives the required velocity in metres per second. For *anticyclones* two velocities are thus indicated, but the *smaller* is the one to take, since it alone is physically possible. See equations C and D, above.

This nomogram should be used in conjunction with the surface distribution of pressure only in the absence of local disturbances (thunderstorms, squalls, etc.), or strong horizontal temperature gradients; where the pressure gradient is well defined—isobars smooth, and for elevations between, roughly, 500 and 2000 metres.

* Folded plate attached to inside of back cover.

Automatic Adjustment of Winds in Direction and Velocity.—In discussing the more extensive winds it is convenient to consider the earth as stationary and the air as moving over it without friction under the influence of three distinct horizontal forces: (1) The deflective force, due to the earth's rotation; (2) the horizontal component of the centrifugal force, due to the curvature of the path, and (3) the horizontal or gradient pressure, due to gravity. The first two are at right angles to the course of the wind and therefore help to control its direction, but do not alter its speed. The latter, however—that is, the gradient pressure—affects both the direction and the speed. Furthermore, as the velocity depends upon the horizontal pressure alone, and as the

FIG. 42.



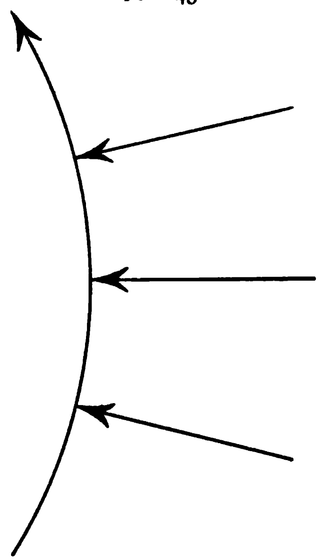
Deflection and path of winds in frictionless flow under a force of constant magnitude and constant geographic direction.

other forces depend in turn upon the velocity, and are zero when it is zero, it follows that of the three forces only the gradient pressure is independently variable.

Consider, then, the result of applying a horizontal pressure *p* of constant magnitude and constant geographic direction to a small mass *m* of air, free, as above assumed, from friction: Let *m*, Fig. 42, be the mass in question initially at rest with reference to the surface of the earth, and let it be acted on by the force *p*, exactly poleward, say. Immediately the mass moves, under the applied pressure *p*, the deflective force *d* becomes operative, thus curving the path (to the right in the northern hemisphere, to the left in the southern) and introducing the centrifugal force *c*. So long, however, as the angle between the path and the force *p* is

less than 90 degrees there will still be a component of the latter in the line of motion; accordingly the speed of m will continue to increase, and therefore also the deflective force d. If this angle should exceed 90 degrees, the force p would have a component opposite to the direction of motion, which consequently would be slowed up and d thereby correspondingly decreased. In the end, therefore, a poleward force along the meridians on an object free

FIG. 43.



Path of winds in frictionless flow under a converging force.

to move gives it an exactly west to east velocity of such magnitude that, except in very high latitudes, the resulting deflective force is nearly equal to the horizontal pressure—the horizontal component of the centrifugal force being then comparatively small, except near the poles. Whatever the direction of the gradient force, whether poleward, as above assumed, or any other, the final motion is normal thereto.

A change in the magnitude but not in the direction of p, above, would only shift the latitude of the path and change the velocity so as to be nearly proportional to p.

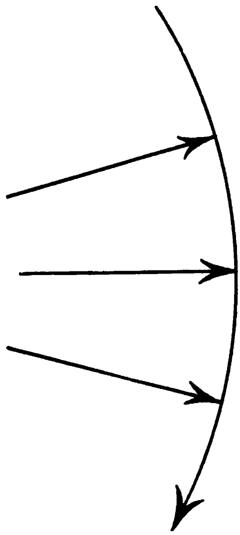
If the horizontal pressure is not everywhere in the same direction, but converges, as in Fig. 43, or diverges, as in Fig. 44, the path, in adjusting itself normally to the directions of this

pressure, obviously curves, as in cyclonic and anticyclonic regions, respectively.

In all cases, then, the wind automatically follows approximately the isobar of its position, with substantially the gradient velocity.

General Relations of Wind to Elevation.—Knowledge of the directions and velocities of the winds of the earth is still frag-

FIG. 44.



Path of winds in frictionless flow under a diverging force.

mentary and incomplete. Over large areas even the surface winds are unknown, and over regions best studied these alone are well known. The continuous records obtained at mountain stations have given much information in regard to air movements, but stations of this nature are comparatively few, and, besides, their data, however valuable, are always affected to an unknown extent by local topography. Cloud observations have also given a large amount of valuable information, but it, too, is only fragmentary. At best a cloud observation seldom gives more than the direction and velocity of the air at one level, nor does such an observation ever apply to the stratosphere, since this region is never visited by clouds. In many respects kites and sounding balloons have

furnished the most valuable data in regard to the movements of the upper air and their causes, but, unfortunately, aerological investigations of this nature, with relatively few exceptions, have been restricted to the northern hemisphere, and even there mainly to the summer season. Nevertheless, by combining the data gathered from these various sources a number of tentative conclusions, subject, of course, to modification, have already been reached in regard to the winds of different parts of the world from the surface up to great elevations. Some of the more important of these conclusions are:

> 1. That there is no continuous and rapid overflow of the atmosphere at all longitudes from the equatorial to the polar regions. At an elevation of 10 kilometres, for instance, the wind of middle northern latitudes seems to have southerly components about as often as northerly.

From this it follows that the equator-polar circulation is irregular and probably complex even at the higher altitudes.

> 2. That the equatorial winds are not always and at all levels from the east; that, on the contrary, west winds occur (how regularly is uncertain) at elevations of about 18 to 20 kilometres, with east winds again prevailing (certainly at times) at still greater elevations.

The cause of this layer of equatorial west wind has never been explained. Indeed, it may be only a local and temporary phenomenon.

> 3. That layers of air in which the temperature increases with increase of elevation, and others in which the temperature is constant, exist at different levels, especially through the first two or three kilometres. This stratified condition of the lower atmosphere appears to be universal. It is found even over tropical oceans, and is exceedingly well developed over the ice plateau of Antarctica.

Each layer usually shows such different humidity and such different wind velocity from those of the adjacent layers as to indicate a distinct origin, which it well may have. For example, as explained in an earlier section, a rising convection current on reaching its equilibrium level flows away substantially at that

particular elevation, and obviously retains its own humidity (provided condensation has not taken place), dust content, and other peculiarities. Its viscosity is not the same as that of the adjacent air, because its humidity or temperature, or both, are different. Hence, as shown by billow clouds, any such layer with a distinctly independent velocity tends to retain its integrity and to glide over another from which it differs physically without rapid intermingling. And there are still other obvious causes of temperature and humidity irregularities and consequent stratification of the atmosphere, such as reflection from, and evaporation of, clouds, surface cooling, and air drainage. Clearly, then, one should expect to find in the lower atmosphere substantially the kind and amount of temperature inversions and other irregularities that it actually shows.

4. That the upper winds are exceedingly variable along the edges of the high-pressure belts, and that marked disturbances occur in the antitrades.

5. That the north-poleward pressure gradient in the upper atmosphere becomes very small long before the arctic circle is reached—in fact, between 50° and 60° N.

6. That in high northern latitudes, where the poleward pressure gradient of the upper atmosphere is small, the westerly winds are not constant.

Local Wind Velocity and Elevation.—Everyone knows that the wind increases with increase of elevation. Even casual observations of such objects as sails of ships, tops of trees, columns of smoke, or isolated clouds suffice to show qualitatively that wind velocity increases with height above the surface; while measurements made by triangulation on freely drifting clouds and balloons, or by anemometry on tethered kites, fully support the conclusions reached by the simpler methods just mentioned. Near the surface of the earth—up to from 2 to 8 metres over an open plain—the condition of the wind, upon whose force this limit depends, may be summarized as follows:

Actual velocity: exceedingly irregular.
Average velocity: increases rapidly with elevation.
Rate of velocity increase: $\begin{cases} a, \text{ increases with average velocity.} \\ b, \text{ decreases with elevation.} \end{cases}$

Above this thin surface layer the wind increases so nearly regularly with elevation that its approximate velocity at any level up to 16 metres may be computed, according to Stevenson,[38] from its observed velocity at some other height by the empirical equation,

$$V = v \sqrt{\frac{H + 72}{h + 72}},$$

in which V is the computed wind velocity for the level H in terms of the known velocity v at the height h, both elevations being expressed in feet.

If heights are given in metres, this equation becomes,

$$V = v \sqrt{\frac{H + 22}{h + 22}}.$$

Other empirical equations expressing the relation of wind velocity to elevation have been given for greater heights. Archibald,[39] for instance, finds that his velocity observations between 100 metres and 600 metres elevation fairly satisfy the simple equation,

$$\frac{V}{v} = \left(\frac{H}{h}\right)^{\frac{1}{4}}.$$

Shaw [40] suggests as a likely formula,

$$V = \frac{H + a}{a} V_o,$$

in which V is the wind velocity at the height H above *ground,* V_o the observed anemometer velocity at a fixed position, and a a constant, obviously depending upon surrounding topography, anemometer exposure, and, perhaps, other factors.

Among the most interesting observations on the relation of wind velocity to altitude are those of Dr. Cesare Fabris,[41] based on some 200 pilot balloon flights made at nearly equal intervals during the year June, 1910, to May, 1911, at Vigna di Valle, the principal aerological station of the Royal Italian Oceanographic Committee. This station is about 25 miles northwest from Rome.

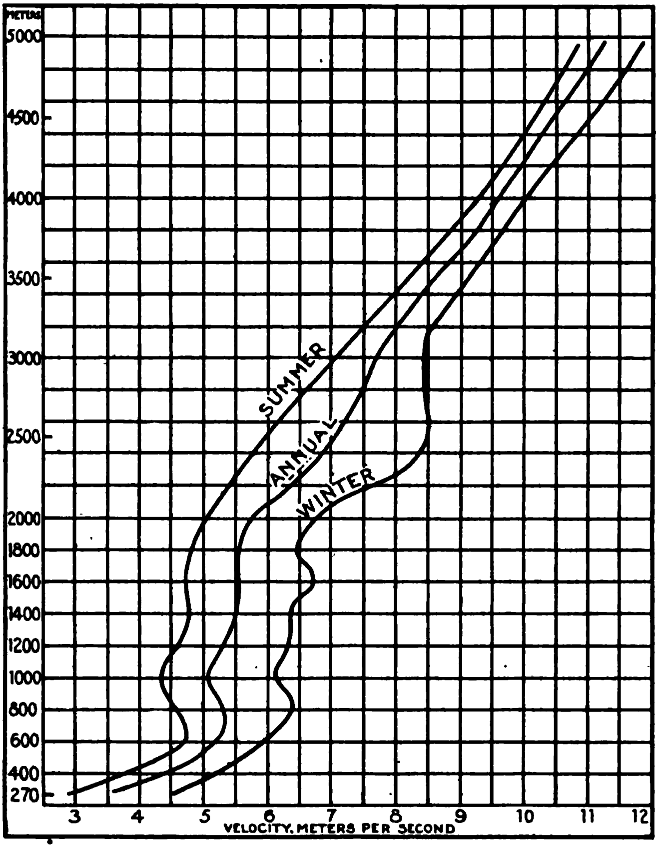
[38] *Jour. Scot. Meteor. Soc.,* 5, p. 348, 1880.
[39] *Nature,* 33, p. 593, 1885.
[40] Advisory Committee for Aeronautics, "Reports and Memoranda," **66,** No. 9, p. 8, 1909.
[41] *R. Comitato Talassografico Italiano,* Memoria 8, pp. 37–46, 1912.

Its coördinates are: Latitude 42° 04′ 41″ N.; longitude, 12° 12′ 43″ E.; altitude, 272.4 metres.

The general results of all the observations are summed up in Fig. 45, which shows four distinct regions:

a. The region of rapid linear increase of velocity with

FIG. 45.



Wind velocity and elevation. (After Fabris.)

increase of altitude, extending from the surface (272 metres above sea level), where the velocity is least, to an elevation above ground of, roughly, 300 to 500 metres. This obviously is the region in which the winds are affected by surface friction and the resulting turbulence. Clearly, too, the average number of eddies and their consequent effect on velocity must rapidly de-

crease with increase of elevation, at least near the surface, substantially as indicated by the given velocity-altitude curves.

b. The region of velocity decrease with increase of altitude; about 200 metres thick and coming immediately above *a*. This decrease probably is not what one at first would expect, but it may be of fairly general occurrence (known to occur in other localities) in analogy to the well-known fact that the maximum velocity in rivers, canals, and other streams is not at the surface, but, roughly, one-third the depth *below* it. This surprising relation between velocity and depth is due to the retarding drag of the bed and banks, together with the viscous pull of the swifter centre that draws the sluggish surface water away from either side; and, since the surface of the earth exerts a similar drag on the atmosphere, it appears that an analogous effect on wind velocity might be suspected, not, of course, because of overflow, for there are no retarding banks, but as a result of convection.

Obviously the amount of interference to the flow of a given stratum of air, exerted by a convecting mass, depends upon the difference of their velocities and the duration of their contact. But near the surface of the earth vertical movements necessarily are slow, and again slow near the limit of convection, and therefore most rapid at some intermediate point. Hence the least interference to its flow and the maximum velocity, or at least a tendency to a maximum, of the turbulent layer of air may occur below its pseudo surface—the upper limit of convection—just as the maximum flow of a river occurs below its surface, though chiefly for a different reason.

c. A region of irregular winds slowly increasing with increase of altitude, extending, roughly, from about 500 to 1500 metres above the surface. These conditions are of very general occurrence between the levels given.[42] The irregularity probably is due to convectional mixing induced during the day by insolation and at night by cloud evaporation.

d. A region of approximately constant increase of velocity with increase of elevation, beginning at about 1500 metres above the surface and extending to at least the maximum height observed, 5000 metres. The wind velocities of this region, being out of the reach both of frictional and convectional disturbances, are determined by the prevailing horizontal pressure gradients.

[42] Berson, *Wissenschaftliche Luftfahrten*, 3, p. 205, 1900.

Cloud and balloon observations show that increase of wind velocity with increase of altitude beyond 1500 to 2000 metres above the surface holds practically to the top of the troposphere, where the velocity in middle latitudes may amount to as much as 90 metres per second (200 miles per hour) or even more.

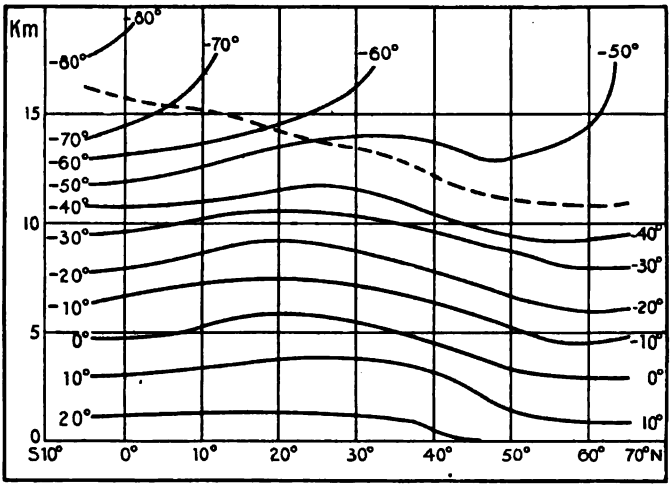
At higher levels—that is, in the stratosphere—the average velocity is decidedly less.

Horizontal Pressure Gradient and Elevation.—All these facts are well known, but there are no generally accepted and satisfactory discussions of the reasons why the average wind velocity at levels above the limit of appreciable surface influence should go on increasing with increase of elevation up to the isothermal base and then decrease. Indeed, data sufficient for a complete solution of this problem are not yet available, and it is only recently that enough facts have become known to indicate at all clearly the several links in the chain of cause and effect that determine the average atmospheric movements in middle and higher latitudes.

Because of the actual distribution of insolation over the earth the temperature of the lower atmosphere, as shown by observation, is warmest, on the average, in equatorial regions and coldest beyond the polar circles, with intermediate values over middle latitudes. Hence, since the temperature of the air above the earth depends mainly upon convection and radiation from below, it follows that the latitude distribution of temperature in the upper air must be substantially the same as that at the surface; that is, warmest within the tropics and coldest in the polar regions, with intermediate values between. And this, indeed, according to kite and balloon records, does apply at each level up to 10 to 12 kilometres, or to fully three-fourths of the air mass. At much higher levels, 15 to 20 kilometres, for reasons that need not be discussed here, the rare atmosphere is coldest over equatorial regions and warmest over high latitudes. This inverse condition, however, does not apply to the winter and summer atmospheres of the same place, nor, presumably, to those of neighboring places on approximately the same latitude. On the contrary, the atmosphere is warmer, on the average, at all explored levels during summer than during winter, and warmer, so far as known, over regions whose temperatures are relatively high than over others of the same latitude that are comparatively cold.

As a crude first approximation to conditions as they actually exist, assume (1) that the temperature distribution is the same along all meridians, (2) that the temperature changes from one latitude to another is the same for all levels, and (3) that sea-level pressure is the same at all latitudes. Assumption (1) approximates the conditions over much the greater portion of the southern hemisphere, but, on account of the irregular distribution of land and sea, has to be modified for any detailed study of the winds of the northern hemisphere. Assumption (2) conforms roughly to average conditions between the thermal equator and

Fig. 46.



Relation of temperature to altitude and latitude. (After Süring.)

latitude 50° to 60°, except near the surface and at altitudes above 10 to 12 kilometres. This is well shown by Fig. 46 referring to the northern hemisphere during its summer, and copied from Süring's paper,[43] on the present state of knowledge concerning the general circulation of the atmosphere. Assumption (3), as applied to normal pressure, is also approximately true except for restricted areas, whose secondary and local effects will not here be discussed.

Consider an atmosphere of the same composition throughout, and having initially the same temperature at any

[43] *Zeitschrift der Gesell. für Erdkunde*, p. 600, 1913.

given elevation, resting on a horizontal plane. Let the temperature be uniformly increased from north to south, say, and by the same amount from top to bottom, thus simulating the temperature distribution that actually obtains in the earth's atmosphere over middle latitudes, as above explained. Find the resulting horizontal pressure gradient at the different levels.

At the height h the horizontal pressure gradient, $\frac{dp}{dn}$, obviously directed from the warmer toward the colder region, is very approximately given by the equation,

$$-\frac{dp}{dn}=p\frac{\Delta h}{HL},$$

in which L is any given horizontal distance along which dn is taken, p the pressure at the level h, above the colder end of L, Δh the difference of vertical expansion of the air below the level in question at the ends of L, or difference of distance through which the level, whose original pressure was p, was lifted at these two places, and H the virtual height of the atmosphere, approximately 8 kilometres, or height it would have above any point if from there up it had the density which exists at that point. The negative sign is used because the pressure decreases as n, measured from a warmer toward a colder region, increases. For simplicity let L be in the direction of maximum rate of horizontal temperature change, north-south, in this case.

Under the assumed conditions

$$\Delta h = a\,\Delta T h, \text{ approximately,}$$

in which a is the average coefficient of volume expansion of the atmosphere below the level h, and ΔT the difference of temperature change at the ends of L.

At any two levels, then, h and h', the horizontal pressure gradients in the same direction are given approximately by the respective equations,

$$-\frac{dp}{dn}=\frac{pa\,\Delta T h}{HL},$$

and

$$-\frac{dp'}{dn}=\frac{p'a'\,\Delta T'h'}{H'L}.$$

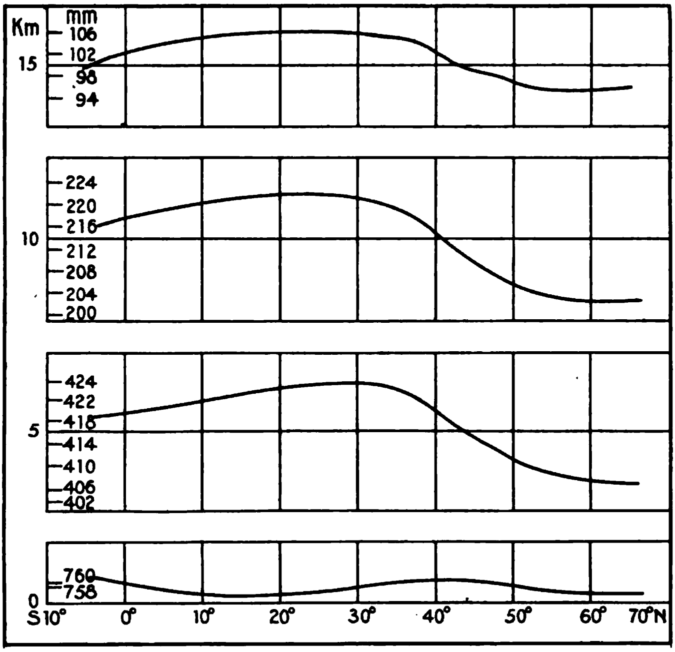
But L may be taken the same in both equations, while a, H, and ΔT generally are not greatly different respectively from a', H' and $\Delta T'$. In reality,

$$\frac{H}{H'}=\frac{T}{T'},$$

and a' is slightly greater than a when T' is less than T. But in this case it appears from observations that actually $\Delta T'$ is slightly less than ΔT, so that

$$\frac{\frac{dp}{dn}}{\frac{dp'}{dn}} = \frac{p\,h}{p'\,h'}, \text{ approximately.}$$

FIG. 47.



Relation of pressure to altitude and latitude. (After Süring)

Again, from the 5- to the 10-kilometre level, and even to some distance below the former and above the latter,

$$\frac{p}{p'} = \frac{h'}{h}, \text{ roughly.}$$

Hence, commonly,

$$\frac{\frac{dp}{dn}}{\frac{dp'}{dn}} = \frac{h'\,h}{h\,h'} = 1, \text{ approximately.}$$

That is, through these levels, or from below 5 kilometres to above 10 kilometres, the horizontal pressure gradient established by the temperature difference between adjacent regions of air

is roughly constant. This conclusion is fully supported by observations, as shown in Fig. 47, referring to the northern hemisphere during its summer, and also copied from Süring's paper.[44]

Level of Maximum Horizontal Pressure-Gradient.—The approximate level of the maximum horizontal gradient may be found as follows:

As just explained, in the equation,

$$-\frac{dp}{dn}=\frac{pa\,\Delta Th}{HL},$$

the factor, $\frac{a\,\Delta T}{HL}$, is roughly constant. Writing G for the gradient and K for the "constant," the equation takes the form,

$$G=Kph.$$

Hence G has a maximum value when

$$p\,dh=-h\,dp.$$

But

$$-dp=p\,\frac{dh}{H}.$$

Hence the pressure gradient is steepest when

$$p\,dh=\frac{h}{H}\,p\,dh,$$

that is, when $h=H=8$ kilometres, roughly.

The following is a slightly different method of arriving at the same conclusion:

The maximum horizontal pressure gradient resulting from a constant temperature difference between two neighboring columns of air obviously is at that level at which the vertical pressure is most changed by the expansion of the air below due to a constant temperature increase.

Let h be any height, and let a be the average coefficient of volume expansion of the air below this level. Then,

$$\Delta h=a\,h,\ \text{nearly},$$

and

$$\Delta p=\rho g\Delta h=\rho g\,a\,h,\ \text{nearly},$$

in which ρ is the density of the air at the level h and g the local gravity acceleration.

But $\rho=Cp$, in which C is a constant, and

$$\Delta p=p\,C.g\,a\,h=K\,p\,h,$$

say, in which K may be regarded as a constant.

[44] *Loc. cit.*

Hence, as before, Δp has its maximum value when

$$p\, \delta h = -\, h\, dp = \frac{h}{H} p\, dh.$$

That is, the horizontal gradient is steepest when $h = H$. But, as is well known, $H = 8$ kilometres, approximately. Hence the horizontal pressure gradient, resulting from a temperature distribution substantially that which actually obtains in the atmosphere, is greatest at a height of about 8 kilometres.

According to this conclusion the maximum seasonal pressure change should occur at the elevation of 8 kilometres, or a little higher perhaps, since the surface pressure is slightly greater in winter than summer. And that is where it does occur, as shown by the table on page 72.

Constancy of Mass Flow—Egnell's Law.—At a distance above the surface of the earth sufficiently great to avoid appreciable retardation due to friction and turbulence—that is, at elevations greater than 2 kilometres (usually less)—the wind obviously must blow in such direction and with such velocity that there is an approximate equality between the pressure gradient on the one hand and the combined centrifugal force and deflection force due to rotation on the other. Hence, at these levels, if $\frac{dp}{dn}$ is the maximum horizontal pressure gradient,

$$\frac{dp}{dn} = -\rho v \left(2\, \omega \sin \phi + \frac{v}{R} \tan \phi\right), \text{ approximately.}$$

in which ρ is the density of the air at the level under consideration, v the wind velocity, ω the angular velocity of rotation of the earth, ϕ the latitude, and R the radius of the earth.

A little calculation shows that in the case of a wind following, roughly, a parallel of latitude the second term in the parentheses is always small, except in very high latitudes, in comparison with the first. Thus for a 22.4-metres-per-second (50 miles per hour) west wind at latitude 45° the first is about 30 times greater than the second. Hence, under these conditions,

$$\frac{dp}{dn} = -\, \rho\, v\, 2\, \omega \sin \phi, \text{ approximately.}$$

But, as just explained, the horizontal pressure gradient, $\frac{dp}{dn}$, is roughly constant between 5 and 10 kilometres elevation and directed polewards. Hence, at any given latitude, $\rho\, v$, the mass flow, or mass of air crossing unit normal area per unit time, tends to remain constant with change of altitude from 4 or 5 kilometres above sea level up to the isothermal region. In other words, through this region, $\rho\, v$, at altitude h, equals $\rho'\, v'$, at altitude h', nearly. This re-

lation between the density and velocity of the atmosphere at different levels is known as Egnell's law,[45] determined empirically by himself, as previously by H. H. Clayton, [46] from cloud observations. Obviously ρv has a maximum value at that level at which the horizontal pressure gradient is a maximum; that is, at about 8 kilometres above sea level.

Relation of Velocity to Altitude Above 5 Kilometres.—Obviously, if the temperature is constant, as, for simplicity, we may assume it to be,

$$\frac{\rho}{\rho'}=\frac{p}{p'}.$$

But, as already seen, under this condition of constant temperature, through a considerable range of altitude—that is, from below 5 to above 10 kilometres—

$$\frac{p}{p'}=\frac{h'}{h}, \text{ roughly.}$$

Hence,

$$\frac{\rho}{\rho'}=\frac{h'}{h}, \text{ roughly.}$$

But, as explained above, $\rho v=\rho' v'$
Therefore,

$$\frac{v}{v'}=\frac{h}{h'}, \text{ approximately,}$$

or the velocity of the wind through the levels in question is roughly proportional to the altitude.

Above the isothermal level over the regions between the thermal equator and latitude 50° to 60° the horizontal temperature gradient decreases, and presently even reverses with increase of elevation, as shown by Fig. 46, and therefore the corresponding pressure gradient also decreases as shown by Fig. 47. Hence, the mass flow, ρv, likewise decreases with elevation above this critical level. Further, the decrease of the horizontal pressure gradient, and consequently of ρv, with altitude in the stratosphere appears usually to be more rapid than that of the density alone, from which it follows that the wind velocity generally must have its maximum value at or below the isothermal level.

Season of Greatest Winds.—From the above discussion it is obvious that the general wind will be swiftest whenever the temperature contrast between the air of higher and lower latitudes is greatest. But the temperature of the atmosphere in low latitudes

[45] *C. R.*, 136, p. 360, 1903.
[46] *Amer. Met'l Jour.*, 10, p. 177, 1893.

does not change through the year nearly so much as does that of higher latitudes. Hence, the maximum horizontal temperature gradient, and therefore the greatest pressure gradient and strongest winds, must occur during winter.

Latitude of Greatest Winds.—The latitude of strongest winds clearly is that at which the horizontal pressure gradient is greatest. In the northern hemisphere, according to Fig. 47, this occurs in the summer at about latitude 45°. It is obvious, however, since the pressure gradient depends in general upon the latitude rate of temperature change, that the belt of maximum winds must shift more or less from season to season—poleward with the coming of summer, equatorward with the onset of winter.

Hours of Greatest and Least Winds.—On land, but not appreciably at sea, the velocity of the surface wind has a well-defined daily period. Over level regions this velocity is least, on the average, about sun-up and greatest from 1 to 2 P.M. The change is larger on clear days than on cloudy, and also most pronounced in summer, when it reaches an average altitude of about 100 metres, and least in winter, when it rises to only about 40 metres.

The physical explanation of this phenomenon was given long ago by Espy. During the night, when there is no vertical convection, surface friction holds the lower air comparatively quiet, while the upper air glides over the lower with but little restraint. During the day, however, and especially during clear, summer days, vertical convection and the accompanying turbulence so mixes the surface layers of the air with those next above as to bring both to a more or less common velocity, which is greater than the undisturbed or night surface velocity, and less than that of the undisturbed upper layers before their mixture with the lower.

Daily changes of wind velocity also occur on mountain tops, where the maximum is at night and the minimum by day, or just the reverse of the velocity changes that occur near the surface over plains. Three factors, possibly more, combine to produce this result: (*a*) Contraction of the lower air by night, thus bringing air of slightly higher levels, possibly 15 metres (50 feet) or so, and therefore of somewhat greater velocity down to the mountain top. (*b*) The presence by day and absence by night of surface disturbances, due to convection, in the air flowing over the mountain. (*c*) Overflow from the region of maximum expansion to the region of maximum compression. Since the greatest

expansion of the lower air usually occurs at 3 to 4 P.M. and its greatest compression at 5 to 6 A.M., it follows that the overflow will be from west to east, or with the prevailing winds, through the night, and from east to west, or against them, during most of the day; that is, from sun-up to 3 or 4 P.M.

Diurnal Shift of the Wind.—The average direction of the wind changes slightly during the day, both over plains and on mountain tops, the tendency being for it always to follow the sun, or, rather, the most heated section of the earth. That is, the wind tends to be east during the forenoon, south (in the northern hemisphere) during the early afternoon, and west during the late afternoon and early evening. This does not mean that at each instant the wind really blows directly from the then warmest region, but that the actual changes through the day in the average hourly wind directions can be accounted for by a velocity component away from that region. The whole sequence results from the thermal expansion of the atmosphere (progressive from east to west), which causes an increase of pressure and consequently an outward flow at all levels above the surface. The area covered is so vast that the time involved, only a few hours, is insufficient for the completion of the convection circuit, so that even the surface winds are *away* from the most heated regions, as stated, and not toward them, as in sea and land breezes, for instance. The compensating or return current occurs at night, when the component, outside the tropics at least, is from the higher latitudes. In reality the entire phenomenon is only a diurnal surge, a flux and reflux, of the atmosphere due to diurnal heating and cooling.

Normal State of the Atmosphere.—From the above explanations of the causes of general winds, especially those that pertain to cloud levels, it appears that the normal state of the atmosphere is one of considerable velocity with reference to the surface of the earth. In middle latitudes, at least, this velocity is from west to east more or less along parallels of latitude and so great as nearly to balance the latitudinal pressure gradient due to the zonal distribution of insolation. Calms, therefore, in this region must be regarded as disturbances of the atmosphere, and indeed often are comparatively shallow, with normal winds above.

Equatorial East to West Winds.—East to west winds are quite as general and constant in equatorial regions as are west to east

winds in middle latitudes. Along its borders, roughly 30° N. and 30° S., this equatorial belt of east to west winds is very shallow. Toward the equator its thickness increases, as a rule, until it reaches at least the limit of vertical convection. There are, however, great irregularities in these winds, just as in those of higher latitudes on either side of it. But the general conditions are as stated and require explanation.

Conceivably the tidal action of the sun and moon on the atmosphere might set it rotating from east to west. But the barometric amplitudes of the atmospheric tides are very small, only about 0.0109 m.m. and 0.025 m.m., due respectively to the sun and the moon.[47] Besides, they have but little phase lag, or follow closely under the disturbing body. Hence the tendency of the tide-producing forces to establish an east to west circulation of the atmosphere must be very small.

The diurnal heating and cooling of the air presumably also tends slightly to produce a planetary circulation, possibly in the same direction as that due to the tidal action, or from east to west. In equatorial regions, and in general wherever and whenever days and nights are approximately equal, the atmosphere is most condensed about daybreak, or, say, at 5.30 A.M., and most expanded at about 3.30 P.M. Hence at such times and places the gradient toward the east is to the pressure gradient toward the west substantially as the greatest and least distances, measured along a parallel, between the meridians of highest and lowest temperature; that is, as 7 to 5. On the other hand, the time of flow along the steeper gradient is to the time along the gentler as 5 to 7. But as the distance through which a given mass is moved, provided it is free to move, is proportional to the product of the force acting by the square of the time, it follows that the diurnal temperature changes, neglecting friction, which indeed may even reverse the sense of motion, should give the atmosphere an east to west velocity whose average magnitude would be determined in part by viscosity. The effect of the double diurnal pressure wave on this velocity is uncertain.

However, this thermal effect on wind velocity, whatever its value, clearly is of secondary importance. Possibly it may have some relation to the unexpected east to west winds, the so-called upper trades, reported in the stratosphere over equatorial regions.

[47] Lamb, Hydrodynamics, 4th edition, p. 552, 1916.

but many more observations and a more rigid discussion of the theory of atmospheric circulation would be necessary to settle this interesting question.

The only other obvious cause of east to west and west to east general or planetary winds is interzonal circulation, to which, indeed, they usually are regarded as being entirely due. Heating in equatorial and cooling in polar regions necessarily produce a more or less vigorous interchange of air, but, as already explained, one that is profoundly modified by earth rotation.

Assume, as initial conditions, non-rotation, smoothness of surface, uniformity of surface temperature, and absence of local convection. Let the temperature of the surface and the atmosphere now be decreased in proportion to the distance from the equator. There will be a poleward overflow and an equatorward underflow. Further, if, as assumed, there is no local convection—no intermingling of the air at different levels—the equatorward surface air obviously must remain at the surface throughout its course to lower latitudes and then, after rising in the general, not local, convection, move poleward at the top of the atmosphere. Similarly, any other layer must retain, in its equatorial course, a nearly fixed height above the surface and in its poleward journey a corresponding distance below the top.

As a further modification assume again an initial uniformity of surface temperature, but let the earth and the atmosphere be everywhere rotating at the same rate, so that there shall be no winds of any kind. As before, let the temperature be decreased in proportion to the distance from the equator, but let there be no local convection. Again there would be established an upper poleward and an under equatorward circulation, but the air that started toward either polar region would quickly assume an eastward component, while the under- or counter-current would have a westward component. In the absence of friction or other disturbing factor, the upper air moving under a pressure gradient from lower toward higher latitudes would approach along an asymptotic spiral a certain limiting parallel, where the deflecting force due to its final velocity would equal the pressure gradient across that circle. At the same time the east to west under-current would give a deflective force counter to the equatorward gradient. Hence, perhaps, equilibrium would soon be reached with west to east upper winds balancing the poleward pressures and east to

west lower winds balancing the equatorward pressures, and thus further interzonal circulation prevented.

But surface friction, viscosity, local convection, and other disturbing factors so restrict the approach to equilibrium velocities that interzonal circulation is continuous, though of varying intensity, both local and general. Hence the eastward moving upper air must gradually reach the surface and reach it as an eastward wind. In its subsequent course to lower latitudes it will become a westward wind, as already explained. In general, then, the areas of planetary atmospheric descent are the regions of west to east winds, while the similar areas of ascent are regions of east to west winds. Doubtless, too, these phenomena are accentuated at the surface—that is, the eastward and the westward surface winds are stronger than they otherwise would be—by the intermingling of the air of different levels through innumerable local convections.

What latitude establishes the boundary between the east and west winds? This important question has no answer, not even a theoretical one, unless height is considered. At the surface the boundary is several degrees nearer the equator during winter than in summer, but its average latitude is, roughly, 30° to 32°. Near its border the east to west winds are very shallow, but in general they increase in depth as the equator is approached until they extend to the limit of vertical convection.

If the temperature distribution were the same along all meridians, and gradually varied from highest at the equator to lowest at the poles, it would seem that the area of ascending air would be substantially the same as the area of descending air (really a trifle larger, because of the higher temperature and consequent greater volume of the ascending air), and therefore that the surface borders between east and west winds would be approximately 30° on either side of the equator. But temperature is not distributed in this ideal way. There are restricted areas which are exceptionally warm, at least during a portion of the year, and others that are exceptionally cold, hence one would expect the area of ascent to be only roughly equal to the area of descent, and therefore the boundaries in question to be, as they are, only approximately at latitudes 30° N. and 30° S.

The velocity of the west to east winds of middle and higher latitudes and the velocities of east to west winds of equatorial

regions obviously depend ultimately upon the rate of interzonal circulation. If this circulation were zero, surface friction, facilitated by local vertical convections, soon would greatly diminish and finally eliminate any cross-meridian velocity that originally might obtain. On the other hand, an extremely vigorous interzonal circulation would lead to violent east to west winds, partly because velocity is not altered by mere deflection and partly because there would then be less time for the latitude (conservation of area) effects on the velocity to be minimized by convectional turbulence—the total number of such disturbances becoming larger and their cumulative effects therefore greater with increase of time. Hence the moderate east winds of equatorial regions and west winds of higher latitudes that actually exist are due to the fact that the interzonal circulation itself is moderate—being largely held in check, as already explained, by the automatic formation of gradient winds in the free atmosphere which roughly follow parallels of latitude.

It appears, then, (*a*) that the temperature gradient, directed in general from the equatorial toward the polar regions, establishes an upper pressure gradient in the same direction and a lower in the opposite direction; (*b*) that in the absence of friction or other disturbance these pressures would produce east to west and west to east gradient winds with but little or no interzonal circulation; (*c*) that as the winds actually are more or less checked by surface friction, turbulence, convection, etc., they fail to attain full gradient velocities, and therefore cross the isobars at a small angle, except near the surface, where this angle is much larger, and thus maintain a correspondingly vigorous interzonal circulation even in the absence of cyclones and anticyclones; (*d*) that the actual west to east winds of the middle and higher latitudes and the east to west winds of equatorial regions are due chiefly to their approach to gradient directions, and, finally, (*e*) that the strength of any steady wind is proportional, approximately, to the gradient pressure, and its direction substantially normal thereto.

Probable Interzonal Circulation of the Stratosphere.—The primary circulation just explained involves all the atmosphere from the surface of the earth up to at least the highest cloud levels, but there is reason to believe that it does not extend to the greatest altitudes. Indeed, it appears probable that far above the uppermost clouds there may be another primary or fundamental circu-

lation in reverse direction to that of the lower. This inference is based on the fact that the stratosphere is so much warmer in high than in low latitudes that seemingly there must be an overflow of air from the former to the latter and a corresponding return; that is, a primary circulation in the stratosphere in which the upper branch is from the polar (in this case warmer) toward the equatorial (in this case colder) regions and the under from the equatorial toward the polar regions, with, of course, longitudinal components in each due to the earth's rotation. In a sense the upper circulation, if it exists as inferred, is the mirror image of the lower, though more regular.

In addition to the primary circulation or circulations of the atmosphere as a whole, there are several secondary circulations or wind systems of magnitude sufficient to bring them markedly under the influence of the earth's rotation. It will be convenient next to consider some of the more important of these winds.

Chapter IX.

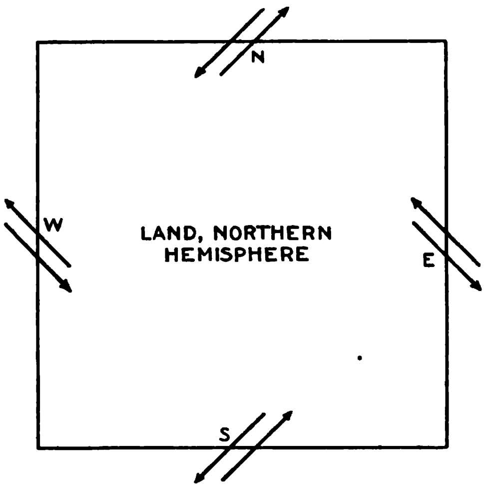
ATMOSPHERIC CIRCULATION (*continued*).

Winds Due to Widespread Heating and Cooling (continued).

MONSOONS.

Summer monsoons and winter monsoons, for convenience discussed under the same head, bear the same relation to summer and winter that sea breezes and land breezes bear to day and night. It is the temperature contrast between land and water that establishes the circulation that manifests itself on the surface as a sea

Fig. 48.



Prevailing directions of monsoon winds, northern hemisphere.

or land breeze in the one case and as a seasonal or monsoon wind in the other. The direction of the surface wind in either case is always from the cooler toward the warmer of the adjacent regions, from the ocean toward the land by day as a sea breeze and during the warmer season as a summer monsoon; from the land toward the ocean by night as a land breeze and during the colder season as a winter monsoon. Hence monsoons may be regarded as sea and land breezes of seasonal duration, and might very well be classed with the latter under some common appropriate caption. However, because of the immense areas involved,

it cannot be said of them, as of sea and land breezes, that they are caused by mere local temperature differences. Besides, the duration of a land or sea breeze is so brief that it covers only a narrow strip along the coast, as already explained, while the monsoon winds extend far from the coast, both inland and to sea and the directions of the former, since their paths are always short, are but little affected by the rotation of the earth, while the courses of the second are greatly modified by this important factor.

The prevailing directions of monsoon winds, except where distinctly modified by the general circulation, are given by the following table and by Figs. 48 and 49:

Direction of Monsoon Winds

Hemisphere	Season	Land south	Land west	Land north	Land east
Northern	Summer	N. E.	S. E.	S. W.	N. W.
	Winter	S. W.	N. W.	N. E.	S. E.
Southern	Summer	N. W.	N. E.	S. E.	S. W.
	Winter	S. E.	S. W.	N. W.	N. E.

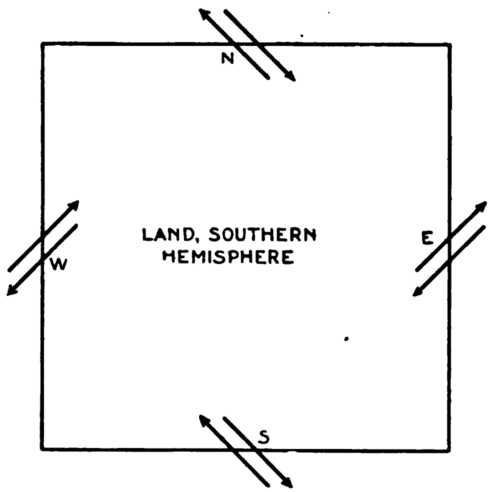
Since monsoons depend upon seasonal temperature contrasts between land and water, it is obvious that winds of this class must be most pronounced where such contrasts are greatest—that is, in temperate regions—and least developed where the temperature contrasts are smallest—that is, in equatorial and polar regions. It is even possible for secondary monsoons to develop, or for a monsoon to occur within a monsoon. This merely requires a favorably situated inland sea, such as the Caspian. In such cases monsoons or seasonal winds prevail between the inland sea and the surrounding land, and in turn between the continent as a whole and the adjacent oceans, just as, and for the same reason that on a still greater scale, there is a constant circulation between the perpetually warm equatorial regions and those about the poles that are continually cold.

Another comparison between these several winds, the semi-daily (land and sea breeze), semi-annual (monsoon), and perpetual (interzonal), that is interesting and instructive concerns

their depth. As already stated, the land and sea breezes seldom reach greater depths than 100 to 500 metres; the winter monsoon of India has a depth, roughly, of 2000 metres, and the summer monsoon 5000 metres; while the general or interzonal circulation involves the whole of the troposphere with a depth of 10 to 12 kilometres, and probably also, though perhaps to a less vigorous degree, even the stratosphere.

If the term monsoon be extended, as it properly may, to include all winds whose prevailing directions and velocities

FIG. 49.



Prevailing directions of monsoon winds, southern hemisphere.

undergo distinct alterations as a result of seasonal changes in temperature, it clearly follows that this class of winds is well nigh universal. Nevertheless, it is generally thought of in connection with only those places where it is most strongly developed, and especially where the seasonal winds are more or less oppositely directed. Among these places are: India (Indian monsoons are the most pronounced of all and have been most fully studied), China, the Caspian Sea, Australia, and portions of Africa.

In the United States the chief monsoon effects are in the eastern portion, where the prevailing winds are northwest in winter and southwest in summer, and in Texas, where the prevailing winds are also northwest in winter, but southeast in summer.

TRADE WINDS.

As previously stated, in equatorial ocean regions, or, roughly, over the oceans between latitudes 30° N. and 30° S., the winds usually have an east-to-west component. In the northern hemisphere they blow rather constantly from the northeast, becoming east-northeast and finally nearly east winds as the equator is approached. Similarly, in the southern hemisphere, starting from the southeast, they gradually back through east-southeast to nearly east. In each case they blow " trade "; that is, in a fixed or nearly fixed direction. It is because of this steadiness of direction and not because of any relation they may have to the paths of commerce that they are called trade winds. Along each border of this belt, or along both the northern and southern horse latitudes, calms are frequent, while such winds as do occur generally are light and variable in direction. Besides, the barometric pressure is high, humidity low, and sky clear. Hence it generally is inferred that throughout the horse latitudes the air is descending. This evidence, however, as applied to places other than the centres of maximum pressure is not quite conclusive—it only shows that the air is not ascending.

Another narrow belt of calms or light variable winds, known as the region of the doldrums, approximately follows the equator (more exactly the thermal equator), where the two systems of trade winds, the northern and the southern, come together. Here, however, the barometric pressure is low, humidity high, and skies often filled with cumulus and other clouds that give conclusive proof of strong ascending currents.

Trade winds in the sense here used—that is nearly constant winds blowing in a westerly direction—do not occur on land except along coasts and over islands. Besides being well-nigh peculiar to the oceans, they are even different from ocean to ocean, and also, since they tend to follow the thermal equator, somewhat different in latitude and intensity from season to season.

According to Shaw the average velocities of the Atlantic trade winds are as follows:

Trade-wind Velocities, Atlantic Ocean.

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year	
N. E. trade...	11.9	13.0	13.5	13.4	12.3	11.4	10.3	8.3	9.6	7.4	9 8	11.6	10.5	miles hour
S E. trade...	14.1	13.0	13.0	12.1	11.0	12.1	12.1	15.0	17.0	15.0	16.1	15.0	13 9	miles hour

From this it appears that the trades are strongest during the winter when their counterpart, the system of westerly winds of

higher latitudes, is strongest; and weakest during the summer when their counterpart is weakest. It also appears that the southeast trades, or those pertaining to the southern hemisphere, are about one-third stronger than the northeast trades, due probably to the greater extent of the southern oceans and consequent less surface friction—the same reason, doubtless, that the westerly winds of the southern hemisphere are stronger, on the average, than the westerlies of the northern hemisphere.

The trade winds of the Pacific Ocean are weaker than those of the Atlantic and not so constant in direction. On the Indian Ocean the trades are confined to the southern hemisphere. North of the equator the winds of this ocean, being controlled by the adjacent continent, are distinctly of the monsoon type.

The seasonal shifting in latitude of the trade regions and belt of doldrums is shown by the following table, copied from Hann's Lehrbuch, 3d edition, p. 463:

Seasonal Latitude Limits of Trade Winds and Doldrums.

	March		September	
	Atlantic	Pacific	Atlantic	Pacific
N. E. trade.	26°–3°N	25°–5°N.	25°–11°N.	30°–10°N.
Doldrums...	3°N.–Equator	5°–3°N.	11°– 3°N.	10°– 7°N.
S. E. trade..	Equator–26°S.	3°N.–28°S.	3°N.–25°S.	7°N.–20°S.

ANTITRADE WINDS.

As the heated and expanded air of equatorial regions overflows to higher latitudes it necessarily is deflected by the rotation of the earth. That portion which goes north changes from an east wind near the equator to a southeast, south, southwest, and, finally, at about latitude 35° N., a more nearly west wind. Similarly, that portion which goes south becomes northeast, north, northwest, and, finally, at about latitude 30° S. a more nearly west wind.

At great altitudes, 10 to 15 kilometres, the east-to-west velocity near the equator is, roughly, 36 metres per second (80 miles per hour). Hence its west-to-east velocity around the axis of the earth is about 428 metres per second (957 miles per hour). As this air, assuming it to start from the equator and neglecting viscosity effects, moves to higher latitudes its west-to-east velocity

must so increase, according to the law of the conservation of areas, that at about 16° N. or S. its angular velocity will be the same as that of the earth, and itself, therefore, be moving only poleward in the plane of the meridian. The exact latitude, however, at which the antitrades move directly poleward depends upon the position of the thermal equator and therefore varies with the seasons. Thus during August and September, when the centre of the doldrums is, roughly, 8° N., the inflection of the northern antitrades occurs somewhere between latitudes 20° N. and 25° N. At other seasons, because the doldrums are then nearer the equator, the place of inflection is also less removed. Beyond the turning point, wherever that may be, these upper or antitrade winds become westerly, and, except as modified by local disturbances, tend, as previously explained, to reach, under the influence of the poleward pressure, a limiting or gradient velocity, and to follow parallels of latitude. However, there are innumerable disturbances, mainly due to the distribution of land and water, that cause constant and abundant interzonal circulation which feeds and indefinitely maintains the antitrade wind portion of the general or planetary atmospheric circulation.

The height of the antitrades (depth of the trades) is greatest, at any given place, during summer and least during winter. It also decreases with latitude, becoming zero, on the average, at about 30° N. and S. Thus during winter their height over Cuba, 22° N., is about 3.5 kilometres; over Hawaii, 19° 30′ N., about 3 kilometres; over Jamaica, 17° N., 6.5 kilometres; and over Trinidad, 12° N., 8 kilometres. But whatever their height it is always the same as the depth of the trades of which they are but the overhead continuation. Indeed, the trade winds as they approach the equator ascend and gradually flow off poleward, thus producing in each hemisphere a great antitrade branch of the general circulation, which in turn becomes the westerlies of higher latitudes. These, in their turn, are confused by storms and other local disturbances, but after few or many vicissitudes, as circumstances may determine, ultimately return to a similar starting-point, only to begin another of their endless cyclic journeys through trades, antitrades, westerlies, and the innumerable secondary winds such a course implies.

TROPICAL CYCLONES.

A tropical cyclone—the cyclone of the Indian Seas, the hurricane of the West Indies and South Pacific, and the typhoon of the West Pacific and China Sea—consists of a vast whirl of rapidly moving air currents surrounding a calm and relatively small centre or vortex.

Distinction Between Tropical and Extra-tropical Cyclones.—Although tropical and extra-tropical cyclones have many similarities, such as low-pressure centres, abundant precipitation, same instantaneous wind directions, and the like, and although it may be impossible to say just when a tropical cyclone on its way to higher latitudes becomes extratropical in character, nevertheless they usually differ from each other in several important respects. Among these differences are: (*a*) The isobars of the tropical cyclone generally are more symmetrical and more nearly circular than those of the extra-tropical. (*b*) The temperature distribution around the vortex of the tropical cyclone is practically the same in every direction, while about the extra-tropical it is very different. (*c*) In tropical cyclones rains are torrential and more or less equally distributed on all sides of the centre; in the extra-tropical rains usually are much lighter and very unequal in different quadrants. (*d*) Tropical cyclones usually have calm rainless centres 10 to 50 kilometres (6 to 31 miles) or more in diameter, while the extra-tropical rarely show this characteristic whirl phenomenon. (*e*) Tropical cyclones are most frequent during summer of the hemisphere in which they occur, while the extra-tropical are strongest and most numerous during winter. (*f*) Tropical cyclones often move to higher latitudes, where they assume, more or less completely, characteristics of the extra-tropical; the extra-tropical, on the other hand, never invade the region of the tropical nor assume its distinctive characteristics. (*g*) The pressure-drop of the tropical cyclone generally begins with the winds; in the extra-tropical it usually begins much sooner. (*h*) The tropical cyclone has no anticyclone companion; the extra-tropical usually has—to the west.

Place of Occurrence.—Tropical cyclones occur over the warmer portions of all oceans except, possibly, the South Atlantic. They are most numerous, however, in the west Atlantic (including the Gulf of Mexico), Bay or Sea of Bengal, and west Pacific (including the China Sea), where their annual frequencies are,

roughly, about 4, 8 and 24 respectively. They seldom originate closer than 5° or 6° to the equator but most frequently between latitudes 10° and 20.° In fact, they seem to originate almost entirely along the belt of doldrums, and therefore, since this belt follows the sun, to appear at higher latitudes during summer and lower, or not at all, during winter.

Size and Shape of Storm.—The diameter of the tropical cyclone varies greatly. Near their origin some storms may be no more than 80 kilometres (50 miles) across, while others, when well developed, may have diameters of 300 to 1500 kilometres (187 to 932 miles). The clouded area incident to typhoons, always much more extensive than the surface storm, may be even 3000 kilometres (1864 miles) across.

The shape of the storm, as given by the isobars, appears usually to be that of an ellipse whose diameters are to each other, roughly, as 2 to 3, with the longer axis in the direction of travel.

Direction of Wind.—The direction of the surface wind is spirally in at an angle of 30 degrees, roughly, to the isobars, counter-clockwise in the northern hemisphere, clockwise in the southern. At an elevation of only 700 to 800 metres the inflow is said to cease, and above this level the circulation is outward. These horizontal motions necessitate a correspondingly strong upward component around the vortex or inner portion of the storm, and a slower downward component over a much greater surrounding area.

Velocity of Wind.—The velocity of the wind in a tropical cyclone also varies greatly from one storm to another, and even more from one to another portion of the same storm. Near the centre, or within the eye of the storm, which may have any diameter from 8 to 50 kilometres (5 to 31 miles) or more, the wind is very light and the sky clear or only partially covered with high clouds. Away from this centre, especially on the poleward side, the winds often reach destructive velocities of 40 to 50 and even 60 metres per second (90 to 112 or even 134 miles per hour), but decrease in violence rather rapidly with increase of distance from the centre; dropping to only moderate winds of 50 to 60 kilometres (31 to 37 miles) per hour at a distance of, say, 300 kilometres (187 miles).

Direction of Travel.—Tropical cyclones of the northern hemisphere first move west, then usually northwest. Many turn

north at latitude 20° to 25°, roughly, and finally move away to the northeast. In the southern hemisphere the corresponding directions of travel of the tropical cyclone are: West, southwest, south, and, finally, southeast.

Velocity of Travel.—The velocity with which tropical cyclones travel varies from almost zero in certain cases, especially at or near the place of inflection when this happens to be abrupt, to perhaps 800 kilometres (497 miles) per day. Over the Bay of Bengal, Arabian Sea, and China Sea the velocity averages about 320 kilometres (199 miles) per day. Over the south Indian Ocean the velocity ranges from 80 to 320 kilometres (50 to 199 miles) per day. Over the west Atlantic the average velocity before and during recurvature is about 420 kilometres (260 miles) per day, but after recurvature—that is, when moving northeast over middle latitudes—about 640 kilometres (398 miles) per day.

Origin and Maintenance.—Since tropical cyclones originate in a belt or region of doldrums where convectional rains are frequent and heavy, and since they rarely occur closer than 5° or 6° to the equator, it follows that both vertical convection and earth rotation are essential to their genesis.

The atmosphere of a doldrum belt becomes very warm and humid, and therefore frequently is in a state of vertical convection. The upward branches of this convection are nearly always limited to very restricted areas, where they break through, as it were, and often give rise to local thunderstorms. Occasionally, however, heating and expansion must take place more or less uniformly over a comparatively extended region. So long as the upward current is gentle and restricted to a small area, the compensating inflow from the sides is also gentle and can produce only a cumulus cloud and perhaps a thunderstorm. In the event that such a storm is formed, the inflowing counter-current to the ascending warm air is replaced by an equivalent column or sheet of descending cold air immediately to the rear. That is, the loss of warm surface air is compensated by a similarly concentrated and vigorous downflow of cold upper air. Hence, rotary circulation since it depends upon horizontal inflow from all, or at least several, sides, is not possible in the case of ordinary thunderstorms, whatever their location.

On the other hand, an approximately equal expansion of air

over a relatively large area, whether caused by an increase of temperature, or vapor density, or by both, must lead to an overflow above and a corresponding surface inflow around the outer borders.

Obviously the rate of volume overflow at any time is proportional to the area in question, while the corresponding inflow is proportional to the boundary multiplied by the average normal component of the wind. If the area is circular with radius R, it follows that the rate of outflow above is proportional to πR^2, and the rate of inflow below to $2\pi R V_n$, in which V_n is the average radially inward component of the wind at the distance R from the centre. But as the two currents compensate each other except as modified by precipitation, explained below, it follows that V_n, other things being equal, is proportional to R. Hence, when the area involved is rather large, 100 miles, say, in diameter, the relatively shallow and spirally moving compensating or return current may become very perceptible. This at once feeds the entire rising column with excessively humid air that renders it an even better absorber than before of both insolation and terrestrial radiation and increases its rate of expansion, thus initiating, perhaps, a widespread condensation. If so, the latent heat thus set free, while it does not actually raise the temperature of the air, reduces the rate of adiabatic cooling from approximately 1° C. to about 0.4° per 100 metres increase of elevation, and thereby establishes within the rising column temperatures distinctly higher than those of the surrounding air at the same level. In this way the circulation is accelerated, and thereby the rate of condensation and freeing of latent heat increased until, through growth of size, restricted supply of water vapor, and other causes, a limiting, somewhat steady, state is attained.

When the conditions here described occur at some distance from the equator the rotation of the earth deflects the inflowing air and establishes a rotation around the region of lowest pressure —an effect all the more likely to occur (perhaps rarely else does occur), when the existing convection takes place along the doldrum boundary between the rather oppositely directed (not opposing) trade winds, the one from higher latitudes, the other from across the equator. But whatever the radius of curvature, the angular momentum remains constant—the law of the conservation of areas obtains—except as modified by friction and viscosity, and there-

fore, since surface-drag is effective at but small elevations, the atmosphere at only 100 to 200 metres above the water may, as it moves inward, soon reach that velocity at which its deflective force is equal to the horizontal pressure gradient. When such velocity is reached, as it obviously may be at any appreciable altitude, inflow at that place necessarily ceases. Near the water, however, this limiting velocity is prevented by surface friction. Hence, as soon as the whirl is well established, it must be fed almost exclusively by the lowest and therefore most humid air. In this way a maximum amount of precipitation, and, through it, a maximum amount of thermal energy, is secured—a condition important to the maintenance of the tropical cyclone, as is evident from the fact that it tends to go to pieces over dry land, especially before it has recurved and become essentially extra-tropical.

Of course, similar atmospheric expansions may, and doubtless do, occur in the doldrums when they are on or very close to the equator, but in this case a whirl is impossible, and therefore a low so initiated will soon fill by gentle, somewhat radial, winds from all sides and at considerable altitudes, or, at most, mere local thunderstorms will develop.

From the above it is evident that the seat, so to speak, of the tropical cyclone is where the sustaining energy is supplied; that is, where condensation is taking place. Hence the movement of the air at this level, and not at the surface, determines the course of the storm, and even carries it athwart shallow surface winds.

Chapter X.

ATMOSPHERIC CIRCULATION (*continued*).

Winds Due to Widespread Heating and Cooling (continued).

EXTRA-TROPICAL CYCLONES.

General Remarks.—The strong winds and heavy precipitations of middle latitudes are associated with the occurrence of low barometric pressure, while gentle winds and clear skies as commonly are associated with the occurrence of high barometric pressure. Hence the cyclone, or that system of winds that accompanies and surrounds any considerable region of minimum pressure, and the anticyclone, or that system of winds that belongs to and encircles a region of maximum pressure, deserve and have received a vast amount of observation and study. Nevertheless, in many respects—in their origin, in their temperature distributions, and in the laws of their movements—cyclones and anticyclones still remain in great measure the meteorological mysteries they have always been.

Although the cyclones and anticyclones of extra-tropical regions are as closely associated and as fully the complements of each other as are hills and hollows, it nevertheless will be convenient to consider them independently. It will also be convenient first to summarize the facts of observations, and then to string these facts together on the thread of a provisional theory.

Size.—The area covered by an extra-tropical cyclone, the largest of all distinctive storms, nearly always amounts to millions of square kilometres. In North America the average diameter of these storms is estimated to be, roughly, 2500 kilometres (1553 miles), which probably is not greatly different from their average diameter on other continents. Over the North Atlantic their diameters are still larger, while the greatest of all in size is the semipermanent or winter Aleutian "low," which appears usually to be much larger than the travelling cyclones of the Atlantic or even the great semipermanent Icelandic "low."

Direction of Movement of the Cyclonic Centre.—The direction the centre of a cyclone travels, wherever it may be located, is substantially the same as that of the higher clouds or of the atmosphere at 4 to 10 kilometres above sea level. In general,

therefore, the cyclones of middle latitudes travel from west to east, with (in the northern hemisphere) a southerly dip over continents and a northerly deflection over oceans. Cyclones, for instance, that develop between latitudes 30° and 45°, in the western United States, usually turn northeast before reaching the Mississippi River. Farther west they may move east, or even somewhat southeast.

Locus of Maximum Cyclonic Frequency, or Chief Paths of Cyclonic Storms.—Probably no part of the earth's surface more than 3 or 4 degrees from the equator is wholly free from cyclonic storms, but the frequency of their occurrence varies greatly with respect to both time and place. Beginning with the West Pacific: During summer and fall many cyclonic storms come from the general region of the Philippines and move northeast across or on either side of Japan. Winter and spring cyclones enter on this same general course at latitudes 30 degrees to 40 degrees; some, presumably, being of oceanic origin, while others obviously either develop within or cross over China. In any case, the general track of these storms is along the Japanese and Kurile Islands, and thence east over the Bering Sea. The main path is then southeast across the Gulf of Alaska, with the storms, including off-shoots from the Aleutian "low," crossing onto the continent anywhere between latitudes 40 degrees and 60 degrees, but apparently most frequently in the general neighborhood of Vancouver Island. These Pacific storms usually cross the continent nearly from west to east, dipping slightly south over the Great Lakes, and finally leave it by way of Newfoundland. A smaller number of storms from the North Pacific dip far south, somewhat like the Mediterranean branch mentioned below, to about latitudes 35 degrees to 40 degrees, but usually recurve west of the Mississippi and join the main course as they reach the Atlantic. Those that originate in or cross over the central and southern portions of the United States, as also those that come from the Gulf of Mexico, move northeast and gradually merge their paths with that of the Pacific storms anywhere from the Great Lakes to the Newfoundland Banks. Other cyclones coming up from the Florida and West Indies regions follow the coast, not far off shore, and also merge their paths with that of the others in the neighborhood of Newfoundland. Half-way or more across the Atlantic the path of maximum

storm frequency breaks up into at least three distinct routes. The main route turns far north, usually by way of the Norwegian Sea, then southeast, entering Russia in the neighborhood of the White Sea, and passing on toward Central Asia. A second route turns southeast and crosses Europe generally along the northern side of the Mediterranean, and then turns north either across Austria toward northwest Russia or by way of the Black Sea toward Central Asia. A third and least frequented route, commonly running just south of Ireland, appears to cross both the North and the Baltic Sea, and then, like the others, to move on toward Siberia and central Asia. The cyclonic storms of central and northern Asia do not appear to be very numerous. Nevertheless, their track of maximum frequency seems to turn south, as does the similar track over North America, as far as Lake Baikal, thence probably to the Sea of Okhotsk and across or to the south of Kamchatka to the main storm path north of the Aleutians, as already explained.

In the southern hemisphere the path of maximum storm frequency appears closely to follow the 60-degree parallel of latitude. Presumably it dips poleward at both the Ross and the Weddell Sea, as each of these is a region of semipermanent low pressure.

It must be remembered, in this connection, however, that in both the southern hemisphere and the northern, cyclonic storms occur almost everywhere, and therefore that the routes above described are only paths of maximum cyclonic frequency and not of exclusive travel.

Velocity of Travel.—The velocity with which the centre of a cyclonic storm moves along its path varies greatly. It depends upon the season, being fastest in winter and slowest in summer; upon location, being faster in America than in Europe, for instance; and, finally, upon the individual storm.

The following table gives average velocities of cyclonic centres for different parts of the northern hemisphere. Those pertaining to the United States were computed from a table of average 24-hour movements as determined by Bowie and Weightman[48] from 16,239 observations, covering the years 1892–1912, inclusive. The others are from Hann's "Lehrbuch der Meteorologie," 3d edition, p. 518.

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[48] *Monthly Weather Review*, Supplement No. 1, p. 8, 1914.

Average Velocity of Cyclones in Metres per Second.
(For the United States the velocity is also given in miles per hour.)

	United States	Japan	Russia	North Atlantic	West Europe	Bering Sea.
Winter......	(29.9) 13.4	12.4	10.8	8.2	8.0	8.5
Spring......	(23.7) 10.6	11.1	9.2	8.3	7.2	8.5
Summer.....	(20.7) 9.3	7.8	8.0	7.4	6.6	10.3
Fall.........	(24.5) 11.0	10.6	9.6	8.3	8.2	9.3
Year........	(24.7) 11.1	10.5	9.4	8.05	7.5	9.1

Frequency.—The frequency of the occurrence of cyclonic storms varies not only from place to place, as already explained, but also at any given place, or even over an extensive area—probably an entire hemisphere—according to season. Tropical cyclones, it will be recalled, are far more frequent during summer and early fall than during winter. Mid-latitude cyclones, on the other hand, have exactly the opposite relation of frequency to season, being, in general, most numerous in winter and least numerous in summer. Exceptions to this rule apply to the paths of tropical cyclones next after the recurvature, provided we regard such storms as having then become extra-tropical. Perhaps exceptions also apply to certain regions on the poleward sides of the main cyclonic routes, since these are farthest north in summer and farthest south in winter. This, however, is not certain. A statistical investigation might show that even the greatest increase due to latitude shift is more than compensated by the general seasonal decrease in frequency.

When all storms are counted that appear in the United States or Southern Canada, whether short or long lived, weak or intense, it appears [49] that the frequency of summer (June, July, and August) "lows" is to that of winter (December, January, and February) "lows" approximately as 5 to 8. On the other hand, if only long-lived cyclones are considered, it appears [50] that in the United States the frequencies of summer to winter storms are about as only 2 to 9, and those of Europe as 3 to 10.

In either case, then—that is, whether only the longer lived and more intense "lows" are counted, or whether all, of whatever magnitude and duration, are included—it seems that cyclonic

[49] Bowie and Weightman, *Monthly Weather Review*, Supplement No. 1, p. 7, 1914.
[50] U. S. Weather Bureau Bulletin A, p. 6. 1893.

storms are most frequent during winter and least frequent during summer. Further, the extra-tropical storms of winter are not only more numerous than those of summer, but also, in general, longer lived, more intense, and faster moving.

Direction of Winds.—From the directions of winds in and about a region of low barometric pressure, as given on synoptic charts, Fig. 99, for instance, it is often, perhaps usually, inferred that cyclonic winds circulate spirally inward and upward and then outward and upward, counter-clockwise in the northern hemisphere, clockwise in the southern, around a storm axis. This indeed is, in general, the course of the winds in tropical cyclones, especially in those that are violent and of small diameter, as the eye of the storm and directions of cloud movements clearly indicate, but it does not apply to extra-tropical cyclones, except, perhaps, to the occasional ones of great violence and small diameter. Extra-tropical cyclones rarely have clear centres, as they would if the circulation about them was closed or along spiral paths of repeated turns. Neither, in general, is a closed circulation indicated by the movements of the clouds. Again, it often happens that the velocity of the forward moving wind of a cyclone is less than that of the storm itself, so that instead of flowing around the storm centre it necessarily is left behind.

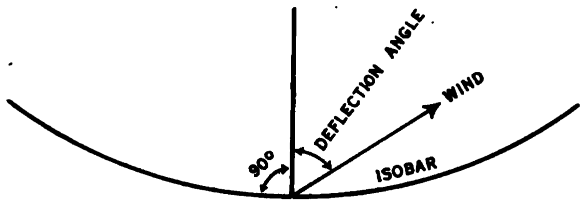
Synoptic weather charts, therefore, show instantaneous wind directions, but not wind-paths. This is because the storm condition itself is moving forward—moving, indeed, with a velocity nearly always comparable to, and at times even faster than, that of the lower winds themselves.

The main body of the storm winds, those below an elevation of 5 or 6 kilometres, except everywhere near the surface, and also generally about the poleward side above 2 to 3 kilometres elevation, blow, in the cyclonic sense, roughly parallel to the surface isobars. This does not mean that the path of any given particle of air is around and around the centre of low pressure, because, as above explained, this centre itself and its system of isobars are both in rapid transit. Near the surface the velocity is so slowed down that the deflection forces no longer balance the horizontal pressure, and therefore the winds of this level are directed inward at a considerable angle across the isobars. Through the poleward half of the storm area the horizontal temperature gradient is nearly always opposite in general direction to the horizontal

pressure gradient at the surface. Therefore, with increase of elevation in this section the pressure gradient usually weakens from the start and later reverses at the height of only a few kilometres—often less than one kilometre; while the winds first increase (where the surface drag rapidly decreases) to a maximum, then decrease, and later more or less reverse in direction.

Deflection Angle.—The angle between the surface wind direction at any place within a cyclonic storm and the normal to the corresponding isobar, the "deflection" angle (Fig. 50), is greatest, or the surface winds most nearly parallel to the isobars, *a*, when the winds are swiftest and thus develop the strongest deflective forces—therefore greatest to the south and east of the storm centre and least to the north and west, *b*, when the velocity

FIG. 50.



Deflection angle.

of the storm as a whole is least, *c*, in the summer time, because during this season the storm movement is less than during other seasons, *d*, over water where it is roughly 80 degrees, because the surface drag is less here than over land where the "deflection" angle averages only 40 degrees to 50 degrees.

It is important to note also that usually the deflection angle does not greatly change with distance from the centre. This follows from the fact that the horizontal pressure, the wind velocity, and the consequent surface friction and percentage loss of gradient velocity all are roughly constant in any given direction from the centre, so long as only points distinctly within the storm area are considered.

With increase of elevation and consequent decrease of surface drag the deflection angle over land gets larger by 25 or 30 degrees in the first kilometre. Beyond this elevation it still gains, but relatively very slowly. At an elevation of several kilometres the velocity of the air is decidedly greater than that of the storm,

and therefore air that may have risen to this level is carried forward. Hence the main outflow of the extra-tropical cyclone is toward the east.

Wind Velocity.—As just stated, the pressure gradient and wind velocity are roughly constant along any given radius from the storm centre. This is because at middle and higher latitudes the deflective force is essentially geostrophic (due to the rotation of the earth) and to only a small extent cyclostrophic (due to

Cyclonic Wind Velocity in Metres per Second and (Miles per Hour).

	Altitude	Surface 122 m.	500 m.	1000 m.	1500 m.	2000 m.	2500 m.	3000 m.	3500 m.	4000 m.
South quadrant	Winter	6.16 (13.8)	14.75 (33.0)	15.09 (33.8)	15.32 (34.2)	15.79 (35.3)	16.85 (37.7)	18.28 (40.9)	19.25 (43.1)	20.30 (45.4)
	Summer	5.42 (12.1)	9.64 (21.6)	10.69 (23.9)	11.31 (25.3)	12.22 (27.3)	12.91 (28.9)	14.08 (31.5)	15.69 (35.1)	17.21 (38.5)
	Year	5.84 (13.1)	12.24 (27.4)	12.93 (29.0)	13.38 (30.0)	14.07 (31.4)	14.94 (33.4)	16.24 (36.3)	17.53 (39.2)	18.81 (42.1)
West quadrant	Winter	6.70 (15.0)	13.47 (30.1)	13.39 (30.0)	13.93 (31.2)	14.75 (33.0)	16.18 (36.2)	17.31 (38.7)	17.31 (38.7)	.... (....)
	Summer	5.66 (12.7)	9.45 (21.2)	9.89 (22.1)	9.91 (22.1)	10.17 (22.7)	10.65 (23.8)	11.40 (25.5)	12.07 (27.0)	12.62 (28.2)
	Year	6.06 (13.5)	11.34 (25.4)	11.52 (25.7)	11.80 (26.4)	12.34 (27.6)	13.29 (29.8)	14.23 (31.9)	14.56 (32.6)	.... (....)
North quadrant	Winter	4.72 (10.5)	8.85 (19.8)	8.95 (20.0)	9.00 (20.1)	9.09 (20.4)	9.07 (20.3)	9.32 (20.8)	8.52 (19.0)	8.02 (17.9)
	Summer	4.84 (10.8)	7.85 (17.6)	8.37 (18.7)	8.45 (18.9)	8.81 (19.7)	9.11 (20.4)	10.26 (22.9)	10.63 (23.8)	10.63 (23.8)
	Year	4.79 (10.7)	8.36 (18.7)	8.66 (19.4)	8.72 (19.5)	8.94 (20.0)	9.08 (20.3)	9.78 (21.9)	9.57 (21.4)	9.32 (20.8)
East quadrant	Winter	4.50 (10.1)	10.45 (23.4)	10.02 (22.4)	10.43 (23.4)	10.58 (23.7)	11.64 (26.0)	12.11 (27.1)	13.37 (29.9)	14.59 (32.7)
	Summer	4.11 (9.2)	8.22 (18.4)	8.64 (19.4)	8.77 (19.6)	8.98 (20.1)	9.50 (21.3)	9.50 (21.3)	9.81 (21.9)	11.86 (26.5)
	Year	4.34 (9.7)	9.37 (20.9)	9.37 (20.9)	9.64 (21.6)	9.82 (21.9)	10.61 (23.7)	10.84 (24.3)	11.62 (26.0)	13.25 (29.7)

circular motion). The winds, however, often are different in different portions of the storm area, and commonly strongest in its southern and eastern quadrants, where the isobars are most crowded and the direction of the winds roughly that of the storm movement.

The actual average velocity of the wind in the different quadrants of a cyclone and at different elevations is given in the pre-

ceding table by Peppler,[51] based on a large number of measurements made during 1903–1908, at Lindenburg; latitude 52° 10′ N.; longitude, 14° 15′ E. Probably other mid-latitude regions have approximately the same average cyclonic wind velocities. This, however, is not certain, nor are there available sufficient data for determining the question.

Convection.—The vertical movements of the air, whether up or down, in an extra-tropical cyclone, or between such a cyclone and a neighboring anticyclone, are not known with much detail and accuracy. However, since the cyclone moves eastward with the air currents directed inward across the isobars, it is obvious that ordinarily the chief air convergence, due in part to increase of latitude, and hence the principal vertical convection, must be on the front or east side. Temperature also usually helps to locate the chief upflow in this quadrant, since its winds necessarily are from lower latitudes, and, therefore, relatively warm.

This localization of the uprising air explains why, other things being equal, most of the precipitation due to cyclonic storms occurs to the east and southeast (northeast in the southern hemisphere) of their centres.

Other things, however, are not always equal. Thus an extensive plain rising gradually to great elevations may slope in such direction that the mechanical or forced convection over it on any side of a cyclonic centre may approach, or even exceed, the thermal convection to the east. The Great Plains east of the Rocky Mountains illustrate this point. Here precipitation in the case of "stagnant" or slow-moving lows usually is most pronounced to the north of the centre where the winds are persistently up the slope. This is an extreme case, but it suffices to show that a rule relating to shifting winds and clear or foul weather that applies well in one place may not apply at all in another. Similarly, on the Pacific coast of North America, for instance, where the ocean is to the immediate west, the heaviest rains are to the south and west of the cyclonic centre.

Velocity of Travel and Amount of Precipitation.—It is well known that the velocity of travel of an extra-tropical cyclone and the amount of precipitation accompanying it are to each other, roughly, in inverse ratio. This is simply because the slower the storm travels, the longer the winds blow into it at any

[51] *Beiträge zur Physik der freien Atmosphäre*, 4, p. 95, 1911.

given place and, therefore, other things being equal, the greater the duration and the amount of the precipitation at that place. In extreme cases very fast moving cyclones may give but little or even no precipitation at all.

Classification.—Cyclones occur in extra-tropical regions with so great frequency that several such storms are nearly always present in each hemisphere. Naturally they have been much studied and therefore variously classified, especially according: to duration, as semipermanent and migratory; to season of occurrence, as summer and winter; to zone of origin, as tropical and extra-tropical; and to the place from which first reported as, for instance (referring to only those within or near the United States), Alberta, North Pacific, South Pacific, Northern Rocky Mountain, Colorado, Texas, East Gulf, South Atlantic, and Central.

All these classifications are useful, but not adapted to the present purpose, which is to group the cyclones, as far as practicable, according to their more important causes. Perhaps this end will be fairly well served by dividing them into *thermal* (identical with semipermanent), *insolational,* and *mechanical.*

Thermal (*Due to Relatively Warm Water*).—The name semipermanent cyclone—for which the alternate name, thermal cyclone, is here proposed for reasons that will appear below—or semipermanent "low," has been given to that system of winds of any region over which the barometric pressure habitually or seasonally averages lower than for the surrounding regions. The term generally is used as though it applied to but one and the same cyclone, however it might wander or even for a time wholly disappear. Thus, one always says *the* Icelandic "low," not *an* Icelandic "low." Similarly, *the* Aleutian "low," not *an* Aleutian "low." But, as stated, this applies only to average conditions. In reality there is no one permanent Icelandic "low," for instance, that retains its identity wherever it may be, but only a series of sluggish or temporarily fixed lows, all of which originate over, or, on invading, become intensified over, practically the same restricted region.

There are several semipermanent cyclones in various parts of the world. The most nearly continuously active of these, at least in the northern hemisphere, and at all seasons apparently productive of many migrating cyclones, lies southeast of Green-

land and southwest of Iceland. Another such region, active during winters only and known as the Aleutian " low," lies along and to the south and southeast of the Aleutians, extending into and including the Gulf of Alaska. The Norwegian Sea and, possibly, the Sea of Okhotsk are other such high-latitude regions. The Gulf of Lyons is a low-pressure haunt during winter, as is also the Black Sea, and the Caspian Sea, as its monsoon winds definitely show. The Gulf of Mexico, over which occasional winter cyclones appear to generate, may likewise be added to the above list.

In the southern hemisphere the regions of most persistent lows are the Ross Sea and its counterpart, the Weddell Sea, on the other side of the continent.

All the above regions have surfaces warmer than those that at least partially surround them. The circulation induced by such temperature distribution is converted into a system of cyclonic winds by the deflective force due to the earth's rotation. The warm waters off the coast of Greenland and Iceland, for instance, necessarily maintain the atmosphere above at higher temperatures, level for level, than that of the neighboring ice-caps. Hence a practically continuous overflow of air from the one place, with compensating drainage and inflow from the other, is enforced by the existing and perpetually maintained distribution of unequal surface temperatures. These temperature contrasts are most pronounced, and the resulting Icelandic " low " most intense, during winter; but it prevails through summer also, for the simple reason that the necessary temperature gradients, though weakened during this season, are neither obliterated nor reversed—the water remains always warm in comparison with the ice-caps of both Greenland and Iceland, which persist from season to season and from year to year.

The Aleutian " low," on the other hand, is merely seasonal: it prevails only while the adjacent Alaskan and Siberian regions are snow-covered and relatively cold. When this snow is gone the temperature gradients are even reversed, and the off-shore drainage of winter is replaced by the on-shore winds of summer. Similar considerations and explanations obviously apply to all the other regions frequented by semipermanent cyclones.

Insolational (*Of Land Origin*).—Since gulfs and seas flanked by relatively cold land areas induce, as explained, more or less

permanent cyclones, it follows that peninsulas flanked by relatively cold water should also be generators of cyclonic wind systems. Similarly, any area of sufficient size that becomes heated through insolation to temperatures above those of the adjacent regions should likewise induce or tend to induce a circulation of the cyclonic type. The Spanish peninsula shows, during summer, the phenomenon in question. It also occurs over the Alaskan peninsula onto which summer winds blow from the Gulf of Alaska, from Bering Sea and from the Arctic Ocean, obviously producing, through rotational deflection, a distinct cyclonic circulation. Similarly, the Great Plains often show daylight or insolational lows from which occasional cyclonic storms appear to originate. Also many start over northwestern Australia.

Of course, entire continents show low average pressure during summer and high during winter, while in each case the opposite condition applies to the oceans. Such conditions, however, are not especially productive of storms, because the areas involved are hyper-cyclonic in size—so large, in fact, that they only modify the general or planetary circulation without producing local disturbances within it. Neither do temperature contrasts between areas that are very small in comparison with that of the average cyclone produce extensive precipitation, but mere local disturbances quickly smoothed out by the general circulation, or, at most, only thunder showers. In short, for the development of cyclones by temperature contrasts the warm area must be neither too large nor too small, neither continental in extent nor in size a mere island or bay.

Mechanical.—The mechanical "low" is divisible into two classes: (1) Permanent—in reality not a cyclone at all in the ordinary sense of a low centre with encircling isobars—and (2) migratory—the characteristic cyclone of middle latitudes. In the first class, certainly, and presumably in the second also, the low pressure is rather the *result* than the *cause* of the associated winds. Indeed, in the case of any steady wind, except those near the surface or close to the equator, its sustaining force (force in its direction) is small in comparison with the deflective force at right angles to its path due to the rotation of the earth.

Mechanical (Permanent).—There are two well-developed. permanent lows of this type (mechanical) and also an imperfectly developed third. These are (*a*) The equatorial low, which

roughly follows the equator through its entire course, due partly to the relatively high temperature of this belt (to that extent an insolational "low") and partly to the right and left deflective forces of the westward winds of the northern and southern hemispheres, respectively. (*b*) The Antarctic trough, encircling Antarctica generally between 60 degrees and 70 degrees S. and having an annual average pressure of about 740 mm., mechanically sustained jointly by the northward pressure of the swift west winds over the oceans and the southward pressure of the east to west component of the vigorous southeast air drainage or fallwinds of Antarctica. (*c*) The Arctic trough, irregular in outline and intensity and apparently only fragmentary.

Mechanical (*Migratory*).—The great majority of extratropical cyclonic storms are migratory, and apparently originate either by breaking off from or in some manner being induced by the semipermanent and insolation lows, or, occasionally, by somehow forming at almost any other place, especially along the more frequented storm paths. The genesis, development, and detailed structure of these storms are by no means well understood, and therefore the following tentative hypothesis in respect to their origin and maintenance is offered chiefly as a convenient mnemonic by which the principal known facts concerning them may be remembered.

Tentative Hypothesis of the Origin and Maintenance of Migratory Cyclones.—It will be recalled that the prevailing wind movement of middle latitudes is from west to east, and that to a first crude approximation (more nearly attained in the southern hemisphere than in the northern) parallels of latitude are followed with such velocity that the poleward pressure gradient is just balanced by the rotational deflection. Obviously, however, numerous surface inequalities, irregularities of temperature distribution, cloudiness, precipitation, and the like, prevent this gradient from being constant along any parallel of latitude or even remaining constant at any given place. Hence the prevailing winds themselves, being primarily under the control of this gradient, correspondingly vary in direction and velocity. Also, because of surface friction, there is much air leakage across the dynamical partition of swiftest winds (usually along parallels of 40 to 60 degrees) to higher latitudes, and, of course, an equivalent return flow.

Now let a disturbance, due to whatever cause, deflect a considerable and rather deep section of the eastward flowing air toward the adjacent pole. Immediately it flows eastward faster than before, in accordance with the law of the conservation of areas, and thus crowds upon the air in front, unless it in turn has sufficient velocity to keep out of the way, a condition that probably does not usually obtain. Because this air comes from lower latitudes and therefore commonly is relatively warm and its absolute humidity often great, and also because it is flowing to regions where the meridians are crowded closer together, it necessarily rises, and in so doing usually yields abundant precipitation, whose latent heat materially aids to perpetuate the storm—continuously to develop a " low " in the forward quadrant.

Simultaneously as this broad body of air sweeps to higher latitudes an equivalent amount, necessarily to the westward, moves in the opposite direction. Indeed, this return branch may have contained the initial impulse; the results would be the same. Here, also, the law of the conservation of areas applies. The return current necessarily lags and in some measure checks the eastward flow of the air to the rear. In this manner the atmosphere between the two components of the horizontal circulation, the forward speeding up, the rearward lagging, is mechanically more or less expanded (stretched), while that on both sides is compressed. The lower air, especially in front of the storm centre, being retarded by friction and turbulence, flows spirally inward, then upward and out, largely in response to the decrease of pressure due to the increased eastward velocity of the upper winds. The " low " thus, or however, formed, constitutes a travelling break in the partition between the mid-latitude and high-latitude circulations. In front of the " low " the primary circulation finds its way to colder regions, while to its rear return currents simultaneously bring equivalent amounts of other air to warmer sections. At any rate, whatever the origin of extratropical cyclones; it is obvious that much of the interzonal circulation between middle and high latitudes, perhaps by far the greater part of it, occurs simultaneously on their opposite sides. Between the tropical and extra-tropical regions the chief intercirculation is through the trades and counter-trades—to lower latitudes by the former, mainly, and to higher chiefly by the latter.

Since the area covered by a cyclone or anticyclone is very great,

averaging, roughly 2×10^7 square kilometres, it would seem, as abundantly supported by cloud movements, that each must directly involve at least the whole depth of the troposphere. On the other hand, the stratosphere, from its different temperature, humidity, and wind velocity, appears to be relatively passive, suffering rather than causing either cyclonic or anticyclonic effects.

According to this conception, the troposphere within a cyclonic area is mechanically expanded and, besides, has a marked upward component. It therefore must be cooled, though in the east quadrant, where the wind is from lower latitudes, the warming on that account may equal or even exceed the expansional cooling. Similarly, in an anticyclone the troposphere is mechanically compressed and also has a downward component, thus producing a temperature increase, except, possibly, on the east or forward side, where this effect may be equalled or exceeded by the transfer of colder air from higher latitudes. Hence, because of opposite directions of convection, upward (cooling) in the cyclone, downward (heating) in the anticyclone; contrary changes in pressure, decrease (cooling) in the cyclonic area, increase (heating) in the anticyclonic; and, presumably, inequality of radiating power due to differences of moisture content—the descending (warming) air of the anticyclone being relatively dry and thus heat preserving—one might expect to find, as observations (referred to later) show, that the troposphere is comparatively cold in cyclones and warm in anticyclones, except, perhaps, near the surface, where convection is less operative.

If, as seems likely, cyclonic and anticyclonic winds involve the troposphere through its whole depth, it follows from Egnell's law, $pv =$ a constant, with change of elevation, that the pressure gradient is also a constant, and, finally, that the pressure difference between a high and its neighboring low may be of the same order of magnitude at all levels up to the top of the troposphere.

Further, if the stratosphere is essentially inert in respect to the genesis and progress of surface storms, it clearly must sink to lower levels over cyclonic areas and be raised to higher over anticyclonic, and thereby itself undergo pressure and temperature changes,[52] and also briefly (during the formative stage) manifest cyclonic and anticyclonic wind systems.

Let a stratospheric column be dropped bodily a distance dh,

[52] Shaw, "Perturbations of the Stratosphere," M. O. 202, p. 47, 1909.

and let the surrounding air come in until equilibrium is again established. At each level there obviously will result a change in pressure directly proportional to the pressure at that level. That is, throughout the column

$$\frac{dp}{p} = K, \text{ a constant.}$$

But, as is well known,

$$\frac{dT}{T} = C\frac{dp}{p},$$

in which T is the absolute temperature, and C a constant, 0.2843 for dry air. Hence, since T is constant, roughly, in the stratosphere, dT is also constant, and the upper air remains vertically isothermal, whatever the pressure increase or decrease. An increase of pressure in the stratosphere, such as presumably takes place over cyclones, *increases* its temperature, while a decrease of pressure, such as probably occurs over anticyclones, correspondingly *decreases* its temperature. In each case the pressure effect presumably is slightly enhanced by the coincident change in the intensity of radiation from below.

Suppose the temperature of the stratosphere over a cyclone should differ from that at the same place over the following anticyclone by 10° C., what, according to the above conception, will be the approximate change of boundary level? Let h be this change, and let the temperature of the stratosphere be 220°, absolute: Then since

$$\frac{dp}{p} = \frac{dh}{H},$$

in which H is the height of the homogeneous atmosphere, about 6450 metres at the assumed temperature, it follows that

$$\frac{10}{220} = 0.2843\,\frac{h}{H}, \text{ roughly,}$$

and

$$h = 1 \text{ kilometre, approximately,}$$

That is, the temperature of the stratosphere will increase or decrease at the rate of approximately 10° C. per kilometre enforced fall or rise, respectively, under the influence of cyclonic and anticyclonic disturbances.

While migratory cyclones of this nature, mechanical or counter-current, may originate almost anywhere outside the tropics, it is evident that some places are far more favorable to their genesis than others. Among such favorable places it is

probable that the Gulf of Alaska is one of the most pronounced, at least during winter, since here the relatively warm water gradually creates a " low " that deflects in a northerly direction the air currents in the south and southeast, and to a southerly direction the air currents in the north and northwest. Hence a low formed over this gulf, or farther west along the Aleutian Islands, is likely to become accentuated on its eastern side through the advent of warm air and the onset of heavy precipitation, and therefore be carried away from its moorings, as it were, and set adrift along the great air currents, where, as already stated, it acts as a travelling centre of vigorous interzonal circulation between middle and high latitudes. Of course, the very process that forces the cyclone away from its place of origin brings in colder air, usually from higher latitudes. But this, in turn, is slowly warmed from the great supply of heat in the water below, and thus all those conditions essential to the breaking off of another storm similar to the previous one are reëstablished, and so on indefinitely, or until the season and consequent temperature distributions so change as to prevent such action.

According to this conception, the permanent " low " around which interzonal circulation tends to be active often produces that local disturbance or deflection of the general circulation necessary to the genesis of a mechanical or dynamical cyclone. Further, although the necessary mathematical demonstrations may not be obvious, it appears in a general way that such a cyclone would travel a course and tend to exhibit characteristics as follows:

a. The storm would travel with the general circulation.

This is fully supported by observations.

b. It would tend, in general, to follow the path of maximum winds, this being the course approached by the west-to-east circulation, and therefore to travel along (not down, but at right angles to) the maximum interzonal temperature and pressure slope.

This inference is also supported by observations.

c. Its average annual course would follow annual isotherms.

This, too, is in accord with observations.

d. Because the winds to the east and southeast (northeast in the southern hemisphere) of the front gen-

erally are relatively warm and humid, and because they are focused in direction, thus leading to congestion that forces the rear winds to climb over those in front, it follows that, except as modified by topography and proximity to oceans, these would be the quadrants of maximum precipitation, as indeed they commonly are, except along west coasts.

e. The rising and, therefore, rain-producing air should flow off eastward with the general circulation.

The cirrus and other high level clouds that forerun the cyclone amply support this inference.

f. The mechanical cyclone should usually be accompanied by a correlative anticyclone to its rear.

There is much observational evidence in support also of this conclusion.

g. The average latitude of cyclones should be greater than that of anticyclones, since the main mass of air of the one has a component toward, and of the other from, the adjacent pole.

A statistical test of this conclusion is not at hand, but it seems to accord with inspection.

h. Since, as here conceived, the storm involves the general circulation from the surface to its top—that is, all the air up to the stratosphere—it seems possible that the pressure contrasts between " low " and adjacent " high " may also extend to this level.

Even this inference is supported by much observational evidence.[53]

i. Since, according to the conception here offered, the atmosphere of the particular type of cyclone under discussion is mechanically rather than thermally expanded, and since the swift winds of the front would tend to draw upward and out such air as leaks into the " low," it seems that the atmosphere of a cyclonic region might be warm in the east quadrant, but cold in the centre and the other quadrants.

[53] Dines, *Jr. Scot. Meteorol, Soc.*, 16, p, 304, 1914.

j. Since, according to this conception, the winds to the west of a cyclone are from higher latitudes, it follows that they must spread out, coming where meridians are more separated, with the upper portions flowing to lower levels, and also, because flowing toward the equator, must lag behind and thus by a damming up process build, through overflowing currents, a " high " in which the temperature of the bulk of the atmosphere, because of its downward component, shall be relatively warm.

k. The expansion of the lower air (below the stratosphere) in the " low " and compression in the " high " leads to lowering and warming the stratosphere over the cyclone, and raising and cooling it over the anticyclone. The difference in intensity of radiation by the moist and dry air of the two regions probably accentuates these conditions. Furthermore, since the northerly winds of the anticyclone act as a partial barrier to the westerlies of the general circulation, the latter must be deflected to unwonted altitudes, thereby cooling at top to a minimum temperature, more or less below that appropriate to the flux of terrestrial and other radiation. That is, a minimum temperature should occur at the base of the stratosphere over the anticyclone, as shown in Figs. 17 and 18.

All the several inferences under *i*, *j*, and *k* (doubtful, because unsupported by analysis) are fully in accord with observations, whatever the physical cause or causes may be. This is shown by Figs. 17 and 18, which give, respectively, the vertical temperature gradients for winter and summer in regions of high (5 mm. and more above normal), neutral and low (5 mm. and more below normal) pressure, as determined by sounding balloons from Lindenberg, Munich, Strassburg, Trappes, Uccle, and Zurich. The figures of the legend give the number of flights from which the curves were determined. Abundant additional evidence of these formerly unsuspected temperature relations

between the cyclone and anticyclone is given by Wagner,[54] Gold,[55] Dines,[56] and others. They must therefore be accepted as definitely established for the British Isles and Continental Europe, and tentatively accepted (until disproved if not true) for other parts of the world.

Probably the above conceptions of the mechanism of the average extra-tropical cyclone could be elaborately developed from the standpoint of hydrodynamics and thermodynamics, but this would be too tedious to include here. The concept, however, even without such support, may be useful in helping to remember the chief facts learned by recent free air observations.

"Tropical."—As previously stated, a considerable percentage of the tropical cyclones (the actual number probably is only 2 to 3 per year in the northern hemisphere) migrate to extra-tropical regions. Shortly after recurving they gradually lose their original characteristics and become extra-tropical in type as well as location. Nevertheless, they generally still are called "tropical cyclones" (West India hurricanes, typhoons, etc.), however high the latitude actually reached. A causal name is not suggested for this cyclone, for the reason that it is not certain what is the chief factor in its origin.

As already explained, these storms originate usually, if not always, in the doldrums, where the air is quiet, hot, and excessively humid. The stillness of the air, if long continued, leads to a high degree of humidity, and the humidity, in turn, decreases the local pressure and also increases the absorptive power of the atmosphere for both solar and terrestrial radiation. Hence an inwardly directed pressure gradient and a corresponding circulation, the latter increased when flanked by oppositely directed trade winds, are slowly established. The resulting precipitation, through the latent heat thus set free, if sufficiently abundant and properly distributed, accentuates the circulation and thus secures the perpetuation of the cyclone until it fails for want of moisture, as it often does on dry land, or spreads and loses itself in the world circulation. As it recurves and gets well away from the tropics, it generally spreads out, becomes less intense, has most of its pre-

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[54] *Beiträge zur Physik der freien Atmosphäre*, 3, 57, 1909.

[55] "International Kite and Balloon Ascents," 1913. (Geographical Memoirs, No. 5.)

[56] *Jr. Scot. Meteorol. Soc.*, 16, p. 304, 1914.

cipitation on the east side, and otherwise gradually acquires the characteristics of a cyclone of extra-tropical origin, and, presumably, is maintained in the same way. The physical cause of these storms, if they originate, as seems probable, in the doldrums and between counter trades or similar winds, appears to be partly thermal and partly mechanical, and their subsequent maintenance, after reaching the middle and higher latitudes, the same (largely mechanical) as that of any other cyclone of the same place.

ANTICYCLONES.

Anticyclones, or "highs," are divisible, with respect to their genesis, into three classes: (1) Mechanical: *a*, permanent, and *b*, migratory. (2) Radiational: *a*, permanent; and *b*, transitory. (3) Thermal.

In this classification the relative "high" that obtains over an entire hemisphere during its winter, and also those seasonal highs of continental (during winter) and oceanic (during summer) extent, have all been excluded. Like the lows of similar great size, they only modify somewhat the course of the general circulation and give direction to monsoon winds.

Mechanical (Permanent).—Since the surface of the ocean is a gravitational equipotential surface, it follows that west winds, by virtue of their excess centrifugal force, will tend to climb up the bulge of the earth toward the equator, and east winds, because of deficiency in centrifugal force, will tend to slide down this bulge toward the nearest pole. Hence along the borders, between trade winds and the west winds of adjacent higher latitudes, the atmosphere must be subject to a mechanical squeeze. In other words, mechanically produced high pressure belts must encircle the earth at about latitudes 30° to 35° N. and S. They must also be best developed over the oceans, where the trade winds, upon which they largely depend, are strongest and steadiest. These belts are clearly shown in Fig. 51. Nevertheless, they often do encroach to some extent onto land areas. Indeed, it seems probable that the high-pressure, droughty weather that occasionally prevails over the southern United States, and even much of the Mississippi valley, frequently is due, in part at least, to such an encroachment, incident to the poleward summer shift of the northern high-pressure belt.

Mechanical (Migratory).—The migratory anticyclone referred to here, and assumed to be generated in the manner ex-

plained in the discussion of mechanical cyclones, is the common one of middle latitudes. The directions of its system of winds, but in no sense the complete paths of the air particles, are given by synoptic charts, such as Figs. 100 and 101. These directions are spirally outward, clockwise in the northern hemisphere, counter-clockwise in the southern. Hence the relation of anticyclonic wind velocity to horizontal pressure gradient is given by the equation,

$$\frac{dp}{dn} = v\left(2\,\omega \sin\varphi - \frac{v}{r}\right).$$

in which r is the radius of curvature, nearly, of the wind-path at the place considered, and the other symbols have the usual significance as previously given. From the negative sign it appears that for a given radius of curvature the possible wind velocity in a " high " is strictly limited, whatever the pressure gradient.

Velocity and Path of Travel.—The velocity and normal path of the migrating anticyclone are by no means as well known as those of the cyclone, except, perhaps, through the studies of Bowie and Weightman [57] in respect to those that cross the United States. However, the size, frequency, and velocity of travel of anticyclones are all roughly the same as those of similarly located cyclones. Furthermore, their most frequented paths, though, perhaps, generally beginning at higher latitudes over continents and running to lower over the oceans than the similar cyclonic routes, are roughly parallel thereto.

Just why these close relations hold is not certain. It may be interesting, however, to note that they appear to support the above assumption that generally the migrating cyclone and its neighboring westerly anticyclone are correlative parts of a single great atmospheric disturbance.

Wind Velocity.—The actual velocity of the wind in the different quadrants of an anticyclone and at different elevations is given in the following table by Peppler,[58] based on a large number of observations made during 1903–1909, at Lindenburg: latitude, 52° 10′ N.; longitude, 14° 15′ E. Probably other mid-latitude regions have approximately the same anticyclonic wind velocities, but this is not certain, nor are there at hand sufficient data to determine the question.

Radiational (Permanent).—There are two extensive regions.

[57] *M. W. R. Supplement*, No. 4, 1917.

[58] *Beiträge zur Physik der freien Atmosphäre*, 4., p. 95, 1911.

Antarctica and Greenland, where the barometric pressure always is high. At each place the high pressure appears to be the result of the very low prevailing temperatures, which in turn are due in part to the great elevations and in part to the free and abundant radiation from the snow surface through the comparatively clear skies, kept generally free from clouds by the descent of the upper air induced and maintained by the vigorous fallwinds. That sur-

Anticyclonic Wind Velocity in Metres per Second and (Miles per Hour).

	Altitude	Surface 122 m.	500 m.	1000 m.	1500 m.	2000 m.	2500 m.	3000 m.	3500 m.	4000 m.
South quadrant	Winter	4.43 (9.9)	8.48 (19.0)	8.82 (19.7)	8.68 (19.4)	8.60 (19.2)	8.92 (19.9)	9.71 (21.7)	10.14 (22.7)	10.97 (24.5)
	Summer	3.92 (8.7)	6.19 (13.9)	6.25 (14.0)	6.36 (14.2)	6.17 (13.8)	5.97 (13.3)	6.19 (13.9)	7.08 (15.9)	7.83 (17.5)
	Year	4.16 (9.3)	7.32 (16.4)	7.52 (16.8)	7.51 (16.8)	7.38 (16.5)	7.44 (16.7)	7.94 (17.8)	8.60 (19.2)	9.39 (21.0)
West quadrant	Winter	3.93 (8.8)	7.60 (17.0)	7.19 (16.1)	7.35 (16.4)	7.23 (16.2)	7.21 (16.1)	7.57 (16.9)	7.75 (17.3)	8.11 (17.9)
	Summer	3.39 (7.6)	5.26 (11.8)	5.41 (12.1)	5.18 (11.6)	5.28 (11.8)	5.39 (12.1)	5.20 (11.6)	4.74 (10.6)	5.04 (11.3)
	Year	3.74 (8.4)	6.51 (14.5)	6.38 (14.3)	6.35 (14.2)	6.34 (14.2)	6.38 (14.3)	6.46 (14.4)	6.32 (14.1)	6.65 (14.9)
North quadrant	Winter	4.21 (9.4)	9.69 (21.7)	9.54 (21.4)	9.78 (21.9)	10.19 (22.8)	10.69 (23.9)	11.59 (25.9)	12.29 (27.5)	14.43 (32.3)
	Summer	4.05 (9.0)	7.01 (15.7)	7.68 (17.2)	8.41 (18.8)	8.85 (19.8)	9.51 (21.3)	10.04 (22.5)	10.68 (23.9)	11.45 (25.6)
	Year	4.13 (9.3)	8.35 (18.7)	8.61 (19.2)	9.09 (20.4)	9.51 (21.3)	10.09 (22.6)	10.80 (24.2)	11.47 (25.6)	12.92 (28.9)
East quadrant	Winter	4.29 (9.6)	8.24 (18.4)	8.83 (19.8)	9.56 (21.4)	10.92 (24.4)	12.44 (27.8)	13.52 (30.2)	14.02 (31.3)	15.68 (35.1)
	Summer	3.92 (8.7)	5.88 (13.2)	6.42 (14.3)	6.55 (14.6)	6.98 (15.6)	7.54 (16.9)	7.31 (16.3)	7.83 (17.5)	8.28 (18.5)
	Year	4.06 (9.1)	7.01 (15.7)	7.57 (16.9)	8.00 (17.9)	8.89 (19.9)	9.93 (22.2)	10.35 (23.2)	10.86 (24.3)	11.91 (26.6)

face radiation is an essential factor in establishing and maintaining these low temperatures is obvious from the fact that air cannot flow down hill, as it does in these regions, unless it has a greater density and therefore lower temperature than the adjacent atmosphere of the same level. It is also obvious from the prevailing and excessive surface temperature inversions, in which, and because of which, those ice fogs that doubtless furnish much of the interior precipitation are so common.

It will be well to remember in this connection that snow, in addition to reflecting about 70 per cent. of the incident solar radia-

tion,[58a] is also a good emitter of those long wave-length (12–15μ) radiations appropriate to its temperature. In this way the low temperatures are maintained, not only during winter when air circulation and, to some extent, cooling ice supply the only available heat, but also during the long-continued insolation of summer.

The air drainage thus produced is manifest in those strong and persistent southeast or anticyclonic winds that characterize the climates of the border and all explored portions of Antarctica, except, of course, near the pole, and, presumably, therefore, of the whole continent. Similar, though less vigorous, anticyclonic winds also prevail over and around Greenland. Each of these great regions but especially Antarctica, by virtue of its strong and continuous refrigeration, obviously is exceedingly effective in its influence on the atmospheric circulation of its respective hemisphere. If there were no such extensive high and snow-covered areas in the polar regions, it is clear that our general circulation would be less vigorous and doubtless very different in many places.

Radiational (*Transitory*).—During winter elevated snow-covered regions often become very cold and thus build "highs" similar to those of Greenland and Antarctica, though usually much smaller in extent, as well as only temporary. Occasionally these give rise to strong and cold surface winds, especially when the existing gradient is accentuated by the passage of a well-developed cyclone along lower latitudes. Examples of such winds are the mistral of the Rhone Valley and the bora of the Adriatic and Black Seas. The Texas norther and, probably, the blizzard of the Great Plains are other and important examples of the drainage of transitory radiational anticyclones. The well-known violent fallwind of the coast of Norway appears to have a similar origin, as indeed have innumerable other drainage winds in all mountainous and high plateau regions outside the tropics.

Thermal (*Semipermanent*).—As is well known, there are five semipermanent "highs," all of which occur on the oceans: Two, as Fig. 51 shows, about 35 degrees north of the equator and three about 32 degrees south of it. Two are on the Pacific Ocean—one west of southern California, the other off the coast of Chile; two on the Atlantic Ocean—near the Azores (known as

[58a] Abbot and Aldrich, *Proc. Na. Acad. Sci.*, **2**, 335, 1916.

the Azores "high") and off the coast of southern Africa; and one on the Indian Ocean, about half-way between Africa and Australia. A sixth oceanic "high" of this same class, but far less persistent than any of the above, often develops, especially during winter, in the region of the Bermudas.

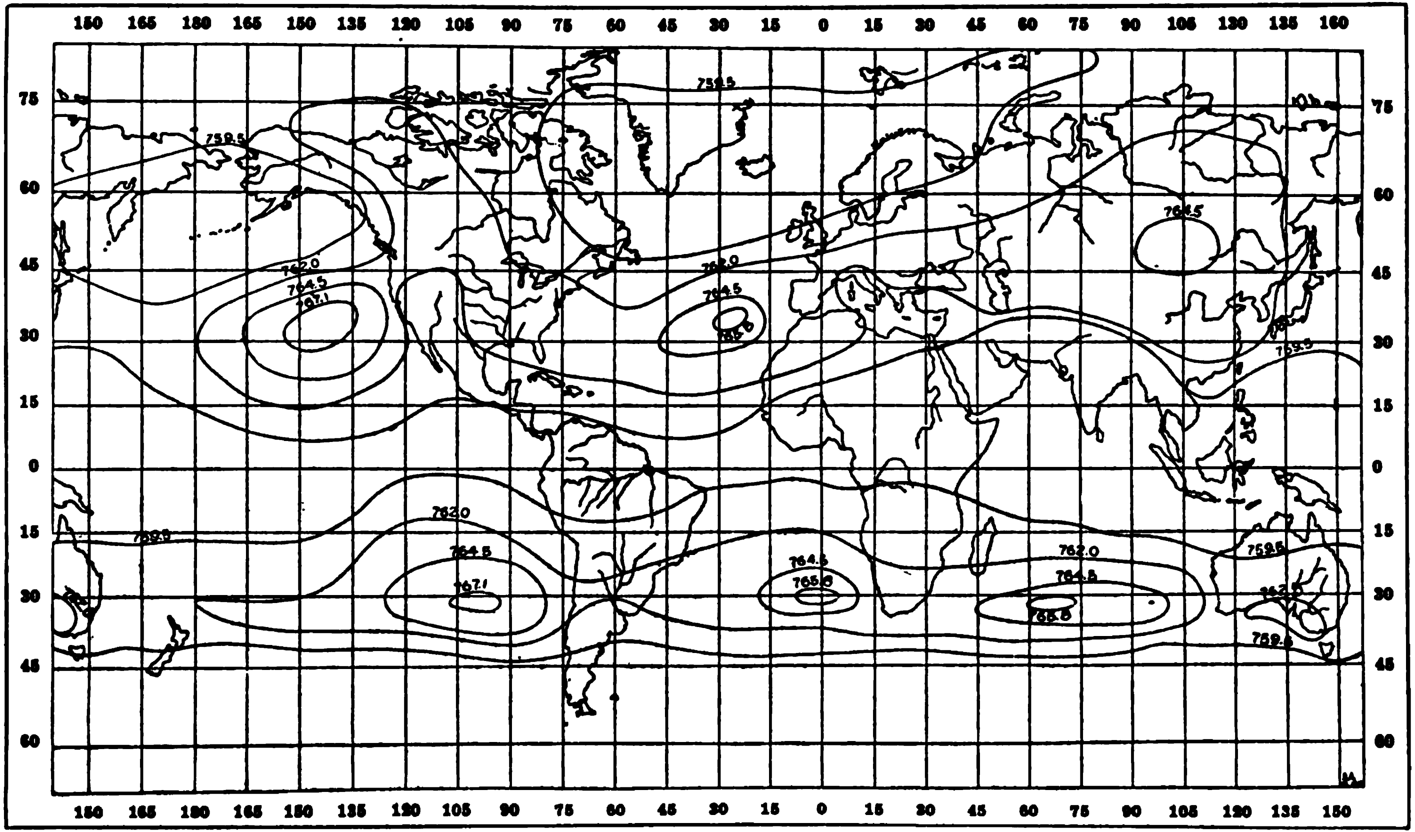
Obviously there must be a close relation between the intensities and locations of these highs and the directions and velocities of the surrounding winds, even to great distances, as shown by Figs. 52 and 53. Hence it is meteorologically important to form some conception in regard to their origin.

It will be seen from Fig. 51 that all these "highs" or centres of maximum pressure occur along the high-pressure belts, and from Fig. 54 that they occur at those places along these belts where the temperature of the air is low for that latitude; that is, where the isotherms are deflected equatorward. At these places, then, there are two causes of high pressure: (*a*) the mechanical pressure that produces the high-pressure belts, as already explained, and (*b*) a relatively low surface temperature which allows the upper air to cool somewhat and correspondingly contract.

It is known from sounding balloon records that the temperature of the atmosphere even to great altitudes follows more or less closely any long-continued temperature changes of the surface. Hence one might reasonably expect the atmosphere over the cold regions, as shown by Fig. 54, to be colder at every level than that of the surrounding atmosphere over warmer regions. A change of 1° C. throughout would change the pressure by 2 mm. or more. Hence, since the regions in question, according to Buchan's charts, are from 1° C. to 3° C. colder than those of the same latitude east or west, it appears that the pressure maxima of 2 mm. to 6 mm. probably are due to the continuous relatively low surface temperatures. In this case the "high" appears to be due to the cooling of the superincumbent atmosphere to that temperature at which its radiation is in substantial equilibrium with the minimum radiation from below.

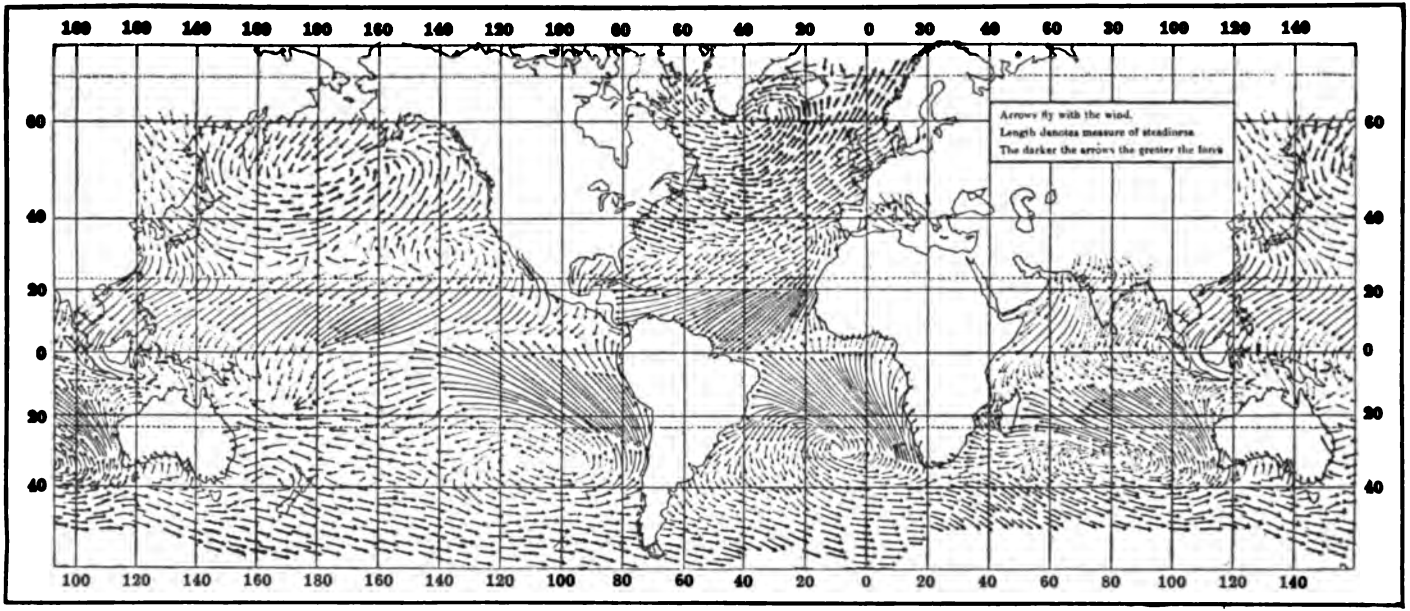
But what is the cause of the local low surface temperatures? Referring to Fig. 55, it will be seen that there are five different places, and only five, where a distinctly cold ocean current crosses a belt of high pressure, and that every one of these is associated with a region of maximum pressure. Neither is there a semi-permanent "high" anywhere else on the oceans. Wherever, then,

Fig. 51.



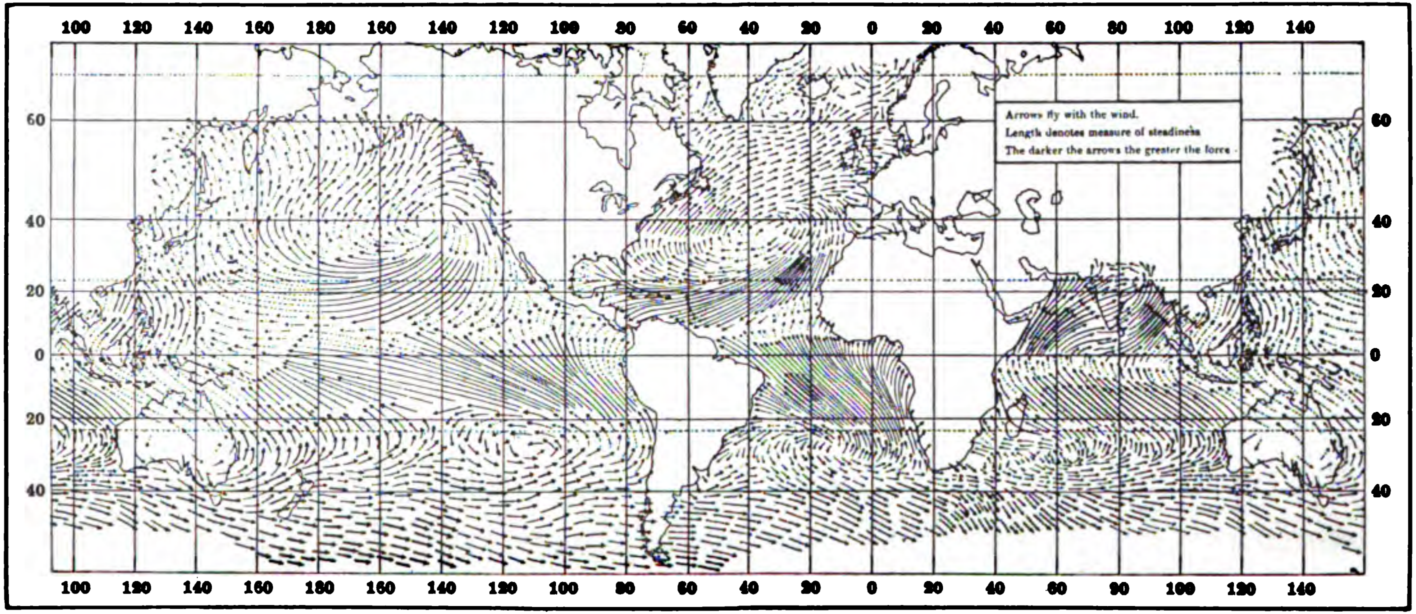
Annual average isobars. (After Buchan.)

FIG. 52.



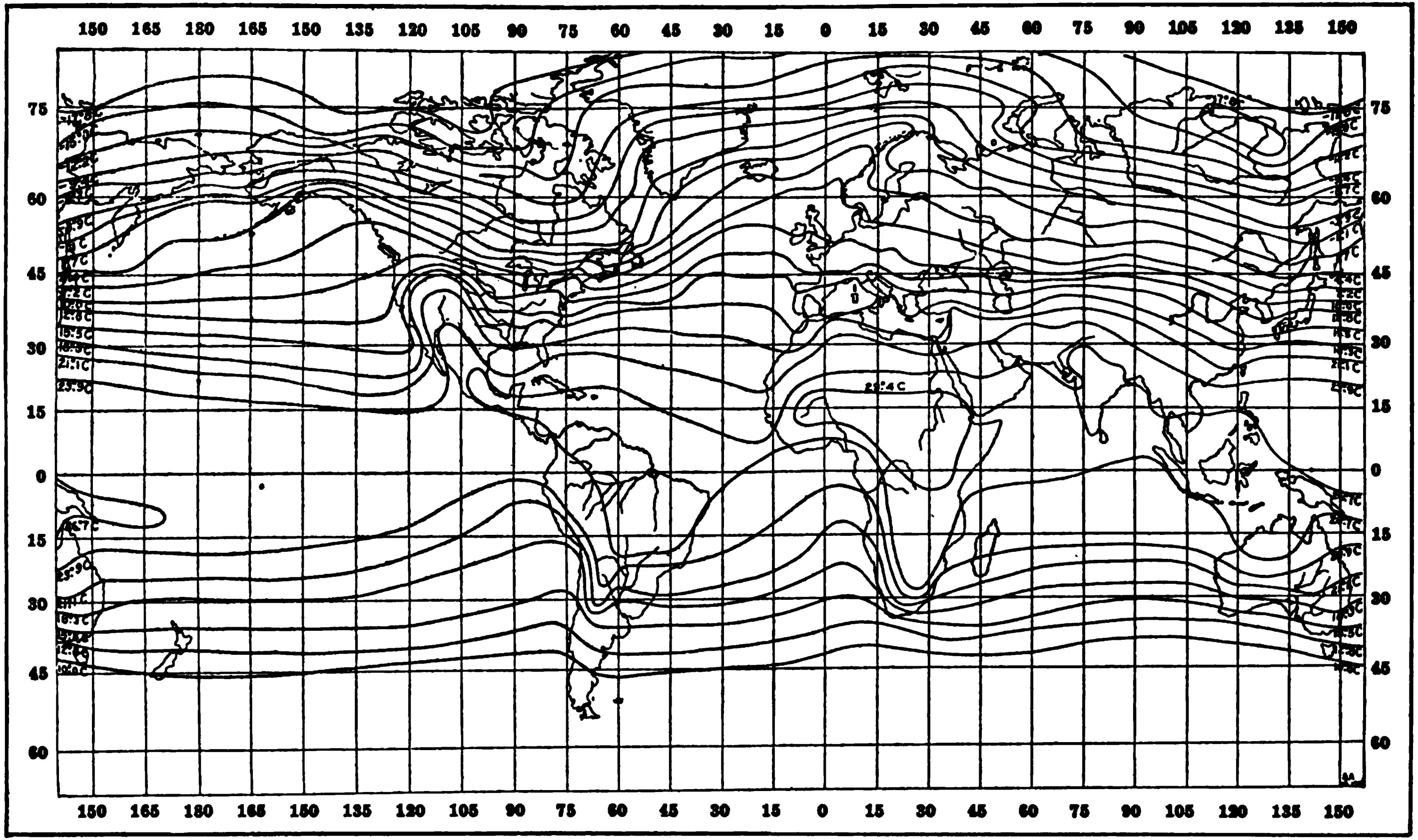
Ocean winds, January and February. (Köppen.)

Fig. 53.



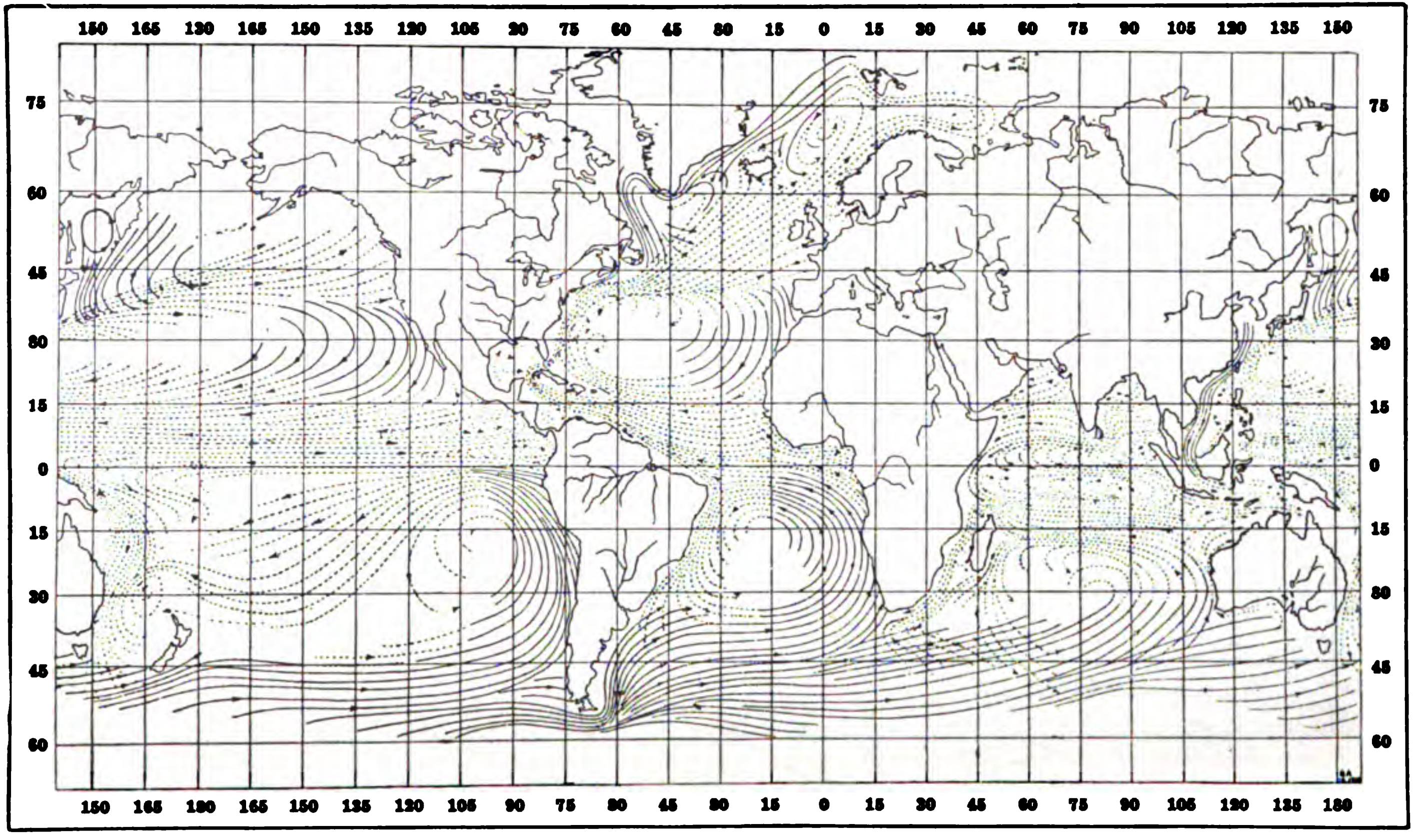
Ocean winds, July and August. (Köppen.)

FIG. 54.



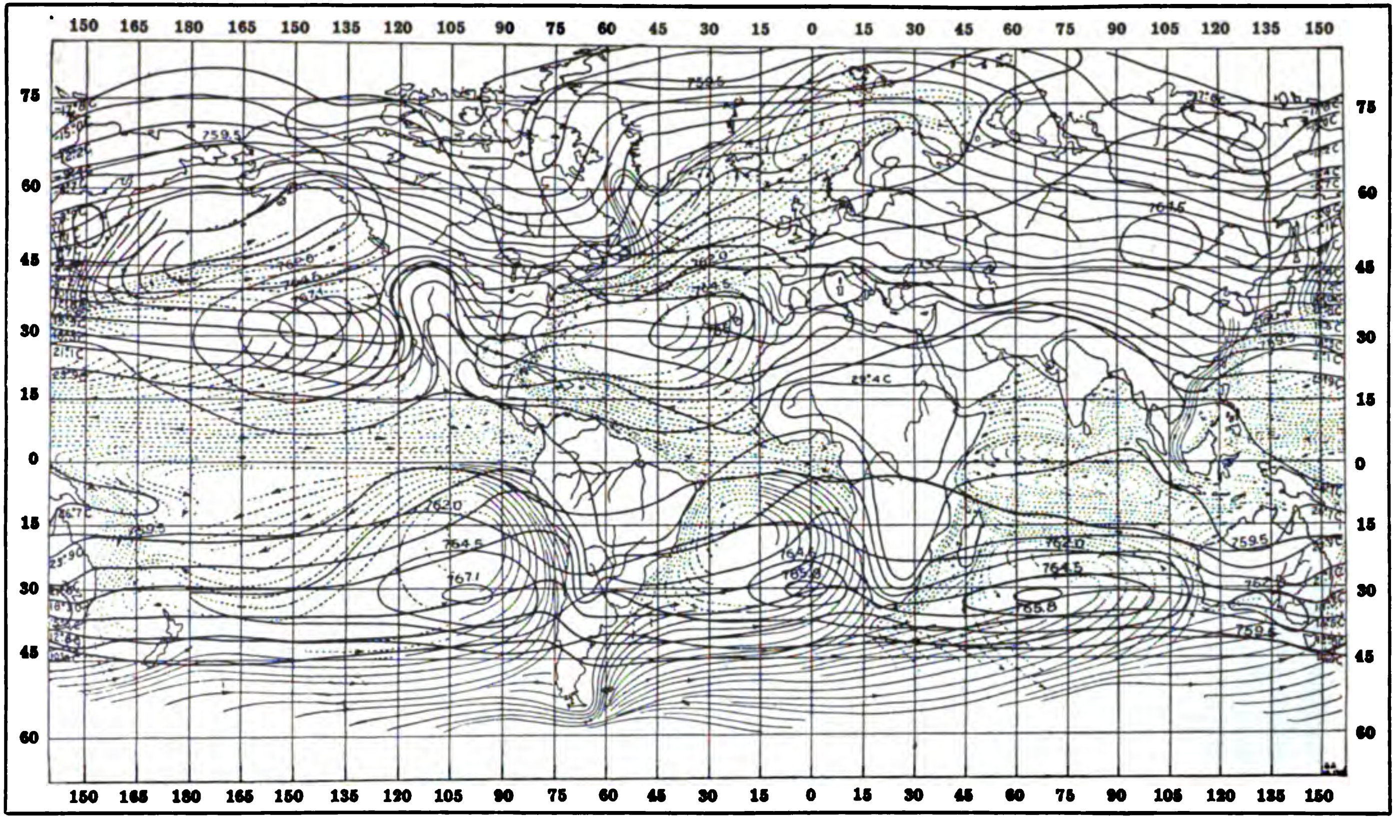
Annual average isotherms. (After Buchan.)

FIG. 55.



Ocean currents.

FIG. 56.



Ocean currents, annual average isotherms, and annual average isobars.

the mechanical effect that produces a belt of high pressure is reinforced by thermal contraction due to cold water, there, and only there, as illustrated by Fig. 56, are found a maximum of atmospheric pressure and the centre of a semipermanent anticyclone. During winter there is also a slight minimum temperature along the North Atlantic high-pressure belt near Bermuda, and a similar one along the South Pacific belt just east of New Zealand, and at each place a corresponding tendency to the maintenance of an anticyclone.

One obvious effect of all these semipermanent highs is the location of branches or channels of interzonal circulation, analogous to those of the cyclones and anticyclones of higher latitudes. Thus, much tropical atmosphere, in addition to that carried by the counter-trades, reaches middle latitudes by flowing around and to the west of the semipermanent "highs." From here the next stage in the general circulation takes the air to still higher latitudes, and even to polar regions, around and to the east either of the semipermanent "lows" or of the migratory cyclones. In its return it passes to the west of the "lows" or east of the travelling "highs," and finally around and to the east of the semi-permanent "highs." These, however, are only general channels and, presumably, average routes, upon which are superimposed innumerable and ever-changing irregularities.

FORCED WINDS.

Although in greater or less measure all winds are interdependent, only a relatively small number obviously are generated and maintained by other and coexisting winds. Among these are eddy winds, Maloja winds, foehn or chinook winds, and, presumably, the winds of the tornado and waterspout.

Eddies.—Wherever the wind blows across a steep-sided hill or mountain, eddies are likely to be formed, especially on the lee side. In such cases the direction of the surface wind is approximately opposite to that of the general or prevailing wind, with a calm between them.

For the practical purpose of the weather forecaster, wind eddies have but little significance, except in one important particular. He must exclude from his forecasting data all reports of wind direction obtained at places where eddies are likely to prevail. Such eddies, however, may be of great importance to the

aviator, since they produce, on their forward sides, troublesome down-currents and also shallow surface winds which, because oppositely directed to the winds, less, perhaps, than 100 metres above, may render landing at such places difficult or even very dangerous.

Maloja Wind.—The " Maloja " wind, named after the Maloja pass in Switzerland, below which, and for some distance along the valley of the Inn, it is well defined, is only a reverse valley breeze—reverse because such convectional tendency as the insolational heating of the valley in question may produce is more than counteracted by the similar heating of a suitable, and suitably situated, neighboring region. It is the controlling pressure distribution due to this latter heating that locates the up-draft and induces the down-valley Maloja wind.

Similarly, the flooding of a basin by gravity winds often produces a forced breeze up a narrow pass, where ordinarily a down-current would be expected.

Foehn, Chinook.—The foehn, or chinook, as it generally is called in North America, is a warm, dry wind blowing down a mountain side onto the valleys and plains beyond. It differs from the typical fallwind in being warm, level for level, and not cold, as is the latter, in comparison with the air of surrounding regions.

Any system of winds, whether of trade, cyclonic, or other origin, extending to or near to the surface and blowing more or less normally across a mountain ridge, necessarily induces up-currents, dynamically cooled, on the windward side and down-currents, adiabatically heated, on the lee side, except along the under portions of such eddies as may be produced, where the directions and consequent temperature changes are just the reverse. Therefore:

1. Foehns occur at all seasons.
2. The relative humidity of the foehn is always low.
3. The rise in temperature is greatest when the original vertical temperature gradient is least; hence greatest, other things being equal, *a,* when the upper air is warmest—that is, when there has been precipitation to the windward; *b,* when the surface air is coldest—that is, when there has been free night radiation (clear skies) on the lee side; and *c,* during winter, when the vertical temperature gradient through the first several hundred metres may

be only 4° C., say, per kilometre, instead of the usual 7° to 8° C. of summer.

The inertia of the wind crossing the mountain tends to carry it on well above the valley, or plain, beyond, but its drag on the lower air, due to viscosity, deflects it downward. Because of this deflection a foehn wind often strikes on the lower slopes, or adjacent region, with great violence, from which, and mainly because of its dynamical heating, it rebounds to higher levels. Along a belt, therefore, well down the mountain, or even a little beyond it, the surface wind may be exceedingly turbulent and violent, while both farther away and also nearer, or on the higher slopes, it is comparatively light. Furthermore, owing to changes in the general direction of the crossing current, or in its strength, or both, the wind belt may shift toward or from, or up or down, the mountain, or even vanish entirely.

During its earlier stages a foehn is often accompanied by a crest cloud, by dissolving scud drawn down out of this cloud, and by a cumulus roll over the rebounding wind; and, a little later, by general precipitation.

Another interesting phenomenon of the foehn is the transmission of sounds from the windward to the leeward side of a mountain, and often miles away, where ordinarily they are not heard at all. The explanation is obvious. The wind, which blows roughly parallel to the slopes, increases rapidly with distance from the surface; hence the sound wave, because it is carried forward in the faster layers more speedily than in the slower, crosses the crest in an approximately vertical position, and then roughly converges, or focuses onto places some distance to the leeward.

Since winds of this origin often are swift, and their dynamical heating pronounced, it follows that under favorable circumstances a very strong foehn may even develop a secondary " low " —on the same side of the mountain, of course, as the centre of the primary one.

TORNADO.

The tornado,[59] in which the air travels in approximate circles, as its name implies, is well nigh peculiar to the United States east

[59] For detailed discussion see: Finley, "Tornadoes," New York, 1887, and Ferrel "A Popular Treatise of the Winds," New York, 1889.

of the Rocky Mountains. Nor is it at all equally distributed over even this area, since it occurs rarely in the Alleghenies, seldom along the Gulf and Atlantic coasts, frequently in central and northern Alabama, Georgia, and South Carolina, frequently also in Ohio, Indiana, Illinois, and southern Michigan, and most frequently in Missouri, Kansas, and Iowa.

The tornado develops only in connection with a thunderstorm, usually just in front of the rain, and especially in connection with those particular storms that form along a valley " low," or between V-shaped isobars where opposing winds of widely different temperatures give rise to that exceptionally strong vertical convection essential to the genesis and growth of the thunderstorm. The season of most frequent occurrence, therefore, is spring and early summer; in fact, during winter it is unknown, except occasionally near the Gulf and in other warm sections. Similarly, the time of most frequent occurrence is 3 to 5 P.M. Also, since it is only a local phenomenon, while the conditions favorable to its genesis are much more extensive, it often happens that a number of tornadoes develop, even close together, in connection with a single distorted cyclone.

The diameter of the tornado averages only about 300 metres (984 feet); the length of its path varies roughly from 100 metres to possibly 500 kilometres (328 feet to 310 miles); its direction of travel is nearly always northeast; its rate of travel, though differing greatly, averages roughly 11 metres per second (25 miles per hour), thus passing a given place in half a minute or less; while its winds, always counter-clockwise, are the swiftest known, estimated at 45 to 225 metres per second (100 to 500 miles per hour). It is therefore by far the smallest, briefest, and severest of all storms. Essentially it is a phenomenon of the middle atmosphere. In its genesis clouds whirl around each other with great velocity and turmoil, while from beneath their centre a funnel-shaped cloud develops, usually tapering off to a long pendent spout that may or may not extend to the earth. Wherever it does reach the surface it produces a deafening roar, and practically everything in its immediate narrow path that can be blown down or torn to pieces is destroyed, though generally but little damage is done on either side. On the other hand, wherever it does not come to the surface its passage produces but little or no effect.

Cause.—From the characteristics of the tornado and from the meteorological conditions that normally accompany it, it appears that its origin must be purely mechanical. Thus its rotation obviously is derived essentially from that of the cyclone in which it occurs, since it is always in the same sense, counter-clockwise, however small its diameter, and never clockwise, as is often the case with large dust-whirls when formed in still air. But how is the rapid rotation started? From the directions of the V-shaped isobars it is clear that at the cloud level, say, there must be, as often observed, neighboring winds flowing in approximately opposite directions and, of course, more or less intermingling and overrunning counter currents. Hence, under such conditions, the inflow occurring at various levels that feeds the strong up-draft always just in front of a thunderstorm must occasionally so deflect these counter currents, by drawing both into the same rising column, as necessarily to produce a violent whirl.

Here, too, as in all other cases of atmospheric motion, the law of the conservation of areas, or the constancy of the product of radius of curvature by linear velocity, applies, except as modified by friction and viscosity. Hence, as the radii of curvature of the opposing currents may at first be comparatively large, and after the deflection relatively small, it follows that the wind velocity within the whirl, in which both the counter currents are taking part, may be very great. This rotation, however, does not check the up-current, hence that convection which is essential, as explained above, to the rotation is maintained, and therefore the rising currents brought in spirally with increasing angular and linear velocity as the axis of spin is approached. Each filament, so to speak, of the spirally rising air viscously drags along its under and slower neighbors, thereby bringing them nearer the axis and increasing their velocity. In this manner the whole of the rising column is set whirling with greater or less spin. Around the axis of rotation the pressure obviously is reduced in proportion to the centrifugal force, the temperature correspondingly lowered, and therefore a cloud spout often formed.

Wherever the inflow of the surface air is greatly checked, or its course so altered by a deflecting hill or other obstacle as greatly to decrease the spin, there the tornado must lift. It may, however, retain its full vigor in the unaffected upper air, and soon reach the surface again.

Why Essentially Peculiar to the United States.—Since the tornado rarely occurs in violent form except in that portion of the United States which is east of the Rocky Mountains, it follows that that combination of meteorological conditions essential to its genesis seldom obtains in other parts of the world. This combination appears to be very simple—only a vigorous convection between strong neighboring counter currents. But since vertical convection, as indicated by thunderstorms, is common enough in most parts of the world, it follows that the other factor—namely, strong counter currents—is the distinctly American phenomenon. That such currents should often occur east of the Rocky Mountains is obvious from the position and trend of these mountains themselves, giving rise to southward winds; and the location of the Gulf of Mexico, from which winds turn northward. No other similar combination of mountain and ocean wind controls exists, and therefore no other place has in all respects the same kinds, frequencies, and intensities of storms.

Waterspouts.—Only its well-established name requires that the waterspout shall be specifically mentioned, since mechanically it does not differ essentially from the tornado. In fact, a tornado becomes a waterspout as soon as it passes to sea, while a waterspout becomes a tornado immediately it invades the land.

The waterspout, like the tornado, originates well up in the atmosphere, and is especially frequent in those regions where adjacent winds, usually the one above the other, have different directions: hence, where the counter trades, overrunning the trades, occur at ordinary cloud or convection levels; along the belt of doldrums, when considerably removed from the equator and flanked by opposing trade winds; and along boundaries of sharp temperature contrasts, such as the northern border of the Gulf Stream. In all such regions vertical convection may induce a more or less violent whirl in the same manner as that explained in the discussion of the tornado.

Occasionally small whirlwinds start from the surface of lakes or other bodies of water during calm hot weather. The strongest of these produce cloud columns, due to expansion, and are often called "waterspouts," though of radically different origin from that of the waterspout above described.

CHAPTER XI.

WINDS ADVERSE TO AVIATION.

SEVERAL local winds to which but little attention formerly was given, so little indeed that some of them are without special names, are now important through the art of aviation. These are here grouped together, however different in origin and type they may be, for the convenience of any one who may have occasion to consider them.

General Statement.—Every aviator experiences in the course of his flights many abrupt drops and numerous more or less severe jolts. The cause of the first—the sudden drops—he has grouped together and called "holes in the air," while to the latter he has given such names as "bumps," "dunts," etc. There are, of course, no holes, in the ordinary sense of the term, in the atmosphere—no vacuous regions—but at various places in the atmosphere there are, occasionally, conditions which, so far as flying is concerned, are very like unto holes. Neither is the air ever "full of bumps," in the sense of spots of abnormal density, but it often is turbulent in such manner as to render flying rough and uncomfortable. Both sets of atmospheric movements, those that produce appreciable drops and those that cause jolts, are indeed real; and the former, because of their general interest and practical importance, will be considered in some detail. The latter, being of little importance, will only be mentioned incidentally. Furthermore, there are no "pockets of noxious gas." No single gas, and no other likely mixture of gases, has at ordinary temperatures and pressures the same density as atmospheric air. Therefore, a pocket of foreign gas in the atmosphere would almost certainly either bob up like a balloon, or sink like a stone in water. It is possible, of course, as will be explained a little later, to run into columns of rising air that may contain objectionable gases and odors, but these columns are quite different from anything likely to be suggested by the expression "pocket of gas."

The above are some of the things that, fortunately, do not exist. The following, however, are some that do exist, and that produce sudden drops; usually small, so as to give only a negligible bump, but occasionally great enough to involve, when

near the surface, an element of danger. For clearness and simplicity these several kinds of air movements will be provisionally classified under terms suggested by water analogies.

Air Fountains.—A mass of air rises or falls according as its density is less or greater, respectively, than that of the surrounding atmosphere, just as, and for the same reason that, a cork bobs up in water and a stone goes down. Hence, any body of air is driven up whenever it is warmer and therefore lighter (less dense) than the surrounding air at the same level; and as the atmosphere is heated mainly through contact with the surface of the earth, which in turn has been heated by sunshine, it follows that these convection currents or vertical uprushes are most numerous during calm summer afternoons.

The turbulence of some of these rising masses is evident from the numerous rolls and billows of the large cumulus clouds they produce, within, and immediately beneath, which, the air is always rough, however smooth it may be either above or considerably below; and it is obvious that the same sort of turbulence, probably on a smaller scale, occurs near the tops of such columns also as do not rise to the cloud level. Further, when the air is exceptionally quiet, a rising column may be rather sharply separated from the surrounding quiescent atmosphere, as has often been reported by aviators, and as evidenced by the closely-adhering tall pillars of smoke occasionally seen to rise from chimneys.

The velocity of ascent of such fountains of air whether continuous, as in the dust whirl, or only intermittent, is at times surprisingly great. Measurements on pilot balloons, and also measurements taken in manned balloons, have shown vertical velocities, both up and down, of more than 3 metres per second (600 feet per minute). The soaring of large birds is a further proof of an upward velocity of the same order of magnitude, while the formation, in cumulus clouds, of hailstones of various sizes shows that uprushes of 10 to 12 metres per second (2000 to 2400 feet per minute), and occasionally much greater, not merely may, but actually do, occur.

There are, then, "air fountains" of considerable velocity whose sides at times and places are almost as sharply separated from the surrounding atmosphere as are the sides of a fountain of water, and it is altogether possible for the swiftest of these to

produce effects on an aeroplane more or less disconcerting to the pilot. The trouble may occur:

1. On grazing the column, with one wing of the machine in the rising and the other in the non-rising air; a condition that interferes with lateral stability, and produces a sudden shock both on entering the column and on leaving it.

2. On plunging squarely into the column; thus suddenly increasing the angle of attack, the pressure on the wings, and the angle of ascent.

3. On abruptly emerging from the column; thereby causing a sudden decrease in the angle of attack and also abruptly losing the supporting force of the rising mass of air.

4. As a result of rotation, if rapid, as it sometimes is, of the rising air.

That flying with one wing in the column and the other out must interfere with lateral stability and possibly cause a drop is obvious, but the effects of plunging squarely into or out of the column require a little further consideration, as does also the effect of rotation.

Let an aeroplane that is flying horizontally pass from quiescent air squarely into a rising column. The front of the machine may be lifted, as it enters the column, a little faster than the rear. If so, and, in any case, owing to the upward trend of the air, the angle of attack—that is, the angle which the plane of the wing, or plane of the wing chords, makes with the apparent wind direction—will be slightly increased. This will carry the machine to higher levels, which, of itself, is not important. If, however, the angle of attack is so changed by the pilot as to keep the machine while in the rising column at a constant level, and if, with this new adjustment, the rising column is abruptly left, a corresponding descent must begin. But even this is not necessarily harmful. Probably the real danger under such circumstances arises from *overadjustments* by the novice in his hasty attempt to correct for the abrupt changes, instead of letting his ship mainly ride out the inequalities.

If the rising column is in fairly rapid rotation (tornadoes are excluded—they can be seen and must be avoided), as sometimes is the case, disturbances may be produced in several ways. If the column is entered on its approaching side, the head-on wind may so decrease the velocity of the plane with reference to the

surrounding air that on emerging there necessarily must be a greater or less drop, as explained below under the caption "wind layers." On the other hand, if entered on the receding side there will be a tendency to drop within the column, which may or may not be fully compensated for by the vertical component of the wind. Finally, such a rotating column, especially, perhaps, if crossed near its outer boundary, may quickly change the orientation of the plane, and therefore the action on it of the surrounding air.

None of these conditions, however, except when encountered near the surface of the earth, is likely to involve any appreciable element of danger to the skilled aviator. But this does not justify ignoring them—no beginner is skillful, and all must start from and return to the surface.

Rising columns of the nature just described occur most frequently during clear summer days and over barren ground. They also occur, even to surprising altitudes, over roads, sandspits, and other places of similar contrast to the surrounding areas. Isolated hills, especially short or conical ones, should be avoided on low flights during warm, still days, for on such occasions their sides are certain to be warmer than the adjacent atmosphere at the same level, and hence act like so many chimneys in producing updrafts. Rising air columns occur less frequently and are less vigorous over water and over level green vegetation than elsewhere. They are also less frequent during the early forenoon than in the hotter portion of the day, and are practically absent before sunrise and at such times as the sky is wholly covered with clouds.

Although, as just explained, rising currents are certain to be more or less turbulent and "bumpy," they, nevertheless, are great aids to climbing. Hence, the experienced aviator often deliberately gets into them, as do soaring birds, when making a quick ascent.

Air Sinks.—The air sink obviously is the counterpart of the air fountain and is most likely to occur at the same time. Indeed, it is certain to occur over a small pond, lake, or clump of trees in the midst of a hot and rather barren region. These cooler spots localize the return or down-branches of the convection currents, and generally should be avoided by the aviator when flying at low levels. Similarly, on calm, clear summer days, down-cur-

rents nearly always obtain at short distances off shore, over rivers, and along the edges of forests. This type of down-current, however, rarely is swift, except in connection with thunderstorms, and, therefore, while it may render flying difficult, or even impossible with a slow machine, it seldom involves much danger.

Air Cataracts.—The air cataract is caused by the flow of a dense or, what comes to the same thing, a heavily-laden surface layer of air up to and then over a precipice, much as a waterfall is formed. Such cataracts are most frequent among the barren mountains of high latitudes. The cold surface winds catch up, and become weighted with, great quantities of dry snow, and then, because of both this extra weight and their high density, often rush down the lee sides of steep mountains with the roar and the force of a hurricane. But the violence of such winds clearly is all on the lee side and of shallow depth. Hence, where such conditions prevail, the aviator should keep well above the drifting snow or other aerial ballast, and, if possible, strictly avoid any attempt to land within the cataract itself.

Cloud Currents.—It frequently happens that a stratum of broken or detached clouds, especially of the cumulus type, is a region of turbulent currents, however quiet the air at both lower and higher levels. In the case of cumuli, at least, the currents within the clouds are upward, and those in the open spaces, therefore, generally downward. Also each branch of this circulation is more or less turbulent. Hence, while passing through such a cloud layer the aviator is likely to encounter comparatively rough flying, though, owing to the height, of very little danger.

Aerial Cascades.—The term "aerial cascade" may, with some propriety, be applied to the wind as it sweeps down the lee of a hill or mountain. Ordinarily, it does not come very near the ground, where indeed there frequently is a countercurrent, but remains at a considerable elevation. Other things being equal, it is always most pronounced when the wind is at right angles to the direction of the ridge and when the mountain is rather high and steep. The swift downward sweep of the air when the wind is strong may carry a passing aeroplane with it, and lead observers, if not the pilot, to fancy that a hole has been encountered, where, of course, there is nothing of the kind. Indeed, such cascades should be entirely harmless so long as the aviator keeps

his machine well above the surface and thus out of the treacherous eddies presently to be discussed.

Wind Layers.—For one reason or another it often happens that adjacent layers of air differ abruptly from each other in temperature, humidity, and density, and, therefore, as explained by Helmholtz, may, and often do, glide over each other in much the same manner that air flows over water, and with the same general wave-producing effect. These air waves are *seen* only when the humidity at the interface is such that the slight difference in temperature between the crests and troughs is sufficient to keep the one cloud-capped and the other free from condensation. In short, the humidity condition must be just right. Clearly then, though such clouds often occur in beautiful parallel rows, Figs. 79 and 80, adjacent wind strata of different velocities and their consequent air billows must be of far more frequent occurrence.

Consider now the effect on an aeroplane as it passes from one such layer into another. For the sake of illustration let the propeller be at rest and the machine be making a straightaway glide to earth, and let it suddenly pass into a lower layer of air moving in the same horizontal direction as the machine and with the same velocity. This, of course, is an extreme case, but it is by no means an impossible one. Instantly on entering the lower layer, under the conditions just described, all dynamical support must cease, and with it all power of guidance. A fall, for at least a considerable distance, is absolutely inevitable, and if near the earth, perhaps a disastrous one. To all intents and purposes, a "hole" has been run into.

The reason for the fall will be understood when it is recalled that the pressure of any ordinary wind is very nearly proportional to the square of its velocity with respect to the thing against which it is blowing. Hence, for a given inclination of the wings the lift on the aeroplane is approximately proportional to the square of the velocity of the machine with reference, *not* to the ground, but to the *air* in which it happens to be at the instant under consideration. If, then, it glides, with propellers at rest, into a wind stratum that is blowing in the same horizontal direction and with the same velocity, it is in exactly the condition it would be if dropped from rest at the top of a monument in still air. It inevitably must fall unless inherent stability, or skill of the

pilot, bring about a new glide after additional velocity has been acquired as the result of a considerable drop.

Of course such an extreme case must be of rare occurrence, but cases less extreme are met with frequently. On passing into a current where the velocity of the wind is more nearly that of the aeroplane, and in the same direction, part of the supporting force is instantly lost, and a corresponding drop or dive becomes at once inevitable. Ordinarily, however, this is a matter of small consequence, for the relative speed necessary to support is again soon acquired, especially if the engine is in full operation. Occasionally, though, the loss in support may be large, and occur so near the ground, as to be more or less dangerous.

If the new wind layer is against, and not with the machine, an increase instead of a decrease in the sustaining force is the result, and but little occurs beyond a mere change in the horizontal speed with reference to the ground, and a slowing up of the rate of descent.

All the above discussion of the effect of wind layers on aeroplanes is on the assumption that they flow in parallel directions. Ordinarily however, they flow more or less across each other. Hence the aviator, on passing out of one of them into the other, as a rule, has to contend with more than a disconcertingly abrupt change in the supporting force. That is, on crossing the interface between wind sheets, an aviator, in addition to suffering a partial loss of support, usually has to contend with the turmoil of a choppy aerial sea in which "bumps," at least, seem to abound everywhere.

Wind strata, within ordinary flying levels, are most frequent during weather changes, especially as fine weather is giving way to stormy. On such occasions, then, one should be on the watch for these strata, even to the extent of making test soundings for them with pilot balloons. It is also well, at such times, to avoid making great changes in altitude, because, since wind strata remain roughly parallel to the surface of the earth, the greater the change in altitude the greater the risk of passing from one stratum to another and thereby encountering at least a "bump," and, perhaps, a "hole." Also, to avoid the possibility of losing support when too low to dive, and for other good reasons, landings and launchings should be made, if practicable, squarely in the face of the *surface* wind.

Wind Billows.—It was stated above that when one layer of air runs over another of different density, billows are set up between them, as is often shown by windrow clouds. However, the warning clouds are comparatively seldom present; hence, even the cautious aviator may, with no evidence of danger before him, take the very level of the air billows themselves, and before getting safely above or below them, encounter one or more sudden changes in wind velocity and direction due, in part, to the eddy-like or rolling motion within the waves, with chances in each case of being suddenly deprived of a large portion of the requisite sustaining force. There may be perfect safety in either layer, but, unless headed just right, there necessarily is some risk in going from one to the other. Hence, flying at the billow level, since it would necessitate frequent transitions of this nature, should be avoided.

When the billows are within 300 metres, say, or less, of the earth (often the case during winter owing to the occurrence then of cold surface air with warmer air above) they are apt to be very turbulent, just as, and for much the same reason that, waves in shallow water are turbulent. For this reason, presumably, winter flying sometimes is surprisingly rough—the air very "bumpy." Fortunately, however, it is easy to determine by the aid of a suitable station barograph whether or not billows are prevalent in the low atmosphere, since they produce frequent (5 to 12 per hour, roughly) pressure changes, usually of 0.1 mm. to 0.3 mm. at the surface, as shown by Fig. 57.

Wind Gusts.—Near the surface of the earth the wind is always in a turmoil owing to friction and to obstacles of all kinds that interfere with the free flow of the lower layers of the atmosphere and thereby allow the next higher layers to plunge forward in irregular fits, swirls, and gusts with all sorts of irrgular velocities and in every direction. Indeed the actual velocity of the wind near the surface of the earth often and abruptly varies from second to second by more than its full average value, and the greater the average velocity, the greater, in approximately the same ratio, are the irregularities or differences in the successive momentary velocities. This is well shown by pressure-tube traces, of which Fig. 31 is a fine example.

Clearly, the lift on an aeroplane flying either with or against a gusty wind is correspondingly erratic, and may vary between

such wide limits that the aviator will find himself in a veritable nest of " holes " out of which it is difficult to rise, at least with a slow machine, and sometimes dangerous to try. However, as the turmoil due to the horizontal winds rapidly decreases with increase of elevation, and as the aviator's safety depends upon steady air conditions, or upon the velocity of his machine with reference to the atmosphere and not with reference to the ground, it is obvious that the windier it is the higher, in general, the minimum level at which he should fly.

Probably, however, the chief disturbance due to gusty wind—excessive tipping and consequent side-slipping—occurs not during straightaway flying, to which the above discussion applies, but as the aviator turns at low levels from flying against the gusts to flying with them. This is not owing to change in direction, since the velocity of an aeroplane with reference to the air, and therefore the sustaining force, is wholly independent of the velocity of either with reference to any third object, the surface of the earth, for instance. It may be, and presumably usually is, caused as follows: The aviator starts turning, suppose, while in and facing a relatively slow-moving portion of air. On banking, the plane is tipped with its under side more or less against the wind, whereupon the *higher* wing often runs into, or for brief intervals is caught by, a much swifter current than that into which the lower wing dips. Numerical values are not at hand, but the phenomenon of overrunning gusts is familiar from the action of winds on isolated tall trees. This obviously increases the tip, and, in extreme cases, sufficiently to induce a dangerous side slip.

On the other hand, when turning from flying with to flying against the wind, the higher wing catches the increased impact on its upper side. Hence, in this case, the result is merely a temporary flattening of the bank, and a consequent skid of but little danger.

Gusts that envelop the whole of an aeroplane while turning obviously affect the lift, as above explained, and even so, to some extent, when the path of the wind is at right angles to the course of the plane, but in this latter case seldom sufficiently to be of much importance.

Wind Eddies.—Just as eddies and whirls exist in every stream of water, from tiny rills to the great rivers and even the ocean currents, wherever the banks are such as greatly to change the

direction of flow, and wherever there is a pocket of considerable depth and extent on either side, and as similar eddies, but with horizontal instead of vertical axes, occur at the bottoms of streams where they flow over ledges that produce abrupt changes in the levels of their beds, so, too, and for the same general reasons, horizontal eddies occur in the atmosphere with rotation proportional, roughly, to the strength of the wind. These are most pronounced on the lee sides of cuts, cliffs, and steep mountains, but often occur also, to a less extent, on the windward sides of such places.

The air at the top and bottom of such whirls is moving in diametrically opposite directions—at the top with the parent or prevailing wind, at the bottom against it—and since they are close to the earth they may, therefore, as explained under "wind layers," be the source of decided danger. There may be some danger also at the forward side of the eddy where the downward motion is greatest.

When the wind is blowing strongly landings should not be made, if at all avoidable, on the lee sides of, and close to, steep mountains, hills, bluffs, or even large buildings; for these are the favorite haunts, as just explained, of treacherous vortices. The whirl is best avoided by landing in an open place some distance from bluffs and large obstructions, or, if the obstruction is a hill, on the top of the hill itself. If, however, a landing to one side is necessary, and the aviator has a choice of sides, other things being equal, he should take the *windward* and not the *lee* side. Finally, if a landing close to the lee side be compulsory he should, if possible, head up the hill with *sufficient velocity* to offset any probable loss of support due to an eddy current in the same direction. He could, of course, avoid loss of velocity with reference to the air, and hence loss of support, by heading along the hill—that is, along the axis of the vortex—but this gain would be at the expense of the dangers incident to landing in a side or cross wind. His only other alternative, heading down the hill, might be correct so far as the direction of the surface wind is concerned, but it probably would entail a long run on the ground and its consequent danger.

Eddies of a very different type, relatively small and so turbulent as to have no well-defined axis of rotation, are formed, as is well known, by a flow of strong winds past the side or corner of

a building, steep cliff, and the like. In reality such disturbances are, perhaps, more of the "breaker" type, presently to be discussed, than like smoothly-flowing vortices, and should be avoided whenever the wind is above a light breeze.

Air Torrents.—Just as water torrents are due to drainage down steep slopes, so, too, gravity winds strong enough to be called "air torrents" owe their origin to drainage down steep, narrow valleys. Whenever the surface of the earth begins to cool through radiation, or otherwise, the air in contact with it becomes correspondingly chilled and, because of its increased density, flows away to lower levels except when held in check, or even driven up, by opposing winds. Hence, when the weather is clear, and there is no counter-wind, there is certain to be air drainage down almost any steep valley during the late afternoon and most of the night. When several such valleys run into a common one, like so many tributaries to a stream, and especially when the upper reaches contain snow, and the whole section is devoid of forest, the aerial river is likely to become torrential in nature along the lower reaches of the drainage channel.

A flying machine attempting to land in the mouth of such a valley after the air drainage is well begun is in danger of going from relatively quiet air into an atmosphere that is moving with considerable velocity, at times amounting almost to a gale. If one must land at such a place and time, he should head up the valley so as to face the wind. If he heads down the valley and thereby runs with the wind, he will, on passing into the swift air, lose his support, or much of it, for reasons already explained, and correspondingly drop.

Air Breakers.—The term "air breakers" is used here in analogy with water breakers as a general name for the rolling, dashing, and choppy winds that accompany thunderstorm conditions. They often are of such violence, up, down, and sideways, in any and every direction, that an aeroplane in their grasp is likely to have as uncontrolled and disastrous a landing as would be the case in an actual hole of the worst kind.

Fortunately, "air breakers" usually give abundant and noisy warnings, and hence the cautious aviator need seldom be, and, as a matter of fact, seldom is caught in so dangerous a situation. However, more than one disaster is attributable to just such turbulent winds as these—air breakers.

Classification.—The above eleven types of atmospheric conditions may conveniently be divided into two groups with respect to the method by which they force an aeroplane to drop.

1. *The Vertical Group.*—All those conditions of the atmosphere, such as air fountains, sinks, cataracts, cloud currents, cascades, breakers (in places), and eddies (forward side), that, in spite of full speed ahead with reference to the *air*, make it difficult or impossible for the aviator to maintain his level, belong to a common class and depend for their effect upon a vertical component, up or down, in the motion of the atmosphere itself. Whenever the aviator, without change of the angle of attack and with a full wind in his face, finds his machine rapidly sinking, he may be sure that he has run into some sort of a down-current. Ordinarily, however, assuming that he is not in the grasp of storm-breakers, this condition, bad as it may seem, is of but little danger. The wind cannot blow into the ground, and therefore any down-current, however vigorous, must somewhere become a horizontal current in which the aviator may fly away or land, as he chooses.

2. *The Horizontal Group.*—This group includes all those atmospheric conditions—wind layers, billows, gusts, eddies (central portions), torrents, breakers (in places), and the like—that in spite of full speed ahead with reference to the ground deprive an aeroplane of a portion at least of its dynamical support. When this loss of support, due to a running of the wind more or less with the machine, is small and the elevation sufficient there is but little danger, but on the other hand when the loss is relatively large, especially if near the ground, the chance of a fall is correspondingly great.

Chapter XII.

Barometric Fluctuations.

The pressure of the atmosphere undergoes changes that may be classified as seasonal, regional, storm, "ripple," diurnal, semi-diurnal, and tidal. Most of these have already briefly been referred to, but they deserve further and separate consideration.

Seasonal Pressure Changes.—Since the atmosphere both expands and becomes more humid with increase of temperature, and when cooled contracts and also loses moisture, it follows that the resulting circulation (due to gravity) decreases the mass of air, and therefore its pressure, over places at or near sea level in any warming region and increases it, and its pressure, at similar levels over cooling regions. Hence, in general, the normal reading of the barometer at sea level is greater during winter than summer. It is not much greater, however—perhaps two or three millimetres on the average—since the viscosity of the atmosphere is too small to enable it to maintain any considerable pressure gradient. At places of high elevation the average actual (not reduced) pressure is *less* during winter than summer because of the increased density, during the colder season, of the lower air.

The approximate level at which January and July pressures, say, are equal may be computed as follows:

Let the sea-level pressures differ by 2.5 mm. and let the January temperature of the lower air be 20° C. colder than that of July. The pressure difference represents a stratum of the lower air about 27 metres thick, while the temperature difference is, roughly, 0.075 of the absolute temperature. Hence, under the above conditions, the height h at which the January and July pressures are the same is given approximately by the equation:

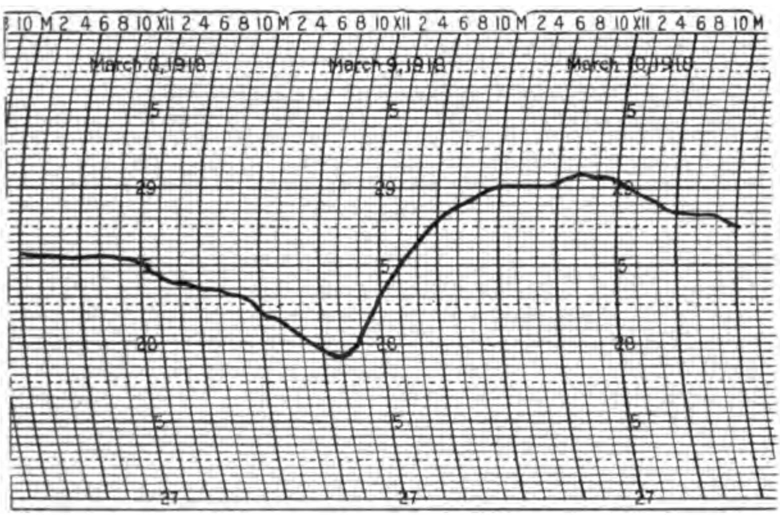
$$h = \frac{27 \text{ metres}}{0.075} = 360 \text{ metres}.$$

In addition to this seasonal pressure change over the whole of the northern and southern hemispheres, complicated to some extent by local conditions and the shifting of the belts of high pressure, there also are similar but greater pressure changes between the continents (high in winter, low in summer) and oceans (high in summer, low in winter) of each hemisphere itself. This

pressure swing between continent and ocean is due to the fact that the summer temperature of the land is much higher and its winter temperature much lower than that of the water.

Regional Pressure Changes.—The great semipermanent lows and highs often shift more or less from their normal positions. These displacements may be in any direction (more frequent in some than in others) and may last for any length of time, from a day or two to a fortnight or even longer. Such pressure changes, whatever their immediate cause, obviously are not seasonal, since

FIG. 57.



Pressure changes (inches) cyclone to anticyclone, Drexel, Nebr.

they occur at all times of the year. Neither are they of the migratory storm type, though themselves contributing to the genesis and development of storms and of great importance in the control of storm courses.

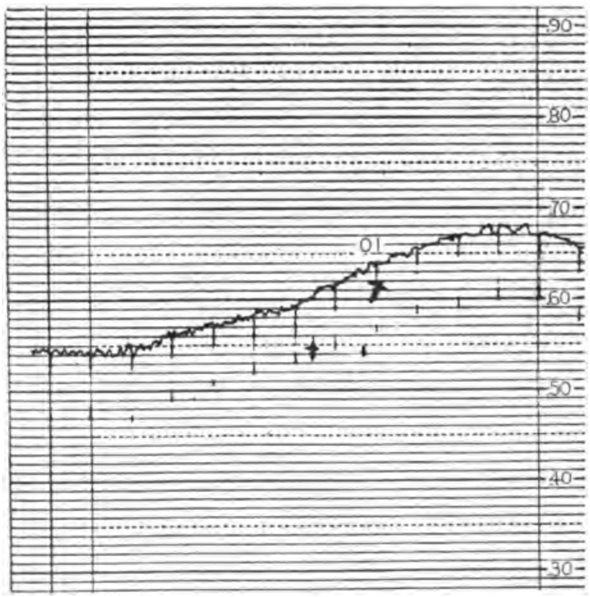
Storm Pressure Changes.—The progressive travel of cyclones and anticyclones, or, rather, of cyclonic and anti-cyclonic conditions, necessarily implies a regular order of pressure changes, through a range often amounting to 25 mm. or more, at each point along the storm path (Fig. 57). This type of change, frequent in extratropical regions at all times of the year, seldom lasts longer than 24 to 36 hours, and averages, perhaps, about 18 hours.

A secondary pressure change, due to the rapid rotation of a tornado or a waterspout, very intense but exceedingly brief—averaging less than one minute—occasionally develops under special conditions.

Perhaps, too, the pulsatory irregularities of the barometer during a thunderstorm should also be included here. Their origin, however, is entirely different.

Barometric "Ripples."—Small pressure changes, amplitude

FIG. 58.



Barometric ripples

usually 0.1 mm. to 0.3 mm. and period of 5 minutes to 10 minutes, Fig. 58 (the regularly spaced vertical lines along the trace are hour marks), and continuing for hours or even days together, are very common during cold weather. As first demonstrated by Helmholtz,[60] whenever layers of air that differ in density at their interface flow over each other long billows, analogous to gravity

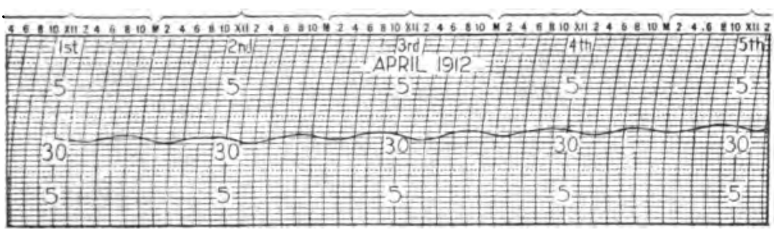
[60] *Sitz. der K. P. Akad. Berlin*, 1889, p. 761; 1890, p. 853. Translated by Cleveland Abbe, "Mechanics of the Earth's Atmosphere," Smithsonian Institution, 1891.

water waves, are produced which conform approximately to the equation,

$$d_1(u-V)^2 + d_2(V-v)^2 = \frac{g\lambda(d_2-d_1)}{2\pi}$$

in which V is the velocity of wave propagation, d_1 and d_2 the densities of the layers whose velocities are u and v, respectively, g the gravity acceleration, and λ the wave-length. If, now, the surface layer is colder than the next above, as it often is during winter, and rather shallow, 100 metres to 500 metres thick, say; the passage of the air billows, like the passage of waves in shallow water, necessarily produces greater or less corresponding changes in the pressure on the bottom—changes that appear as a series of

FIG. 59.



Barogram, Grand Turk Island, West Indies.

ripples in the record of a sensitive barograph. Furthermore, such shallow air billows, like shallow water waves, doubtless are turbulent—a condition that accounts, presumably, for the surprisingly rough flying the aviator often experiences during winter at low levels—300 metres and less.

During summer, when air billows rarely form near the surface, though frequently at greater altitudes, especially that of the cirrus cloud, neither barometric ripples nor shallow turbulences of the kind just mentioned often occur. This, doubtless, is because wave disturbances in air as in water do not penetrate far beneath the wave level.

Diurnal and Semidiurnal Pressure Changes.—It has been known now for two and a half centuries that there are more or less regular daily variations in the height of the barometer, culminating in two maxima and two minima during the course of 24 hours. The phenomenon in question is well illustrated by Fig. 59, a direct copy of a barograph trace obtained April 1–5, 1912, on

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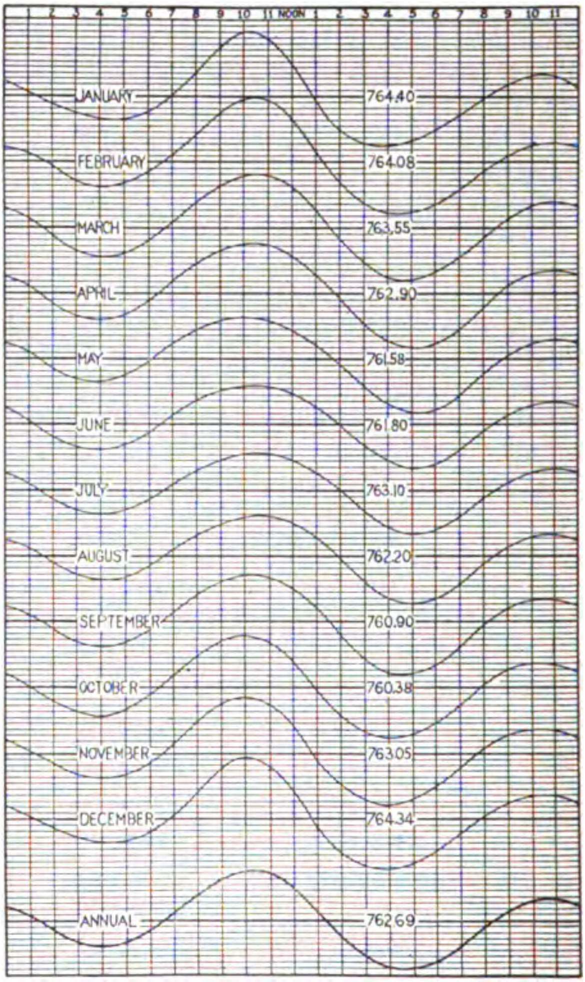
Grand Turk Island, latitude 21° 21′ N., longitude 70° 7′ W. It is further illustrated, and shown to persist through all the seasons, by Fig. 60, which gives, from hourly values, the actual average

FIG. 60.



Average daily barometric curves, Key West, Florida.

daily pressure curve for each month, and also for the entire year, as observed at Key West, latitude 24° 33′ N., longitude 81° 48′ W., during the 14 years, 1891–1904. The actual values are given in the accompanying table.

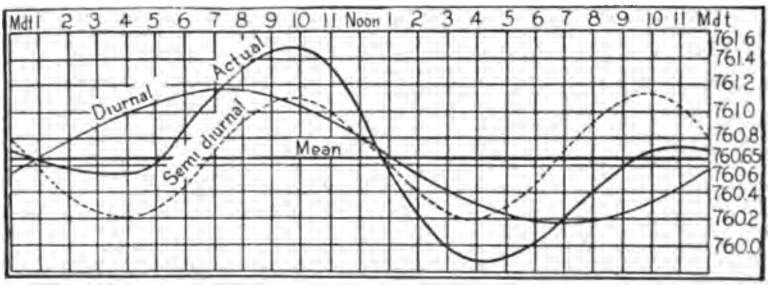
Average Hourly Readings of the Barometer, 1891–1904, at Key West, Latitude 24° 33′ N., Longitude 81° 48′ W., Elevation, 7 Metres.

75th Meridian time	January	February	March	April	May	June	July	August	September	October	November	December	Year
Average	764.40	764.08	763.55	762.90	761.58	761.80	763.10	762.20	760.90	760.38	763.05	764.34	762.69
1 A.M.	+.07	+.15	+.17	+.15	+.12	+.02	+.12	+.15	+.14	+.02	+.05	+.05	+.10
2 A.M.	−.11	−.05	−.10	−.14	−.14	−.20	−.14	−.10	−.10	−.21	−.16	−.11	−.13
3 A.M.	−.27	−.30	−.38	−.39	−.34	−.38	−.34	−.30	−.35	−.39	−.34	−.31	−.34
4 A.M.	−.37	−.41	−.48	−.45	−.37	−.41	−.39	−.40	−.42	−.46	−.39	−.38	−.41
5 A.M.	−.39	−.38	−.43	−.34	−.29	−.34	−.34	−.35	−.35	−.34	−.36	−.38	−.36
6 A.M.	−.29	−.20	−.20	−.11	−.06	−.13	−.19	−.15	−.15	−.13	−.16	−.26	−.15
7 A.M.	+.02	+.13	+.12	+.25	+.27	+.17	+.10	+.15	+.19	+.20	+.17	+.08	+.15
8 A.M.	+.40	+.46	+.31	+.50	+.50	+.45	+.32	+.34	+.44	+.50	+.52	+.43	+.43
9 A.M.	+.83	+.82	+.74	+.72	+.65	+.58	+.50	+.54	+.67	+.78	+.83	+.84	+.71
10 A.M.	+1.10	+1.05	+.87	+.82	+.70	+.63	+.57	+.66	+.80	+.88	+.96	+1.04	+.83
11 A.M.	+.98	+1.05	+.89	+.82	+.67	+.65	+.62	+.69	+.77	+.78	+.83	+.81	+.81
Noon	+.54	+.71	+.68	+.62	+.55	+.55	+.52	+.56	+.56	+.42	+.45	+.50	+.56
1 P.M.	−.08	+.18	+.23	+.32	+.29	+.30	+.30	+.31	+.24	−.05	−.11	−.16	+.15
2 P.M.	−.57	−.33	−.20	−.06	−.06	+.00	+.05	−.02	−.20	−.46	−.51	−.59	−.26
3 P.M.	−.80	−.68	−.56	−.50	−.44	−.33	−.31	−.40	−.60	−.72	−.77	−.77	−.56
4 P.M.	−.83	−.86	−.84	−.80	−.75	−.59	−.59	−.71	−.86	−.85	−.87	−.84	−.79
5 P.M.	−.73	−.83	−.89	−.90	−.88	−.74	−.72	−.79	−.86	−.77	−.75	−.74	−.79
6 P.M.	−.57	−.71	−.76	−.85	−.85	−.69	−.65	−.71	−.71	−.59	−.54	−.53	−.69
7 P.M.	−.34	−.48	−.50	−.60	−.60	−.41	−.41	−.45	−.42	−.31	−.26	−.26	−.41
8 P.M.	−.01	−.15	−.18	−.21	−.22	−.08	−.11	−.12	−.05	+.07	+.10	+.05	−.08
9 P.M.	+.19	+.10	+.12	+.12	+.04	+.15	+.12	+.15	+.24	+.32	+.30	+.28	+.17
10 P.M.	+.32	+.26	+.36	+.32	+.27	+.33	+.30	+.34	+.39	+.42	+.40	+.38	+.36
11 P.M.	+.32	+.34	+.41	+.37	+.34	+.40	+.35	+.39	+.39	+.40	+.40	+.38	+.38
Midnight	+.22	+.26	+.33	+.30	+.27	+.30	+.30	+.28	+.29	+.25	+.25	+.23	+.27

Probably the earliest observations of these rhythmical daily changes in the atmospheric pressure were made by Doctor Beal[61] during the years 1664–65, and therefore very soon after the invention, 1643, of the mercurial barometer. Since Beal's discovery the same observation has been made and puzzled over at every station at which pressure records were kept and studied, but without success in finding for it any adequate physical explanation. In speaking of the diurnal and semidiurnal variations of the barometer, Lord Rayleigh[62] says:

"The relative magnitude of the latter [semidiurnal variation], as observed at most parts of the earth's surface, is still a mystery, all the attempted explanations being illusory."

FIG. 61.



Average daily barometric curve and its components, Washington, D. C. (After W. J. Bennett.)

Obviously the average hourly pressures for a decade or longer at any given place are practically free from storm and other irregular effects, but contain all diurnal and shorter period disturbances that may exist. On being analyzed these actual data show two well-defined sine curves, a diurnal and a semidiurnal, as illustrated by Fig. 61,[63] each of which requires a special explanation. Higher harmonics of very small amplitude have also been found, but, as neither the diurnal nor the semidiurnal disturbances can have true sine values throughout, it follows that the higher harmonics indicated by this type of analysis may not represent any actual force of the same periodicity.

[61] *Phil. Trans.*, 9 (1666), p. 153.
[62] *Phil. Mag.*, 29 (1890), p. 179.
[63] Bennett, *Monthly Weather Review*, 34 (1906), p. 528.

Diurnal Pressure Changes.—There are two classes of well-defined 24-hour pressure changes. One obtains at places of considerable elevation and is marked by a barometric maximum during the warmest hours and minimum during the coldest. The other applies to low, especially sea level, stations and is the reverse of the above, the maximum occurring during the coldest hours and the minimum during the warmest.

The first class of changes just mentioned, the one that concerns elevated stations, is due essentially to volume expansion and contraction of the atmosphere caused by heating and cooling respectively. Thus the lower atmosphere over that side of the earth which is exposed to insolation becomes more or less heated, and therefore, because of the resulting expansion, its centre of mass is correspondingly raised. Conversely, during the night the atmosphere cools and contracts and the centre of mass is proportionately lowered. Hence, so far as this effect alone is concerned, a mountain station, 1000 metres, say, above sea level, will have the greatest mass of air above it when the atmosphere below is warmest or most expanded, and the least when the lower atmosphere is coldest or most contracted—that is to say, this effect tends to produce, at such stations, barometric maxima during afternoons and minima about dawn.

There is, however, another effect resulting from the volume expansion and contraction of the atmosphere to consider; namely, its lateral flow. To this, mainly, is due that daily barometric swing at sea level, as shown by harmonic analysis, the early evening minimum and the early morning maximum, that is the reverse of the high-level oscillation.

The expansion and consequent vertical rise of the air on the warming side of the earth, together with the simultaneous contraction and fall of the atmosphere on the cooling side, establishes a pressure gradient at *all* levels of the atmosphere directed from the warmer toward the cooler regions, a gradient that obviously causes the well-known heliotropic wind—the wind that turns with the sun—and thus leads to maximum pressures at the coldest places and minimum pressures at the warmest. But as these regions are along meridians, roughly, 10 hours, or 150 degrees apart, and perpetually move around the earth at the rate of one revolution every 24 hours, there must be a corresponding perpetual flow of air, or change of flow, as above described, in a

ceaseless effort to establish an equilibrium which, since the disturbance is continuous, can never be attained.

Semidiurnal Pressure Changes.—Both the actual barometric records and their harmonic analyses show conspicuous 12-hour cyclic changes that culminate in maxima and minima at approximately 10 o'clock, A.M. and P.M., and 4 o'clock, also both A.M. and P.M., respectively—the exact hour in each case depending somewhat upon season, elevation, and, presumably, weather conditions.

Some of the observed facts in regard to this 12-hour cyclic change of pressure are:[64]

(*a*) The amplitude, when other things are substantially equal, varies with place approximately as the square of the cosine of the latitude.

(*b*) The amplitude is everywhere greatest on equinoxes and everywhere least on solstices.

(*c*) The amplitude is greater at perihelion than at aphelion.

(*d*) The amplitude is greater by day than by night.

(*e*) The amplitude is greatest on clear days and least on cloudy.

(*f*) The day amplitude is greater over land than over water.

(*g*) The night amplitude is greater over oceans than over continents.

(*h*) Over the tropical Pacific Ocean the forenoon barometric maximum is about 1 mm. above and the afternoon minimum 1 mm. below the general average pressure.

Obviously, other things being equal, both the daily change in temperature and the resulting change in convection are greater in the tropics than elsewhere; greater at perihelion than at aphelion; greater during clear weather than cloudy; greater over land than over water; and greatest when the time of heating and the time of cooling (day and night) are equal, and least when these are most unequal or at the times of solstice. Hence all the above facts of observation strongly favor, if they do not compel, the conclusion that the daily cyclic pressure changes are somehow results of daily temperature changes. There are, however, a

[64] Angot, "Étude sur la marche diurne du baromètre," *Annales du Bureau Central Météorol.*, 1887.

Hann, "Untersuchungen über die tägliche Oscillation des Barometers," *Denkschriften der Wiener Akademie*, Bd. 55, 1889.

Hann, *Meteorol Zeit.*, 15 (1898), 361.

number of other causes of slight pressure changes,[65] but apparently only the following have any appreciable value:

1. *Horizontal flow of the atmosphere from the regions where it is most expanded toward those where it is most contracted.*

The exact hour at which the atmosphere is warmest and most expanded depends upon a variety of circumstances, but on the average it is approximately at 4 o'clock in the afternoon. Hence, in general, at about this time the amount of air overhead, counting from sea level, should be least, and therefore at this hour a sea-level barometer should have its lowest reading. On the other hand, lowest temperatures and maximum contractions obtain soon after dawn, or shortly before 6 A.M. throughout the year near the equator, and everywhere at the time of equinox.

The 24-hour swing of the barometer, therefore, does not appear to be of even period, but rather of intervals that are to each other, roughly, as 5 to 7. To be sure, the barometer is lower at 6 o'clock in the afternoon than at the same hour of the morning, and hence one may assume an even period 24-hour swing, with a morning 6 o'clock maximum and evening 6 o'clock minimum, and partially correct this regular curve (the correction is never perfect) by the superposition of one or more additional sine curves of convenient periods. But this approximation to the true curve does not prove the existence of actual forces with the periods assumed.

It appears, then, that the physical causes of the 24-hour component of the diurnal pressure changes are such as to give a morning maximum at about 6 o'clock and an afternoon minimum at about 4 o'clock. The above causes of pressure change, however, do not account for either of the 10 o'clock maxima.

2. *Interference by vertical convection with free horizontal flow.*

It was long ago suggested by Abbe[66] that convectional interference is the principal cause of the forenoon maximum pressure, which indeed it probably is, as the following consideration shows:

Let the mass m of air be near the ground and have the horizontal velocity v, and let the larger mass M be at a higher elevation and have the greater velocity V in the same direction. If now these two masses should mingle in such manner as to be free from all disturbance, except their own mutual interference, the

[65] Humphreys, *Bul. Mt. Weather Obsy.*, 5 (1912), p. 132.

[66] "Preparatory Studies" (1890), pp. 8 and 56.

resulting final velocity, U, in the same direction, would be given by the equation—

$$U = \frac{mv + MV}{m + M}$$

and there obviously would be no check in the total flow—no damming up and consequent increase of pressure. But this simple mixing of the two masses is by no means all that happens in the case of vertical convection. The rise of the mass m is simultaneously accompanied by an equivalent descent of air from a higher level, which in turn loses velocity, directly or indirectly, by surface friction. If the falling mass is also m, and if its velocity is reduced by friction to v, then from a single interchange, due to vertical convection, the total momentum becomes—

$$2\,mv + (M - m)\,V$$

and the total flow is reduced by the amount

$$m\,(V - v)$$

But as this is for a single interchange, it is obvious that the more active vertical convection becomes, the greater will be its interference with the flow of the atmosphere, the more the winds will be dammed up, and the higher the barometric pressure. As convection increases, reaches a maximum, and then decreases, so, too, will the resulting interference go through the same changes.

Now the general movement of the atmosphere is from east to west within the tropics and from west to east at higher latitudes. Therefore in either case such damming up of the air as vertical convection may produce will be essentially along meridians, and thus a function of the time of day. But, in general, convection increases most rapidly during the forenoon, say 8 to 9 o'clock, is most active at 10 to 11 o'clock, and reaches its greatest elevation about 4 o'clock in the afternoon. Hence the damming up of the atmosphere, due to vertical convection, and the resulting increase of barometric pressure must increase most rapidly during the forenoon, and come to a maximum about 10 o'clock. After this the convectional interference decreases, while at the same time the amount of air in a vertical column of fixed cross-section diminishes as a result of expansion and overflow, until at about 4 o'clock in the afternoon the barometric pressure, as already explained, has reached a minimum.

To form some idea of the magnitude of the barometric change due to convectional turbulence, consider the atmosphere between two parallels of latitude near the equator. This limited quantity may be regarded as a stream flowing around the earth, having its minimum velocity and maximum depth where convectional interference is greatest, and maximum velocity with minimum depth where convection is absent. And since the linear velocity of a point on the equator is approximately 1670 kilometres per hour, while during the forenoon the rate of increase of the barometric pressure at the same place is, roughly, 0.2 mm. per hour, it follows that a damming up, or check in the flow, of the given stream at the rate of 0.44 kilometre per hour would be sufficient of itself to account for the observed rise in the barometer. But if the average velocity of the wind, or flow of the stream in question, is 10 metres per second, which it may well be, the rate of decrease in velocity requisite for the given rate of pressure increase could be produced by having only 1 part in 80 of the whole superincumbent atmosphere brought to rest per hour, or the equivalent thereof, an amount that perhaps is reasonable. At any rate, the assumed velocity decrease is of the same order of magnitude as that observed to take place during, and as the result of, diurnal convection.

Summing up the effects of all the above causes of barometric changes, it appears:

(*a*) That the afternoon minimum is caused essentially by overflow from the region where the atmosphere is warmest, or better, perhaps, from the meridian along which the temperature increase has been greatest, toward that meridian along which there has been the greatest decrease in temperature.

(*b*) That vertical convection interferes with the free horizontal flow of the atmosphere and to that extent dams it up and correspondingly increases the barometric pressure; also, that the time of this interference agrees with the forenoon changes of the barometer, and that its magnitude is of about the proper order to account for the forenoon barometric maximum.

The afternoon barometric minimum and the forenoon maximum, therefore, are to be regarded as effects of temperature increase; the minimum as due to expansion and consequent overflow; the maximum as caused by vertical convection and consequent interference with the free circulation of the atmosphere.

The forced afternoon minimum would occur in an otherwise stagnant atmosphere, and substantially as at present; but not so with the forced forenoon maximum, since the interference or damming effect depends upon a flow or circulation of the atmosphere, parallel roughly to the equator.

It remains now to account for the night 10 o'clock maximum and 4 o'clock minimum.

3. *Natural or free vibration of the atmosphere as a whole.*

This subject has been discussed by several mathematical physicists of great eminence. The latest and most complete of these discussions, and the one to which those interested in this phase of the barometric problem are especially referred, is by Lamb,[67] who concludes:

"Without pressing too far conclusions based on the hypothesis of an atmosphere uniform over the earth, and approximately in convective equilibrium, we may, I think, at least assert the existence of a free oscillation of the earth's atmosphere, of 'semi-diurnal' type, with a period not very different from, but probably somewhat less than, 12 mean solar hours."

Hence any cause of pressure change, having a semidiurnal period or approximately so, would, if of sufficient magnitude and proper phase, account for the 12-hour barometric curve. Such a cause, many think, may be found in the irregular daily march of temperature, since the curve expressing this march is more or less approximately resolvable into a diurnal and a semidiurnal sine curve. But the resolution is not perfect and, besides, there is no obvious cause for a temperature increase by night, and hence the reality of the semidiurnal component in the temperature curve is equally doubtful.

All that is needed, apparently, to give the semidiurnal pressure curve is a pressure impulse of the same period, 12 hours, as that of the free vibration of the atmosphere as a whole. And this is furnished by the forced forenoon barometric maximum, followed six hours later at the same place by the forced afternoon barometric minimum. In other words, taken together, the forenoon and afternoon forced disturbances appear to occur with the proper time interval necessary to set up and maintain the 12-hour free vibrations of the atmosphere.

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[67] *Proc. Roy. Soc.*, A. **84** (1911), p. 551.

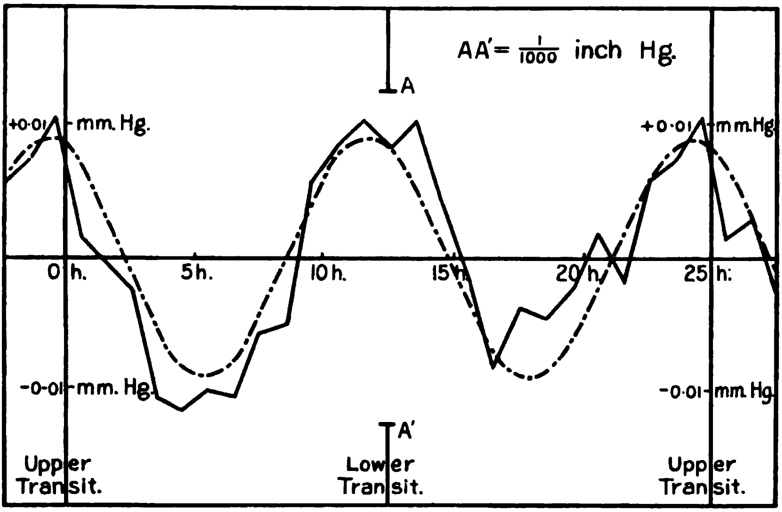
The course of events at each locality appears to be substantially as follows:

1. A forced forenoon compression of the atmosphere, followed by its equally forced afternoon expansion, the two together forming one complete barometric wave, with a 10 o'clock maximum and a 4 o'clock minimum, in harmony with the free vibration of the entire atmospheric shell.

2. Nondisturbance through the night or during the time of a single free vibration.

3. Repetition the following day of the forced disturbances in

FIG. 62.



Lunar semidiurnal atmospheric tide Greenwich, 1854–1917.

synchronism with, and therefore at such time as to reënforce, the free vibrations.

The series of disturbances is continuous, forced by day and free by night, but the resulting amplitudes of the barometric changes are limited, through friction and through the absence of perfect synchronism, to comparatively small values. Each point upon the atmospheric shell receives at every alternate swing a forced impulse in phase with the free vibration, and therefore at such time and in such manner as indefinitely to maintain the vibrations of the atmosphere as a whole.

The forenoon maximum and the afternoon minimum are pri-

mary disturbances equally forced but in different ways by the daily increase of temperature, while the evening maximum and the morning minimum are secondary disturbances caused by the joint action of the forced primaries through the 12-hour free vibration of the atmosphere. In short, the semidiurnal swing of the barometer is a result of merely fortuitous circumstances—of the fact that the mass of the atmosphere happens to be such that the period of its free vibration is approximately just one-half that of the earth's rotation.

Tidal Pressure Changes.—The theory of atmospheric tides is too tedious to include here, especially as it is easily accessible [67a] to all who may have any occasion to look it up. According to this theory the barometric amplitudes in equatorial regions, due to the gravitational action of the sun and the moon, should be about 0.0109 mm. and 0.025 mm., respectively, and rapidly decrease with increase of latitude. These, of course, are not easily disentangled from the numerous other barometric changes. Nevertheless, efforts to do so have been made and, apparently, with fair success, notably by Chapman,[67b] whose results from the Greenwich data of 1854–1917 are shown in Fig. 62.

[67a] Lamb, "Hydrodynamics."

[67b] *Qr. Jr. Roy. Meteorol. Socy.* 44, p. 271, 1918; 45, p. 113, 1919.

Chapter XIII.

EVAPORATION AND CONDENSATION.

INTRODUCTION.

The presence of water vapor in the atmosphere is of such vital importance in the economy of Nature, and the source of so many phenomena, as to demand a study of, among other things: evaporation, by which the vapor is gotten into and rendered a portion of the atmosphere, mainly from free surfaces, but also from vegetation and damp soil; and condensation, by which in various forms it is removed from the air.

EVAPORATION.

Evaporation, the process by which a liquid becomes a vapor, or gas, is a result of the kinetic energy of the individual molecules. Some of the molecules at or near the surface have such velocities and directions that they escape from the liquid and thus become an integral part of the surrounding gas or atmosphere; and as the chance of escape, other things remaining equal, increases with the velocity, it follows (*a*) that the average kinetic energy of the escaping molecules is greater than that of the remaining ones, or that evaporation decreases the temperature of a liquid, and (*b*) that the rate of evaporation increases with increase of temperature.

Just as the kinetic energy of some of the molecules of the liquid carries them into the adjacent space, so, too, the kinetic energy of some of the molecules of the gaseous phase causes them to penetrate into and thus become a part of the liquid. In reality, therefore, evaporation from and condensation onto the surface of a liquid, though necessarily taking place by discrete molecular units, practically are continuous processes whose ratio may have any value whatever. As popularly used, however, and even as very commonly used scientifically, the term "evaporation" refers to the net loss of a liquid, and "condensation" to its net gain, so that, in this sense, both are said to be zero when, as a matter of fact, they are only equal to each other.

In the sense of net loss, which admits of accurate measurement, evaporation has been the subject of numerous investiga-

tions. Vegetation, soil, and the free water surface each offers its own peculiar and numerous evaporation problems. In what follows, however, only the free surface will be considered.

Evaporation into Still Air (a) from Tubes.—The rate of loss of a liquid by evaporation and diffusion, through a tube of fixed length and constant cross-section, into a still atmosphere has been carefully studied by Stefan.[68] Obviously, when a steady state has been attained, the rate at which the vapor escapes per unit area of the cross-section of the tube is constant, directly proportional to the driving force and inversely proportional to the resistance. These in turn are proportional, respectively, to the pressure gradient of the vapor along the tube and the partial pressure of the foreign gas at the same place. In symbols,

$$v = -\frac{k}{P-p}\frac{dp}{dn} = k\frac{d}{dn}\log(P-p),$$

in which v is the volume at 0° C. and 760 mm. pressure of the vapor that escapes per second per unit area of the cross-section of the tube, P the total pressure, a constant, $\frac{dp}{dn}$, the vapor pressure gradient along the tube at and normal to the cross-section at which the partial pressure due to the vapor is p, and k the coefficient of diffusion, whose value depends upon the nature of the vapor and the gas through which it is passing, and their temperature.

But as a steady state is assumed, it follows that both the rate of flow and the coefficient of diffusion k are independent of the distance n along the tube above the liquid surface. Hence,

$$V = \frac{kA}{h}\log\frac{P-p''}{P-p'}$$

in which V is the rate of total evaporation, A the area of the cross-section of the tube, h its height, or the distance of its top (tube supposed vertical) above the liquid, p'' and p' the partial pressures of the vapor at the free end and evaporating surface, respectively.

All the terms in this equation except k may easily be measured, and thus k itself evaluated. But with k known, the rate of evaporation of the same liquid (water, say) from a circular tube or well of any given cross-section and length, provided the length is equal to or greater than the diameter, may be computed from

[68] *Sitzungsberichte der K. Akad. der Wis. Wien.* 68 (1873), 385–423.

the total gas pressure and the vapor pressures at the surface of the liquid and top of the tube.

Evaporation into Still Air (b) from Flush Circular Areas.—The rate of evaporation into still air from a circular tank or pond filled flush with a relatively extensive plane which itself neither absorbs nor gives off any vapor has also been found by Stefan[69] susceptible of complete analysis.

From the general equation

$$v=-\frac{k}{P-p}\frac{dp}{dn}$$

it follows that

$$v=-k\frac{d}{dn}\log\frac{P-p_0}{P-p}$$

in which p_0 is the constant partial pressure of the vapor, during a steady state, at a given point. Hence if

$$u=\log\frac{P-p_0}{P-p}$$

$$v=-k\frac{du}{dn}$$

But this is identical with the flow of heat when $\frac{du}{dn}$ is the temperature gradient and k the thermal conductivity. Similarly it represents the flow of electricity, and also the field of force in the presence of a charged plate. If σ is the density of the surface charge on a plate, then

$$-\frac{du}{dn}=4\pi\sigma.$$

Further, if E is the total charge,

$$E=Cu_1$$

in which C is the capacity of the plate and u_1 its potential. Hence the total diffusion, confined to *one* side, is given by the expression

$$V=2\pi kCu_1$$

But

$$u_1=\log\frac{P-p_0}{P-p_1}$$

in which p_0 is the vapor pressure of the free air at a great distance from the evaporating surface and p_1 its pressure at the surface, or saturation pressure at the surface temperature.

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[69] *Sitzungsberichte der K. Akad. der Wis. Wien*, 73 (1881), 943–954.

The capacity of a circular disk of radius a is

$$C=\frac{2a}{\pi}$$

Hence

$$V=4ak\log\frac{P-p_0}{P-p_1}$$

If p_0 and p_1 are both small in comparison to P,

$$V=4ak\frac{(p_1-p_0)}{P}, \text{ nearly.}$$

The real importance of this equation is its proof that evaporation, under the restricted conditions assumed, is proportional to the *diameter* (or other linear dimension) of the evaporating surface and *not,* as one might suppose, to its area. Obviously, therefore, evaporation in the open under ordinary conditions cannot be directly proportional, as often assumed, to the area involved.

Evaporation into Still Air (c) from Elliptical Areas.—Evaporation from an elliptical surface is slightly faster than from a circular one of equal area, but the difference is small until the major axis of the ellipse becomes several times longer than the minor; being only 1.11 as fast when the ratio of the axes is 1 to 4. Hence, when the axes do not greatly differ, a close approximation to the rate of evaporation from an elliptical surface is given by the equation

$$V=4\sqrt{ab}\,k\log\frac{P-p_0}{P-p_1},$$

or, when p_0 and p_1 are small with reference to P,

$$V=4\sqrt{ab}\,k\frac{p_1-p_0}{P}$$

No exact mathematical expression has yet been obtained for the rate of evaporation into still air from surfaces of any other outline than the above—circle and ellipse.

Evaporation into a Steady Horizontal Wind.—Significant progress towards the complete solution of this difficult problem has been made by Jeffreys,[69a] whose discussion of it is substantially as follows: Let ρ be the density of the atmosphere at any point and D the fraction of this density due to water vapor; let the wind be in the direction x parallel to the evaporating surface, and let its velocity at some distance above this surface be u. The

[69a] *Phil. Mag.,* 35, p. 273, 1918.

components v and w of the wind velocity in the directions y and z, respectively (z being normal to the surface and y at right angles to both x and z) are, therefore, both zero. For moderate winds the velocity of the air may be assumed to increase rapidly through a thin shearing layer from zero at the surface to perhaps half value, $u/2$, a millimetre or so above it. Through this same layer the vapor density will rapidly decrease, if the general air is comparatively dry, from saturation at the surface, where $D = D_0$, say, to some decidedly less value. Beyond this layer the transfer of water vapor, of heat, and of momentum, are all owing essentially to turbulence, as fully explained by Taylor,[69b] and the coefficient k of this "eddy diffusion" is practically independent of position.

Therefore, in analogy to heat conduction, molecular diffusion, etc.

$$\frac{dD}{dt} = \frac{\partial}{\partial x}\left(k\frac{\partial D}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial D}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial D}{\partial z}\right).$$

Also

$$\frac{dD}{dt} = \frac{\partial D}{\partial t} + u\frac{\partial D}{\partial x} + v\frac{\partial D}{\partial y} + w\frac{\partial D}{\partial z}.$$

Hence, as the density gradient changes only with elevation ($v = w = 0$), and as k is constant, it follows that, when a steady state has been attained,

$$u\frac{\partial D}{\partial x} = k\frac{\partial^2 D}{\partial z^2}$$

or, putting $\frac{k}{u} = h^2$, a constant, that

$$\frac{\partial D}{\partial x} = h^2\frac{\partial^2 D}{\partial z^2}$$

An integral of this equation is[69c]

$$D = D_0\left(1 - \frac{2}{\sqrt{\pi}}\int_0^q e^{-q^2}dq\right)$$

in which

$$q = \frac{z}{2h\sqrt{x}}$$

[69b] *Phil. Trans. Roy. Soc. A.*, 215, p. 1, 1915.

[69c] Van Orstrand and Dewey, Professional Paper 95-G, U. S. Geological Survey, 1915.

On taking the origin at the windward edge of the liquid surface,

$$D = 0, \text{ when } x \text{ is negative.}$$

Hence, at the surface, where $z = 0$,

$$\frac{\partial D}{\partial z} = \frac{D_0}{h\sqrt{\pi x}}$$

Therefore, the rate of evaporation is

$$k\rho\frac{\partial D}{\partial z} = \rho D_0\sqrt{\frac{ku}{x}} \text{ per unit area.}$$

and, for a strip of width dy, extending from $x = 0$ to $x = x$

$$\rho D_0\int_0^x \left(\frac{ku}{x}\right)^{\frac{1}{2}} dx = 2\,\rho\,D_0\left(\frac{kux}{\pi}\right)^{\frac{1}{2}} dy.$$

If, now, the length of the strip from margin to margin be l, neglecting end corrections due to sidewise diffusion, the rate of total evaporation is

$$2\,\rho\,D_0\left(\frac{ku}{\pi}\right)^{\frac{1}{2}} l^{\frac{1}{2}} dy,$$

taken over the whole area.

In the case, therefore, of free, unruffled liquid surfaces of medium dimensions, roughly, 20 centimetres to 500 metres across,[69d] it appears, in the case of "eddy diffusion":

1. That the rate of evaporation is proportional to the square root of the wind velocity.
2. That the rates of total evaporation from surfaces of the same shape and same orientation to the wind are to each other as the three-quarter powers of their respective areas.

If, for instance, the surface is a circle of radius a, the rate of evaporation from it is

$$2.44\, D_0\,(kua^3)^{\frac{1}{2}};$$

which accords with the observations of Thomas and Ferguson.[69e]

The equation, therefore, that expresses the rate of total evaporation from a given surface by "eddy diffusion" is very different from the corresponding equation when the diffusion is wholly molecular, nor are they reducible the one to the other.

[69d] Jeffries, *Loc. cit.*

[69e] *Phil. Mag.* 34, p. 308, 1917.

The first applies, approximately, at least, when there is an appreciable wind of the kind specified, namely, steady and strictly horizontal; the second, only when the air is absolutely quiet. The problem, however, of evaporation into imperceptible to very light winds is more difficult, and, as yet, unsolved.

Evaporation in the Open.—Several hundred papers,[70] many of them giving the results of elaborate investigations, have been published on the evaporation of water from free surfaces, vegetation, and soil, and, while no equation has been found that expresses in terms of easily measurable quantities the rates of evaporation in the open, nevertheless several factors that control these rates have been discovered and more or less approximately evaluated. In the case of free, clean surfaces the principal factors are:

(a) *Salinity.*—It has repeatedly been observed that the evaporation of salt solutions decreases with increase of concentration, and that sea-water evaporates approximately 5 per cent. less rapidly than fresh water under the same conditions.

(b) *Dryness of the Air.*—Many observations have shown that, to at least a first approximation, the rate of evaporation is directly proportional, other things being equal, to the difference in temperature indicated by the wet and dry bulb thermometers of a whirled psychrometer.

According to the psychrometric formula developed by Apjohn, Maxwell, Stefan, and others,

$$p_1 - p_0 = AB(t_0 - t_1)$$

in which t_0 is the temperature and p_0 the vapor pressure of the free air, t_1 the temperature of the wet bulb (and surface of evaporating liquid), p_1 the saturation vapor pressure at temperature t_1, B the barometric pressure, and A a constant, provided ventilation is sufficient. But evaporation is proportional to the ratio of vapor pressure gradient to total pressure; that is,

$$V = k\frac{p_1 - p_0}{B}$$

Hence, other things being equal,

$$V = C\ (t_0 - t_1),\ \text{approximately.}$$

But $t_0 - t_1$ increases with the dryness, and hence so does evaporation.

[70] Livingston, "An Annotated Bibliography of Evaporation," M. W. R., June, September, and November, 1908, and February, March, April, May, and June, 1909.

(c) *Velocity of the Wind.*—All observers agree that evaporation increases with wind velocity, presumably through increase, by the action of eddy diffusion, of the vapor pressure gradient near the surface. As above explained, it is now known that in the case of a strictly horizontal and steady wind evaporation from an area of medium size is proportional to the square root of the wind velocity. But, in general, these conditions are not fulfilled in nature. The wind usually has a variable vertical component, and, besides, is irregular in strength and direction. There is not, therefore, any constant relation of evaporation to the *average horizontal* component of wind velocity—the value usually measured.

(d) *Barometric Pressure.*—Since the presence of any gas retards the diffusion of other gas molecules, whether of the same or different nature, it follows that when the vapor tension is comparatively small, evaporation must vary inversely, nearly, as the total barometric pressure, if temperature is constant.

(e) *Area of Surface.*—Obviously the total amount of water evaporated must increase with the area of the evaporating surface, but not necessarily at the same rate. In fact, as already explained, if the evaporation is from a circular area into still air, it increases as the square root of the area; and as the ¾ power of the area in the case of a strictly horizontal wind. Under outdoor conditions, however, it is much more nearly, though probably by no means exactly, proportional to the first power of the surface.

(f) *Temperature of the Water.*—Evaporation increases rapidly with the temperature of the water, roughly in proportion to the saturation pressure at that temperature, provided the general humidity of the air is low. When, however, the water surface is colder than the dew-point temperature of the air the evaporation becomes negative; that is, condensation occurs. When the air is colder than the water surface, evaporation may continue into it after saturation has been reached and thereby produce fog, the process being one of distillation and condensation.

Even when the water is frozen, it still continues slowly to evaporate (sublime) whenever the air is sufficiently dry, but the laws governing this sublimation are not well known.

Empirical Evaporation Equations.—Various equations, each at least partially empirical, have been devised to fit evaporation

data obtained under special conditions. But the "constants" of these equations generally are not constant under other circumstances. Indeed, it may be that no simple equation of this kind, applicable to a wide range of conditions, is possible, and that therefore the most expeditious way to obtain useful evaporation data would be to note the daily, monthly, annual, etc., loss from standard exposures in each climatic region, and, wherever practical, to supplement such data by similar observations on lakes, ponds, and reservoirs. Controlled wind-tunnel experiments would also be interesting and useful.

One of the earliest experimenters to make a careful study of evaporation was John Dalton,[71] who says:

1. "Some fluids evaporate much more quickly than others."

2. "The quantity evaporated is in direct proportion to the surface exposed, all other circumstances alike."

3. "An increase of temperature in the liquid is attended with an increase of evaporation, not directly proportionable."

4. "Evaporation is greater where there is a stream of air than where the air is stagnant."

5. "Evaporation from water is greater the less the humidity previously existing in the atmosphere, all other circumstances the same."

All these are important observations, but they do not fully justify the so-called Dalton equation which Dalton himself apparently never wrote.

Weilenmann and Stelling, working independently and at different places, obtained evaporation equations of the general form[72]

$$\frac{dn}{dt}=(cb+kw)\ (p_s-p_0)$$

in which c and k are constants, b the barometric pressure, w the wind velocity, p_s the saturation vapor pressure at the temperature of the water surface, and p_0 the actual vapor pressure in the free air at some distance from the water.

Fitzgerald[73] finds the rate of evaporation, E, in inches per hour, given approximately by the equation

$$E=\frac{(p_s-p_0)\ (1+1/2w)}{60}$$

[71] *Mem. Manchester Lit. and Phil. Soc.*, 5, 574, read October, 1801.

[72] Hann, "Lehrbuch der Meteorologie," 3d edition, p. 214.

[73] *Trans. Amer. Soc. Civ. Eng.*, 15 (1886), 581–645.

in which p_s and p_0 have the meanings, respectively, given above, and w is the average wind velocity in miles per hour.

Various other equations have been found or proposed, but they either contain unevaluated functions or else were constructed to fit a special set of observations. The multiplicity of such equations, each of but limited use, emphasizes the difficulty of the evaporation problem, if not even the impossibility of finding for it a practical, universal equation.

CONDENSATION.

Condensation, the process by which a vapor is reduced to a liquid or solid, is induced by: (*a*) reduction of temperature, volume remaining constant; (*b*) reduction of volume, temperature remaining constant; (*c*) a combination of temperature and volume changes that jointly reduce the total vapor capacity. In the open, water vapor is condensed: (1) by contact cooling; (2) by radiational cooling; (3) by the mixture of masses of air of unequal temperatures; (4) by expansional or dynamic cooling due to vertical convection, or, occasionally, other causes, especially rotation, as in tornado and waterspout funnels.

Condensation Due to Contact Cooling.—During clear nights the surface of the earth, including vegetation and other objects. loses much heat by radiation, and thus both it and the air in contact with it are reduced to lower temperatures, obviously more pronounced the gentler the winds. After the dew-point has been reached all further loss of heat, producing now a much smaller proportionate decrease of temperature, results in the deposition, respectively, of dew and hoar-frost at temperatures above and below freezing. Similarly, relatively warm, moist air moving over a snow bank, for instance, may deposit some of its moisture.

In any typical case of surface cooling the deposition of dew, say, is caused partly by temperature reduction and partly by decrease of volume. Let the air, saturated at the absolute temperature T_0, be cooled, without change of volume, to T_1, and let the water vapor per unit saturated volume at these temperatures be w_0 and w_1, respectively. Then the quantity of water, $w_0 - w_1$, will be deposited per unit volume as a result of cooling alone,

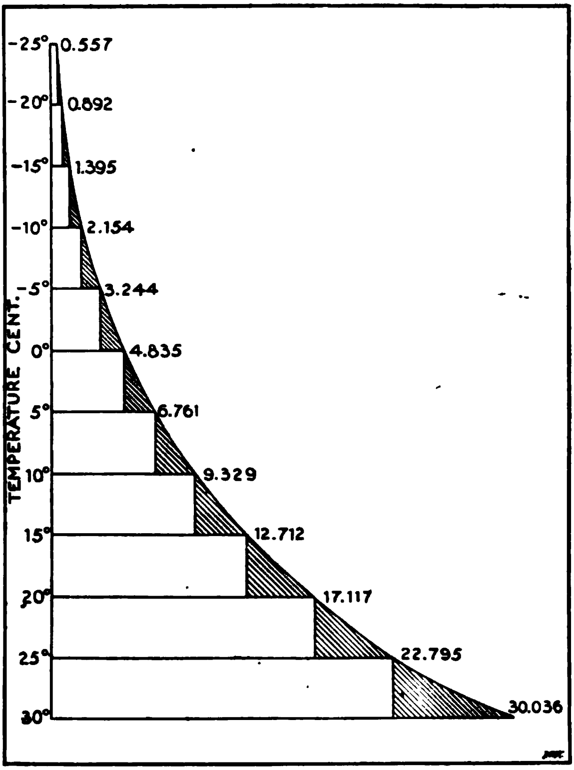
while if the pressure remains constant, as it does, approximately, the volume will be reduced in the proportion

$$\frac{V_0}{V_1}=\frac{T_0}{T_1}$$

and an additional quantity of water

$$w_1\,\frac{T_0-T_1}{T_0}$$

FIG. 63.



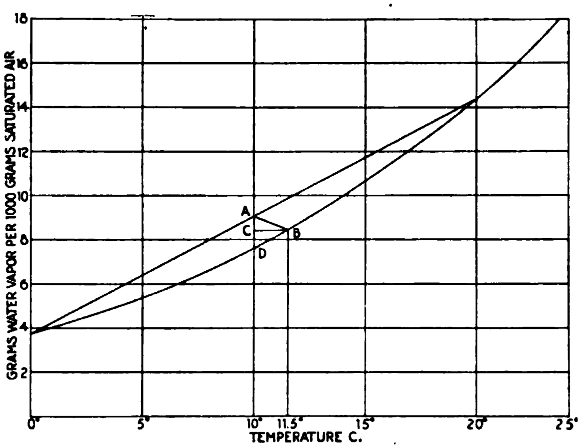
Grammes of water vapor per saturated cubic metre, at different temperatures. Bases of shaded portions proportional to precipitations per 5° C. cooling from the temperatures indicated.

deposited per unit volume at temperature T_0. Hence the quantity q of water deposited per original unit volume due to both processes combined, decrease of temperature and decrease of volume, is given by the equation

$$q=w_0-w_1\,\frac{T_1}{T_0}$$

Condensation Due to Mixing.—Since the amount of water vapor per saturated unit volume decreases with temperature more rapidly than the absolute temperature itself, at least through the range of atmospheric temperatures (see Fig. 63), it follows that the mixture of two saturated masses of air of unequal temperatures must produce some precipitation. The amount of precipitation induced in this manner, however, is surprisingly small; indeed, it seldom can be sufficient to produce more than a light cloud or fog. If the resulting temperature were the proportionate mean of the known temperatures of the quantities of air mixed, the amount of precipitation could easily be computed from the

FIG. 64.



Precipitation due to the mixing of saturated equal masses of warm and cold air

initial humidities. But the latent heat of the condensation prevents this simple relation from obtaining, so that the actual amount of precipitation can better be determined graphically than by direct calculation.[74] To this end use a humidity temperature curve, such as Fig. 64, drawn to scale. For example, let equal masses of saturated air at 0° C. and 20° C. be mixed at normal pressure—certainly an extreme case. As a first approximation it may be assumed that the final temperature is 10° C., and, since there are 3.75, 7.52, and 14.34 grammes of water vapor per 1000 grammes of saturated air at normal pressure and 0° C., 10° C.,

[74] Hann, "Lehrbuch der Meteorologie," 3d edition, p. 249.

and 20° C., respectively, the precipitation per 1000 grammes of the mixed air would seem to be

$$\frac{3.75+14.34}{2}-7.52=1.53 \text{ grammes,}$$

represented by *AD* in the figure.

But the latent heat of condensation causes the final temperature to be above the average, and the amount of precipitated water, therefore, less than that just computed. But since the latent heat of vaporization at 10° C. is approximately 591 calories per gramme, and the specific heat of the air at constant pressure about 0.24, it follows that the warming of the air will be at the rate of 2.5° C., nearly, per gramme of water vapor condensed per 1000 grammes of air. Hence a second approximation to the final temperature and condensation is found by drawing from *A* a line in such direction that it shall indicate a change of 2.5° C. per gramme of condensate, and prolonging it until it meets the humidity temperature curve in *B*. This second approximation gives 11.5° C. very closely, instead of 10° C., as the temperature of the mixture, and 0.6 gramme, instead of 1.53 grammes, as the amount of condensation per 1000 grammes of air, a quantity which, as the figure shows, would be condensed by a temperature decrease of less than 1° C.

Obviously similar graphical solutions may easily be made for mixtures of unequal masses of air, for unsaturated air, for other pressures, and for other temperatures; though for temperatures slightly below 0° C. a greater latent heat of vaporization, approximately 680, must be used.

Since 1000 grammes of saturated air at 10° C. and normal pressure occupies very approximately $\frac{25}{31}$ of a cubic metre, it follows that the condensation above described is about 0.74 gramme per cubic metre, a quantity capable of producing only a light cloud through which objects would be visible to a distance of about 70 metres.[75] Further, assuming the diameter of each cloud particle to be 0.033 mm., Wagner's average value, it follows that the condensation in question could produce only about 39 such fog particles per cubic centimetre.

Even if such a cloud were 1 kilometre thick and all its droplets should be brought down, they would produce a water layer only

[75] Wagner, *Sitz. der K. Akad. der Wis. Wien*, 117 (1908), p. 1290.

0.074 cm. deep. Obviously, therefore, the mere mixing of masses of humid air at different temperatures cannot produce any appreciable precipitation in the form of rain or snow.

Condensation Due to Dynamic Cooling.—Dynamic cooling incident to vertical convection is by far the most effective method of inducing precipitation, but even when the convection is adiabatic it is not immediately obvious, from the initial temperature, humidity, and pressure, just how much water will be precipitated as the result of a given increase of altitude, nor even for a given decrease of temperature. This is because the rate of cooling with elevation is affected by the latent heat of vaporization, and the amount of condensation in turn decreased by the increase of volume, which itself is a function of the temperature and pressure. The problem is further complicated, on passing to temperatures below 0° C., by the latent heat of fusion and by the abrupt considerable change in the heat of vaporization.

It therefore will be convenient to consider independently four possible stages in the dynamic cooling of a quantity of moist air: (*a*) the unsaturated; (*b*) the saturated at temperatures above 0° C.; (*c*) the freezing; and (*d*) the saturated at temperatures below 0° C.

This subject has been studied by several investigators, especially Hann,[76] Guldberg and Mohn,[77] Hertz,[78] and Neuhoff.[79] Of these Neuhoff's paper appears to be the most explicit, and it therefore will be used as the basis of the following brief discussion.

Dry (Unsaturated) Stage.—Let the humidity be such that the mass ratio of dry air to water vapor is $1 : w$. Then the number of calories, dQ, necessary to change the temperature of $1 + w$ grammes of this atmosphere by dT and its volume by dV is given by the equation

$$dQ = (C_v + wC'_v)dT + ApdV,$$

in which C_v and C'_v are the specific heats at constant volume, respectively, of dry air and *unsaturated* water vapor, A the

[76] *Met. Zeit.*, 9 (1874), 321, 337.

[77] "Études sur les Mouvements de l'Atmosphère," part 1, Christiania, 1876, revised 1883; translation by Abbe, "Mechanics of the Earth's Atmosphere," Smithsonian Institution, 1910.

[78] *Met. Zeit.*, 1 (1884), p. 421; translation by Abbe, "Mechanics of the Earth's Atmosphere," Smithsonian Institution, 1891.

[79] *K. Prus. Meteor. Inst.*, 1 (1900), p. 271; translated by Abbe, "Mechanics of the Earth's Atmosphere," Smithsonian Institution, 1910.

reciprocal of the mechanical equivalent of heat, and p the pressure.

But, for n grammes,

$$pV = nRT,$$

in which R is the well-known gas constant and T the absolute temperature, numerically, 273 + reading of centigrade thermometer. Hence,

$$dQ = (C_v + wC'_v)dT + (R + wR')AT\frac{dV}{V}$$

Since pressures in the open air are easily measured, while volumes are not, it will be more convenient to have this equation expressed in terms of the former. This may be done by substitutions from the equations

$$pdV + Vdp = RdT$$

and

$$C_v = C_p - AR$$

in which C_p is the specific heat at constant pressure.

If the convection is adiabatic—that is, if

$$dQ = 0$$

these substitutions give the equation

$$(C_p + wC'_p)\frac{dT}{T} = A(R + wR')\frac{dp}{p},$$

or, by integration,

$$(C_p + wC'_p)\log\frac{T}{T_0} = A(R + wR')\log\frac{p}{p_0},$$

or, more simply,

$$\log\frac{p}{p_0} = K\log\frac{T}{T_0}, \quad K = \text{a constant},$$

in which p_0 and T_0 are, respectively, the initial surface pressure and temperature.

Obviously this equation is applicable only until saturation is attained.

Let e_0 and e be, respectively, the initial and saturation vapor pressures corresponding to the total pressures p_0 and p, and absolute temperatures T_0 and T.

Then

$$\log\frac{e}{e_0} = K\log\frac{T}{T_0}$$

and

$$\log e - K\log T = \log e_0 - K\log T_0 = C, \text{ a constant.}$$

If e_0 and T_0 are both known, C is also known, and since saturation vapor pressure depends upon temperature alone, and is known through a wide temperature range, it is obvious that both $\log e$ and $K\log T$ may be tabulated for many values of T, and

that with such a table it is easy to pick out that value of T which gives the equation

$$\log e - K \log T = C,$$

the equation that determines the limit of the non-saturated or dry stage.

If the convection has been adiabatic it is obvious that the height h of the dry stage is given by the equation

$$h = 100\ (T_o - T) \text{ metres, approximately.}$$

A crude estimate of the saturation, or cloud, height may also be made from the current temperature T_o and dew point T_d. Thus, when owing to convection

$$T_o - T = 1^\circ\ C.,$$

the new volume is, roughly, one part in 80 larger than it would be under the initial pressure. But this increase in volume lowers the dew point 0.2° C., roughly, for average temperatures, as shown by vapor-saturation tables.

Hence

$$T_0 - T_d = \frac{4}{5}\ (T_o - T), \text{ roughly,}$$

and

$$h = 125\ (T_o - T_d) \text{ meters, roughly.}$$

It should be distinctly noted that in general vertical convection does not follow a fixed plumb-line. In cyclonic areas, for instance, the horizontal travel of the air doubtless often is hundreds of times the vertical. Hence in the quadrant of such a region where the clouds are from lower latitudes the vertical temperature gradient at any given place is likely to indicate a greater departure from adiabatic expansion than actually has occurred. This, as explained, is because the proper p_o and T_o to use in the above equations are those that obtained when and where the mass of air in question started to rise, and not those at the surface beneath its position at the time for which the equations are given.

Under such circumstances the true values of p_o and T_o are not accurately known, but that does not affect the validity of the above discussion; it only emphasizes the complexity of the problem as frequently presented in Nature.

Rain (Saturated, Unfrozen) Stage.—After saturation has been attained any further convectional cooling leads to precipitation. It will be assumed that this water is carried along with the ascending current (never strictly true and less nearly so as the drops grow in size), thus leaving the process adiabatic and reversible, and that the volume of the liquid water is negligible in comparison to the space from which it was condensed.

Let p be the total pressure, made up of the two partial pressures, air pressure p' and saturated water vapor pressure e, a function of the temperature alone, and let the mass ratio of air to total water, condensed and uncondensed, be $1 : w$. Then

$$p = p' + e = \frac{RT}{V} + e.$$

As before, the quantity of heat necessary to change the temperature of 1 gramme of air by an amount dT and its volume by dV is

$$dQ' = \left(C_v dT + ART\frac{dV}{V}\right).$$

Let w' be the grammes of uncondensed water vapor per gramme of dry air. Then $w - w'$ is the corresponding number of grammes of liquid water. Hence the heat necessary to bring about the temperature change dT and the vapor change dw' is

$$dQ'' = w's_2dT + (w - w')\,s_1dT + Ldw'$$

in which s_2 is the specific heat of *saturated* water vapor (that is, its specific heat when the volume so changes with the temperature as to maintain saturation and avoid condensation—a negative quantity), s_1 the specific heat of water, and L the latent heat of vaporization.

With T constant,

$$dQ'' = L\,dw' = Td\phi, \quad \text{or,} \quad \left(\frac{d\phi}{dw'}\right)_T = \frac{L}{T}, \ \phi \text{ being entropy.}$$

With w' constant,

$$dQ'' = w's_2dT + (w - w')\,s_1dT = Td\phi, \quad \text{or,} \quad \left(\frac{d\phi}{dT}\right)_{w'} = \frac{s_1(w - w') + s_2w'}{T}$$

But dQ is a perfect differential, therefore

$$\frac{d}{dw'}\left(\frac{s_1(w - w') + s_2w'}{T}\right) = \frac{d}{dT}\left(\frac{L}{T}\right)$$

and

$$(s_2 - s_1) = T\frac{d}{dT}\left(\frac{L}{T}\right)$$

Hence

$$dQ' = ws_1 dT + T\frac{d}{dT}\left(\frac{Lw'}{T}\right)dT,$$

and

$$dQ = \left(C_v dT + ART\frac{dV}{V}\right) + T\frac{d}{dT}\left(\frac{Lw'}{T}\right)dT + ws_1 dT.$$

$$= \left(C_p dT - ART\frac{dp'}{p'}\right) + T\frac{d}{dT}\left(\frac{Lw'}{T}\right)dT + ws_1 dT.$$

Hence, since the process is adiabatic,

$$(C_p + ws_1)\frac{dT}{T} + \frac{d}{dT}\left(\frac{Lw'}{T}\right)dT = AR\frac{dp'}{p'}.$$

By integration, using the subscript o for initial conditions,

$$\log\frac{p'}{p'_o} = \frac{C_p + ws_1}{AR}\log\frac{T}{T_o} + \frac{M}{AR}\left(\frac{Lw'}{T} - \frac{L_o w'_o}{T_o}\right)$$

in which M is the modulus of the system of logarithms used.

But

$$w' = \frac{Re}{R'p'}$$

Therefore

$$\log\frac{p'}{p'_o} = \frac{C_p + ws_1}{AR}\log\frac{T}{T_o} + \frac{M}{AR'}\left(\frac{eL}{p'T} - \frac{e_o L_o}{p'_o T_o}\right) = b\log\frac{T}{T_o} + \left(\frac{a}{p'} - \frac{a_o}{p'_o}\right).$$

in which b, a, and a_o obviously are determinable numerical quantities for given values of w, T, and T_o.

Hence

$$\log p' - \frac{a}{p'} - b\log T = \log p'_o - \frac{a_o}{p'_o} - b\log T_o = \text{a constant}.$$

From this equation a table may be constructed giving the relation between p' and T, and also, since e is known through a wide range of temperatures, between p and T. The value of w', or grammes of water per gramme of dry air, is given for any temperature by the equation,

$$w' = \frac{Re}{R'p'},$$

and the condensed water, w'', per gramme of dry air by the equation,

$$w'' = w - \frac{Re}{R'p'}.$$

Hail (Freezing) Stage.—Further lowering of the pressure beyond that at which the temperature reaches 0° C. causes, so long as there is any liquid water present, both freezing and evaporation. The latent heat of fusion keeps the temperature constant, while the increase of volume under the reduced pressure increases the vapor capacity and thus leads to evaporation.

To each gramme of dry air let there be w, w', and w'' grammes, respectively, of water, vapor, and ice. Then, as there is no change of temperature through this stage,

$$dQ = ART_0\frac{dV}{V} + Ldw' - Fdw''$$

in which F is the latent heat of fusion, and T_0 the absolute temperature at 0° C. The negative sign is used because the heat of fusion is added, or becomes sensitive with freezing; that is, with decrease of pressure and increase of volume.

Assuming the process adiabatic, dividing by T, as before, and integrating, the above equation reduces to

$$\frac{AR}{M}\log\frac{V_1}{V_0} + \frac{L}{T_0}(w'_1 - w'_0) - \frac{F}{T_0}(w''_1 - w''_0) = 0.$$

Let the subscript o indicate the condition when the temperature reaches 0° C. with no ice, and subscript 1 the condition when all the water is just frozen. As the temperature is constant, e will be the same at the beginning and end of the freezing process. At the end of the freezing $w''_1 = w - w'_1$.

Also,

$$\frac{V'}{V_0} = \frac{p'_0}{p'_1};\ w'_0 = \frac{R}{R'}\ \frac{e}{p_0 - e};\ w'_1 = \frac{R}{R'}\frac{e}{p_1 - e};\ \text{and } w''_0 = 0.$$

Hence

$$\log p'_1 - \frac{e}{p'_1}\ \frac{M(L+F)}{AR'T_0} = \log p'_0 - \frac{e}{p'_0}\frac{M}{AR'}\ \frac{L}{T_0} - w\ \frac{M}{AR}\ \frac{F}{T_0}$$

This equation gives, in terms of known quantities, the relation between the partial pressures of the air at the beginning and end of the "hail stage," and therefore the depth of this stage, obviously determined by the amount of water to be frozen, which in turn depends on the original temperature and humidity.

Snow (Frozen) Stage.—At temperatures below 0° C. there will be present in the air only ice and enough water vapor to produce saturation. Hence the discussion applicable to this stage is identical with that for the "rain stage," though two of the constants, specific heat and latent heat, will be different. The specific heat is now of ice, roughly one-half that of water, while

the total latent heat is due to two distinct processes, fusion and vaporization. The equation, therefore, applicable to the snow stage is,

$$\log \frac{p'}{p'_o} = \frac{C_p + ws_i}{AR} \log \frac{T}{T_o} + \frac{M}{AR'}\left(\frac{e(L+F)}{p'T} - \frac{e_o(L_o+F_o)}{p'_oT_o}\right)$$

in which s_i is the specific heat of ice, and the other terms have the meanings previously given.

It will be interesting to note that the form of the adiabatic equation is:

1. For the dry stage,

$$\log p - a \log T = C, \text{ a constant.}$$

2. For a condensation stage,

$$\log p' - \frac{b}{p'} - a \log T = K, \text{ a constant,}$$

in which a and b are numerical coefficients, p the total pressure, and p' the partial air pressure.

The short hail or freezing stage is distinct from either of the others, though it may be represented approximately by an equation of the second or condensation type.

"Pseudoadiabatic" Convection.—Adiabatic expansion of the atmosphere obviously implies that all cloud particles, rain drops, and snowflakes are carried along with the identical mass of air out of which they were condensed. This condition cannot rigorously obtain in Nature at any level; neither do all the products of condensation, especially the smaller droplets, rapidly fall away immediately they are formed. Hence the actual process, if conduction, radiation, and absorption were negligible, would lie somewhere between the adiabatic, with all condensation products retained, and that special type of the nonadiabatic which Neuhoff and others have called pseudoadiabatic, where all such products are immediately removed, probably much nearer the latter than the former.

To reduce adiabatic to "pseudoadiabatic" equations it evidently is only necessary to drop the water and ice terms. This of course, automatically excludes the hail stage—it eliminates all water and therefore renders freezing impossible. Nevertheless the differences between the temperatures and pressures given by the two processes generally are small.

For convenience of inter-comparison the two sets of equations, adiabatic and "pseudoadiabatic," are here grouped together.

Dry stage

Adiabatic, $\log \frac{p}{p_0} = \frac{C_p + wC'_p}{A(R+wR')} \log \frac{T}{T_0}$

"Pseudoadiabatic," Does not exist, there having been no condensation.

Rain stage

Adiabatic, $\log \frac{p'}{p'_0} = \frac{C_p + ws_1}{AR} \log \frac{T}{T} + \frac{M}{AR'}\left(\frac{eL}{p'T} - \frac{e_0L_0}{p'_0T_0}\right).$

"Pseudoadiabatic," $\log \frac{p'}{p'_0} = \frac{C_p}{AR} \log \frac{T}{T_0} + \frac{M}{AR'}\left(\frac{eL}{p'T} - \frac{e_0L_0}{p'_0T_0}\right)$

Hail stage

Adiabatic, $\log \frac{p'_1}{p'_0} = \frac{M}{AR'T}\left(\frac{e}{p'_1}(L+F) - \frac{e}{p'_0}L - \frac{R'}{R}wF\right)$

"Pseudoadiabatic," Does not exist, there being no water and therefore no freezing.

Snow stage

Adiabatic, $\log \frac{p'}{p'_0} = \frac{C_p + ws_i}{AR} \log \frac{T}{T_0} + \frac{M}{AR'}\left(\frac{e(L+F)}{p'T} - \frac{e_0(L_0+F_0)}{p'_0T_0}\right)$

"Pseudoadiabatic," $\log \frac{p'}{p'_0} = \frac{C_p}{AR} \log \frac{T}{T_0} + \frac{M}{AR'}\left(\frac{e(L+F)}{p'T} - \frac{e_0(L_0+F_0)}{p'_0T_0}\right)$

It will also be convenient to have listed the several constants of these equations and their numerical values. If the unit of heat is 1 calorie, the heat necessary to raise the temperature of 1 gramme of water from 0° C. to 1° C., the values of these constants are:

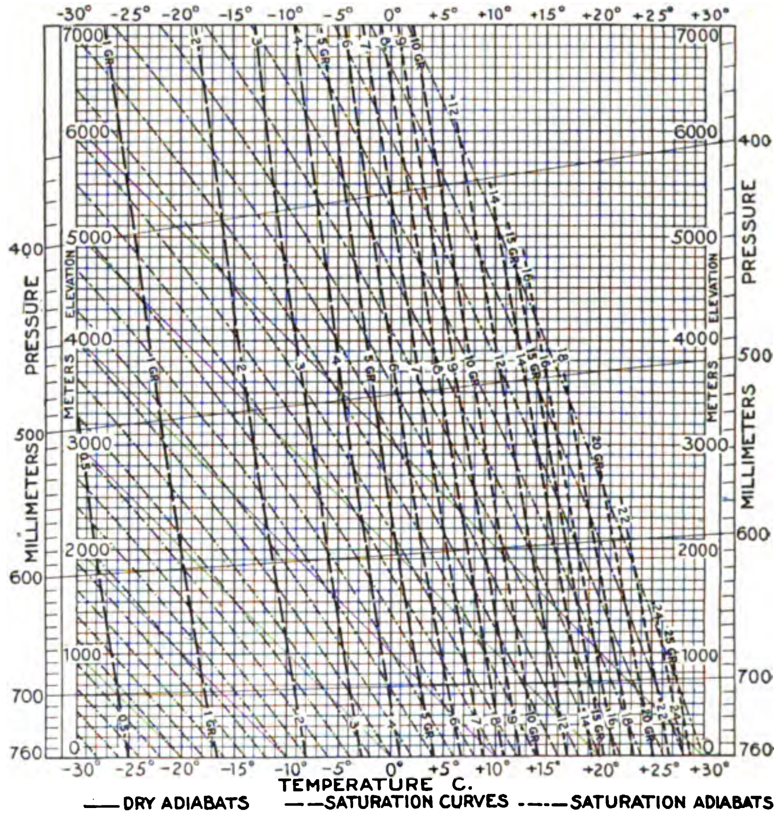
$A = \frac{1}{4.19} \times 10^7$ ergs, ~~nearly~~.
$F = 80$ calories, about.
$L = 600$ calories, approximately.
$M = 0.43429448$, for base 10.
$T = 273 +$ reading of centigrade thermometer.
$R = 28.71 \times 10^6$ ergs per gramme 1° C.; ~~nearly~~.
$R' = 46.42 \times 10^6$ ergs per gramme 1° C.; closely.
$C_p = 0.241$, ~~about~~.
$C'_p = 0.46$, roughly.
$s_1 = 1$, ~~closely~~.
$s_i = 0.5$, ~~approximately~~.

With these values various tables may be constructed for convenient use of the formulæ, as has been done by Neuhoff.[80] Proper hypsometric formulæ give the elevations above sea level corresponding to different conditions of the atmosphere with respect to temperature, pressure, and humidity. Hence it is possible to construct diagrams more or less accurately embodying all such calculations. Fig. 65, copied from Neuhoff's paper, is an especially good adiabatic diagram of this kind.

[80] *Loc. cit.*

As is obvious from inspection, this diagram applies to all altitudes from 0 (sea level) to 7000 metres, and from $-30°$ C. to $+30°$ C. The temperature and altitude differences are equally spaced, and the pressure differences, therefore, unequally in respect to both the other terms. It is assumed that the adiabatic cooling

FIG. 65.



Adiabatic diagram (Neuhoff).

of non-saturated air is at the rate of 1° C. per 100 metres increase of elevation, an approximately correct value, hence the dry adiabats, given in full lines for intervals of 10° C., are straight diagonals, while the saturation adiabats, represented by dot and dash, are considerably curved. The saturation moisture content, in terms of grammes of water vapor per kilogramme of dry air, is given by the broken lines.

Interpolations are readily made on the diagram and approximate values easily obtained by always starting from the intersection of the given temperature and pressure coördinates. For example, let the temperature be 20° C., the barometer reading 760 mm., and the relative humidity 55 per cent. Since, as the diagram shows, saturation at the given temperature and pressure would require about 14.6 grammes of water vapor per 1000 grammes of dry air, it follows that under the assumed conditions only about 8 grammes would be present. Hence the temperature, pressure, and altitude of such a mass of air rising adiabatically are given, through the first convective stage, by that dry adiabat that starts at the intersection of the initial temperature and pressure ordinates, 20° C. and 760 mm. The first stage terminates when saturation is attained, and therefore, in the present case, at the intersection of the given adiabat with the 8-gramme humidity curve at an elevation, as inspection shows, of rather more than 1100 metres and where the pressure corresponds to a barometric reading of about 625 mm. From this level up the conditions of the rising mass of air are given by a saturation adiabat, according to which the temperature will have fallen to 0° C. and the humidity to about 5.25 grammes at an elevation of approximately 2700 metres. The humidity decrease, 2.75 grammes per 1000 grammes of dry air, is the amount precipitated as water in the form of cloud particles and rain drops. If all this water is carried along, its latent heat of fusion will maintain the temperature at 0° C. through an additional rise of about 80 metres, but, as much of this water obviously must drop out, it follows that the actual conditions presumably are rather better represented by omitting the "hail stage," or by a continuous rather than a broken adiabat.

While this diagram gives approximately the relations between temperature, pressure, humidity, and altitude that obtain in regions of strong vertical convection, it does not closely represent them as they normally exist at other places. This is due partly to the horizontal component of air movement, as above explained, and partly to that constant emission and absorption of radiation that always precludes the existence in the atmosphere of strictly adiabatic conditions.

Principal Forms of Condensation.—Condensation assumes many forms, of which the chief are: (*a*) *free drops*, varying in

size all the way from the fog or cloud particle up to the largest rain drop, or from .03 mm., roughly, to about 5 mm. in diameter; (*b*) *dew,* water that has condensed on objects that by any process have attained a temperature below the current dew-point of the air immediately in contact with the bedewed objects. The cooling necessary to the formation of dew usually results from loss of heat by radiation; (*c*) *frost,* a light feathery deposit of ice caused by the same process that produces dew, but occurring when the temperatures of the objects on which it forms are below freezing; (*d*) *rime,* a frost-like deposit of ice, often several inches deep on the windward sides of exposed objects. It is formed from impinging undercooled fog particles, and hence grows straight into the wind; (*e*) *glaze* (ice storm), a coating of clear, smooth ice on the ground, trees, etc. It generally is caused by the falling of rain on cold (below freezing) surfaces; (*f*) *snow,* tabular, and columnar particles of ice formed in the free air at temperatures below freezing. All are hexagonal in type but of endless variety in detail—many exquisitely beautiful; (*g*) *sleet,* ice pellets, mere frozen rain drops (or largely melted snowflakes refrozen)—frozen during the fall of the precipitation through a cold layer of air near the surface of the earth—that rattle when they strike a window, for instance; (*h*) *hail,* lumps of ice more or less irregular in outline and generally consisting of concentric layers of clearish ice and compact snow. It occurs only in connection with thunderstorms and may be of any size up to that, at least, of a baseball or large orange such as fell in considerable quantities at Annapolis and other points in Maryland on June 22, 1915.[80a] Indeed, much larger stones have occasionally been reported and presumably have occurred. At any rate, in some instances stock in the fields have been killed by blows from hailstones of unusual size.

Other forms of precipitation that should, perhaps, be mentioned are: *graupel* soft snow pellets; *mist,* a thin fog of relatively large particles; and *drizzle,* a light rain of very small drops.

How Raindrops Are Formed.—As already explained, the amount of condensation resulting from given temperature and pressure changes can easily be computed, but this is not sufficient to account for the formation of ordinary raindrops. The difficulty lies in the fact that the number of nuclei per unit volume of the

[80a] Fassig, *Monthly Weather Review*, September, 1915.

air (hundreds, or thousands, usually, per cubic centimetre) is so great that the condensation of even all the water vapor present would produce nothing larger than minute fog particles.

It is natural, of course, to suppose that some of the droplets in any cloud are relatively large, and that all of this kind that happen to be in the upper portion will, on falling, grow by collision into full-sized raindrops. But this simple assumption is beset with several difficulties. In the first place, nothing of the kind occurs, at least not to any appreciable extent, in a fog, nor even in the average cloud. In fact, clouds may continuously cover the sky for days without yielding any rain whatever. Secondly, it is far from obvious that the droplets will coalesce, as assumed, on collision. Indeed, if they are not electrified, it is highly probable that, on the contrary, they will rebound from each other, as do the drops of a spraying jet of water,[80b] due, presumably, to the air film between them. Finally, whether one considers a dense sea fog, diameter of particle 10μ, 1200 particles per c.c.,[80c] or a cloud, diameter of particle 33μ, 120 particles per c.c.,[80d], a little calculation shows that not enough water could, on the average, be accumulated by simple collisions, in the manner assumed, to produce medium-sized raindrops. However, this latter difficulty is more apparent than real, as will be explained presently.

A factor in the growth of drops that needs to be examined is the relation of the saturation vapor pressure to their size, by virtue of which the larger tend to increase at the expense of the smaller. From the equation (see page 12)

$$\Delta p = \frac{2T\rho_v}{R(\rho_w - \rho_v)}$$

it appears that the excess pressure above that of normal saturation about fog droplets of diameter 10μ is 2.76 dynes per square centimetre. Now, at 10° C. saturation pressure balances a column of mercury 9.14 centimetres high; hence at 10° C. the ratio of the excess pressure about droplets of 10μ diameter to normal saturation pressure is, approximately, 1 to 44,000; nor is this ratio greatly different at other ordinary temperatures.

[80b] Lord Rayleigh: *Proc. Roy. Soc.*, 38, p. 406, 1879.

[80c] Wells and Thuras: U S. Coast Guard, Bulletin 5, 1916.

[80d] Wagner: *Sitz. der k. Akad. der Wiss., Wien.*, 117, p. 1281, 1908.

Obviously, therefore, the growth of the larger drops at the expense of the smaller, as a result of the difference between their saturation pressures, is entirely negligible.

Another factor is the difference between the temperature of the relatively cool falling drop and that of the adjacent atmosphere. But the amount of condensation thus induced is very small owing to the large value of the latent heat of vaporization.

The difficulties, then, that have to be considered in an attempt to explain the formation of raindrops, seem to be as follows:

a. Drops large enough to fall at an appreciable rate—drops upon which any subsequent coalescence and resulting rain must depend—do not form in the average cloud.

This merely amounts to saying that such formation occurs only in rain clouds—that is, in clouds formed essentially by vertical convection. But how can convection produce drops of the necessary drizzle size in view of the hundreds, or, usually, thousands of nuclei per cubic centimetre? This seemingly insuperable difficulty is overcome (for it does rain), presumably in the following manner: All droplets of whatever size are actually falling in respect to the air in which they happen to be. Hence the rising air leaves more and more of its own nuclei behind as the droplets are abandoned—passes on as progressively filtered air—and thus contributes its subsequent condensation to only the fewer and fewer particles that exist along its upward path. Not only do the droplets fall out of the air in which they were formed, but they also aid in filtering the next portions of the rising column; and so on continuously. In this way the droplets well up within a cloud that is formed by vertical convection eventually become relatively very few, and therefore grow comparatively large as a result of the continuous condensation onto them.

b. Water drops commonly do not unite on collision, but rebound, as shown by the scattering of a jet.

This difficulty is met by the fact that, when slightly electrified, drops do unite on collision,[80e] together with the further fact that rain is always more more or less electrified.

c. A cloud particle in passing straight down through a cloud from top to bottom would not, on the average, touch enough other particles to form by union with them a medium-sized drop.

But this is not important since rains are caused by rising air

[80e] Lord Rayleigh: *l. c.*

through which drops can not approach the earth until they have grown beyond a certain size which increases with the upward velocity. Besides, the variations in this velocity produce repeated rises and falls of drops of the appropriate sizes until growth through coalescence, or condensation, or both, sufficient for their final fall does occur.

The rain process may, therefore, conveniently be divided into the following stages:

1. Vertical convection and the consequent formation of innumerable cloud particles about the condensation nuclei immediately after the dew-point is passed, or, in the case of the more hygroscopic nuclei, slightly before it is reached.

2. The continued rise of the saturated air, but now in a progressively filtered condition as more and more cloud particles are abandoned.

3. Continuous condensation on the fewer and fewer droplets that remain in the "filtered" air, and their consequent growth to appreciable size, accelerated, presumably by coalescence, though at what stage this phenomenon becomes important is not known.

4. The growth of the small drops by further condensation and, especially, by coalescence with other drops (all, on the average, being more or less electrified) during their fall through the cloud and to the earth.

In short, the rising air automatically filters itself immediately it passes the dew-point; the remaining water vapor condenses on relatively few nuclei (transported cloud particles) and thus produces droplets of appreciable or drizzle size; these, being electrified, unite on collision and form raindrops.

Persistent vertical convection does not occur in fogs, nor in the average cloud. Hence they exhibit neither progressive filtering nor continuous condensation. Drops of appreciable size do not, therefore, form within them, nor rain fall from them.

Velocity of Fall of Raindrops.—If the diameter of the drop is very small, 0.1 mm. or less, its approximate steady, or terminal, velocity of fall can be computed by Stokes's well-known equation,[80f]

$$V = \frac{2}{9} gr^2 \frac{(\sigma - \rho)}{\mu}$$

[80f] *Phys. and Math. Papers,* Vol. iii, p. 59.

in which V is the velocity in centimetres per second, g the value, in C.G.S. units, of gravity acceleration, r the radius of the drop in centimetres, σ the density of the drop, ρ the density of the air, and μ its viscosity.

The equations of fall in the case of raindrops of average and larger sizes, 1 mm. to 5 mm. diameter, are, however, very different, and but little more than empirical.[80g] Several of the more important velocities, and sufficient for approximate interpolations, are given in the table below of Precipitation Values. The maximum velocity in air of normal density (velocity proportional, nearly, to square root of density), at which the larger drops break up, is about 8 metres per second.

Intensity of Precipitation.—The intensity, or rate, of rainfall varies from zero up to several inches per hour, and, like the strength of the wind, has been popularly divided into several more or less definite grades. Most of these, together with other roughly average values they imply, are given in the accompanying table.

PRECIPITATION VALUES

(Air density as at 0° C. and 740 mm. pressure.)

Popular name.	Precipitation intensity, mm. per hour.	Diameter of drop, mm.	Velocity of fall, metres per second.	Milligrams of liquid water per cubic metre of air.	Height of cloud above surface, metres.
Clear	0.00	....	.....	0.00	....
Fog	Trace	0.01	0.003	6.0	0
Mist	0.05	0.1	0.25	55.5	100
Drizzle	0.25	0.2	0.75	92.6	200
Light rain	1.00	0.45	2.00	138.9	600
Moderate rain	4.00	1.0	4.00	277.8	600
Heavy rain	15.00	1.5	5.00	833.3	1000
Excessive rain	40.00	2.1	6.00	1851.9	1200
Cloudburst	100.00	3.0	7.00	5401.4	1200
		5.0	8.00		

Why the Atmosphere Generally is Unsaturated.—It may, perhaps, seem strange that, in spite of the continuous and rapid evaporation from nearly all parts of the earth's surface, the atmosphere as a whole never becomes even approximately saturated. This condition, however, is a necessary result of vertical convection. Obviously whatever the temperature and relative

80g Liznar: *Met. Zeit.*, Vol. xxxi, p. 339, 1914.

humidity of a given mass of air at any point of its convectional route, its absolute humidity is less then, in general, than when its ascent began, by the amount of rain or snow already abandoned by it. That is, on the average, air in a convection circuit descends to the earth drier than when it previously ascended from it. In short, convection, because it induces abundant precipitation, is therefore a most efficient drying process; and because comparatively little precipitation is produced in any other way, convection alone prevents the atmosphere from becoming and remaining intolerably humid.

Summer and Winter Precipitation.—Vertical convection, essential, as above explained, to all considerable condensation, results from three distinct causes: (*a*) superadiabatic temperature gradients, due often to surface heating; (*b*) converging winds, as in the front half of cyclones; and (*c*) forced rise from (1) flow over land elevations and barriers of cold air, (2) underrunning of cooler winds. The first, or thunderstorm, type of convection causes much of the summer precipitation of temperate regions, as also nearly all the rain of the tropics, while the second, or cyclonic, convection produces by far the greater part of winter precipitation, except, perhaps, that which occurs along the windward sides of the most favorably situated barriers. Also, during the colder season precipitation usually occurs lower down the barrier slope and may be induced by feebler cyclones or other storms than in the warmer. This is owing in part to the fact that generally there is less difference between the actual and dew-point temperatures during winter than during summer (a condition determined by the great seasonal temperature changes of continents with reference to the ocean), and therefore a less convection required in the first case than in the second to induce condensation, and partly to the greater rate of decrease of temperature with increase of latitude while the days are short than while they are long, a condition that favors winter precipitation by causing a greater fall of temperature during the winter season than any other for a given travel of the wind on the front or rainy side of a cyclone. That is, usually a less vertical convection and a less horizontal travel of the air—a feebler storm—suffices to induce precipitation during winter than during summer.

The contrasts, then, between summer and winter precipitation

are manifold. The more important differences are listed in the following table:

Contrast Between Summer and Winter Precipitation.

	Summer	Winter
Rain	Usually.	Often.
Snow	Never.	Frequent.
Hail (ice lumps)	Occasionally.	Never.
Sleet (frozen rain)	Never.	Occasionally.
On barrier	High.	Low, and up.
Type of storm	Thunderstorm frequently	Cyclone.
Strength of convection	Strong, generally essential	Feebler, often sufficient.
Intensity of cyclone	Decided, usually essential	Slight, often sufficient.

CHAPTER XIV.

FOGS AND CLOUDS.

THE deposition of dew, the forming of hoar-frost, and the sweating of ice pitchers, all examples of surface condensation, show that atmospheric moisture promptly condenses upon any object whose temperature is below the dew-point. Similarly, volume condensation takes place in the form of a fog or cloud of innumerable droplets, or ice spicules, throughout the body of ordinary air whenever by expansion or otherwise it is sufficiently cooled. But this is not equally true of all air. Thus, while the first considerable rapid expansion, and therefore decided volume cooling, of humid air in a receiver, if recently admitted unfiltered, is quite certain to produce a miniature cloud, subsequent expansions of the same air produce fewer and fewer fog particles. If the old air is removed and unfiltered fresh air admitted, the condensations again occur as before; but if the fresh air enters through an efficient filter, such as a plug of cotton wool a few centimetres long, condensation remains as difficult as in the exhausted air. The admission, however, of a little smoke restores to the exhausted, and confers upon the filtered, air full powers of condensation.

Obviously, then, cloud droplets form about nuclei that cannot easily pass through mechanical filters of fine texture, and microscopic examinations of the residue left on the evaporation of these droplets have shown the nuclei to consist in large measure of dust particles, both mineral and organic. Hygroscopic gases, such as the oxides of sulphur and of nitrogen, may also act as condensation nuclei, but ordinarily there is abundant dust in the atmosphere (thousands of particles per cubic centimetre) to provide for all precipitation. It is often urged that free electrons in the air also act as nuclei about which water vapor condenses, but, as this type of condensation requires about a fourfold supersaturation, its occurrence in the open seems extremely improbable.

As stated, volume condensation may be induced in the atmosphere by any cooling process: whether by radiation, as on clear nights; mixing warmer with colder masses of air; movement of relatively warm air over cold surfaces, as in the case of winter

south winds (northern hemisphere); or expansion, owing either to convection or barometric depression. But the cooling process has much to do with determining the extent of the condensation, the kind and amount of precipitation from it, and its general appearance, according to which, chiefly, it is classified.

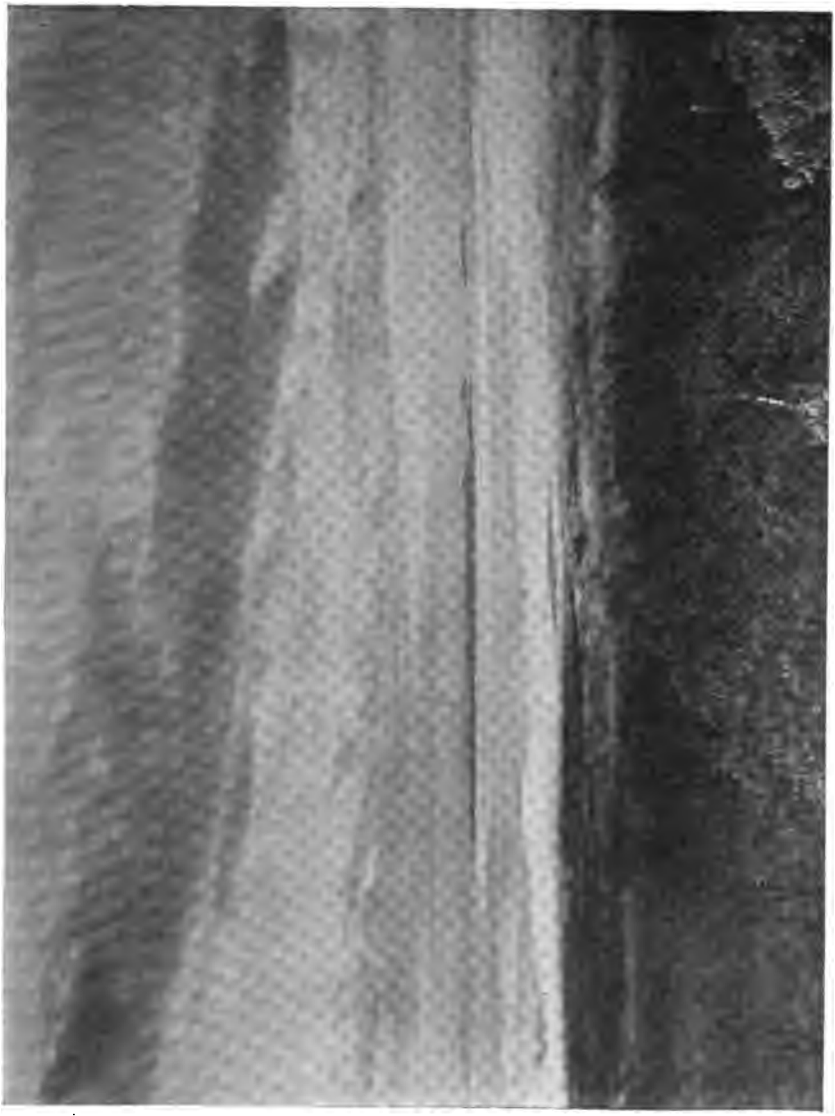
Distinction Between Fog and Cloud.—Volume condensation is divided primarily into fog and cloud, but a sharp distinction between them that would enable one always to say which is which is not possible. In general, however, a fog differs from a cloud only in its location. Both are owing, as explained, to the cooling of the atmosphere to a temperature below its dew-point, but in the case of the cloud this cooling usually results from vertical convection, and hence the cloud is nearly always separated from the earth, except on mountain tops. Fog, on the other hand, is induced by relatively low temperatures at and near the surface, and commonly itself extends quite to the surface, at least during the stage of its development. In short, fog consists of water droplets or ice spicules condensed from and floating in the air near the surface; cloud, of water droplets or ice spicules condensed from and floating in the air well above the surface. Fog is a cloud on the earth; cloud a fog in the sky.

FOGS.

According to the conditions under which they are formed, fogs may be divided into two general classes—radiation fogs and advection fogs.

Radiation Fog.—Fog is likely to form along rivers and creeks and even in cleared mountain valleys during any still, cloudless night of summer and, especially, autumn. In the course of a calm warm day and earlier portion, at least, of the night much water is evaporated into the lower atmosphere of such regions, where in large part it remains as long as there are no winds. Hence this air, because it is humid, and the adjacent surface of the earth lose heat rapidly during the night by radiation to the clear sky. In many cases they cool in the end to a temperature below the dew-point, and thus induce a greater or less volume condensation, on the always-present dust motes, that results in a correspondingly dense fog (Fig. 66). Such fog, however, is not likely to occur during cloudy nights, because the air seldom then cools sufficiently, nor during high winds, since they dissipate the

FIG. 66.



Radiation fog, Loudoun Valley, Va. (A. J. Weed, photo.)

moisture and also through turbulence prevent the formation of excessively cold aerial lakes.

The distinctive factor in the formation of this type of fog is the free radiation of the ground and the lower air by which the latter is sufficiently cooled to induce condensation. Hence fogs formed in this manner are properly termed "radiation fogs," sometimes also called "land fogs" and "summer fogs."

A frequent incidental phenomenon in connection with fogs of this class is their accelerated growth well after daybreak, which occasionally continues until after sun-up when radiation gain exceeds the corresponding loss. It has been suggested that this phenomenon is due to hygroscopic compounds formed in the air by insolation, either direct or diffused, but there is as yet no proof that these compounds are more than a contributing factor, perhaps an entirely negligible one, to the observed result. Another factor, that at times and places may be of some importance, is the soot and hygroscopic compounds discharged into the foggy air from numerous breakfast fires. Usually, however, the sole appreciable cause is the gradual onset of convectional disturbances in the quiescent valley air incident to the insolational warming of the mountain tops and sides. This mixes the cool surface layer with that next above and thereby often increases the fog depth. Furthermore, it drags the river of fog up the valley walls, and thus also increases its width. However, before either process has gone very far evaporation becomes manifest, and generally within an hour or two the fog has totally vanished.

Advection Fog.—Whenever warm, humid air drifts over a cold surface its temperature is reduced throughout the lower turbulent layers by conduction to that surface and by mixture with remaining portions of the previous cold air and a correspondingly dense fog produced. Hence fog often occurs, during winter, in the front portion of a weak cyclone; also whenever air drifts from warm water to cold—from the Gulf Stream, for instance, to the Labrador Current; and wherever gentle ocean winds blow over snow-covered land—circumstances that justify the terms "winter fog" and "sea fog" (drifting on shore in places, and even some distance inland, Fig. 67). Similarly, a cold wind drifting or spreading under and through a body of warm, humid air also produces a fog, though usually a comparatively light one. This explains the fog that frequently forms,

FIG 67.

Advection fog, seen from Mount Wilson, Cal. (F. Ellerman, photo.)

during winter, along the front of a "high," and the thin fog that occasionally is seen over lakes on frosty autumn mornings, when the water appears to be steaming—actually evaporating into air already saturated and thus inducing condensation. It also explains the frequent occurrence of "frost smoke" on polar seas.

If the wind is strong the turbulence extends through a comparatively deep layer. Hence in the case of warm air drifting over a cold surface if the movement is rapid the total duration of contact between any portion of the air and that surface is likely to be so brief that but little cooling can take place and no fog be formed. Similarly, it usually also happens that fog does not form when the cold wind blowing over a warm, humid region is even moderately strong. Here the turbulence mixes the excessive moisture near the surface through so large a volume that saturation commonly is not produced, nor, therefore, any trace of fog.

From the above, it appears that all fogs that result from the drifting of warm, humid air over cold surfaces, as also those that are produced by the flow of cold air over warm, humid regions, are but effects of temperature changes induced by the horizontal transportation of air; hence the proposed general name, "advection fog." The term advection is preferred to convection because the latter is practically restricted, in meteorological usage, to a change of level, whereas in the case under consideration only horizontal movements are concerned. The contradistinction, therefore, between "advection fog" and "convection cloud" is obvious, and, presumably, worth while.

CLOUDS.

The cooling of the atmosphere by which cloud condensation is induced is, perhaps, most frequently produced by vertical convection, either thermal or forced; often, presumably, by the mixing of winds of different temperatures; occasionally by pressure changes, elevation remaining the same; occasionally, also, by radiation; and rarely, in the case of very thin clouds, by diffusion and conduction.

Radiation, though productive of many fogs, is excluded from the list of principal cloud-forming processes for the reason that, as explained elsewhere, any mass of free air that cools in position, as it must whenever its radiation exceeds its absorption, imme-

diately gains in density and falls to a lower level where, when equilibrium is reached, it actually is *warmer* than it was before the cooling began, and its relative humidity, therefore, lower. Hence it seems that radiation could produce clouds only when equally active, or nearly so, over an extensive layer of practically saturated air. If radiation is unequally distributed it tends to evaporate clouds rather than to produce them.

Classification.—It is not practicable, however desirable, to classify clouds according to their causes, as in the case of fogs, for it often happens that the exact cause is not obvious. Hence other bases of classification have been adopted, especially form or appearance, activity, and position. Most, but not all, clouds belong to one or other of the four distinct types, *cirrus, stratus. cumulus, nimbus,* including their alto-, fracto-, and combination forms; alto-stratus, alto-cumulus; fracto-stratus, fracto-cumulus, fracto-nimbus; cirro-stratus, cirro-cumulus, strato-cumulus, cumulo-nimbus.

Cirrus (Ci.).—The name cirrus, literally a curl or ringlet, has been given to those fibrous white clouds that resemble great wisps of hair (mares' tails), giant curling plumes (feather clouds), tangled skeins, and various other things (Figs. 68 and 69). These are the highest, often 10 to 12 kilometres above the earth in middle latitudes and still higher in tropical regions, the most tenuous, and among the most familiar of all clouds.

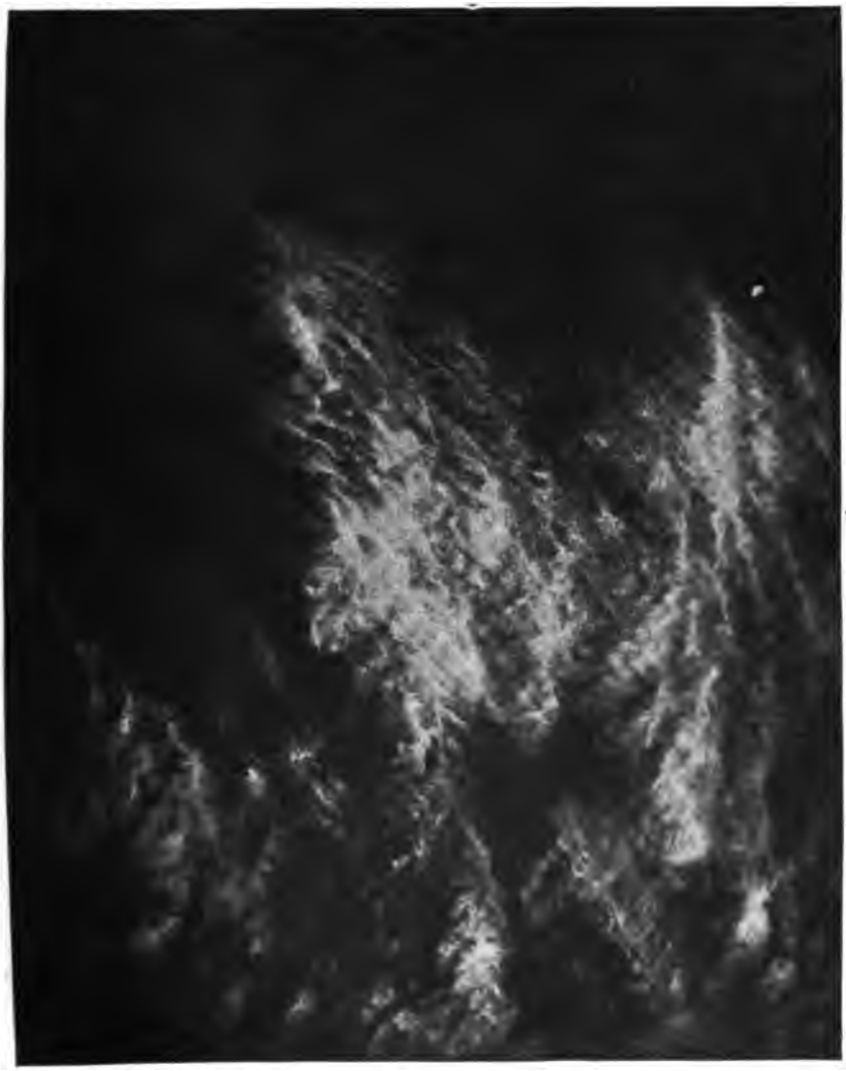
Since cirri usually run far ahead of the rainy portions of a cyclonic area, often even well into the preceding anticyclone, and grow denser as the storm approaches, it is obvious that they frequently result from cyclonic convections that extend nearly or quite to the stratosphere, where, and for some distance below which, the rising air is carried forward much faster than the storm centre. But they also are fairly common as isolated clouds in the midst of "highs," due, presumably, to a mechanical or bodily lifting of the upper air of these regions, or overrunning of air in the general circulation, and, consequently, dynamical cooling not only of the stratosphere, as abundantly shown by the records of sounding balloons, but also of the topmost portion of the troposphere where cirri usually form.

It has been suggested that cirri often are caused by cooling in place by radiation, but, as already explained, this appears to be improbable for clouds so broken and discontinuous. On the

FIG. 68.

Cirrus. (F. Ellerman, photo.)

Fig. 69.



Cirrus. (F. Ellerman, photo.)

contrary, however, it seems likely that through free radiation and cooling at night they often sink to lower levels, get *warmer*, and evaporate. Thermal and mechanical convection, therefore, the first prevailing in tropical regions, the second, presumably, in extratropical, appear to be the only abundant causes of cirri.

The excessively low temperatures at which cirri are formed, generally −30° C. to −50° C., necessitate their being tenuous (at such temperatures there is but little water vapor to condense) and practically insure (exceptions have been reported[81]) that they shall consist of ice needles, or, in some cases, of small snowflakes. The fibrous and feathery structures of the highest cirri may perhaps be explained as follows: Since diffusion is a very slow process, it is clear that moisture is carried into the upper atmosphere mainly by vertical convection, and, as this often occurs sporadically, it appears that through the increase of winds with elevation, the rising and generally humid air is likely to be drawn out into long threads and bands, and to float away in filaments at the convective limit, just as during the early hours of calm autumn mornings chimney smoke in mountain valleys often is drawn out into streaks and ribbons at or near the inversion level.

Any cloud, therefore, produced in this fibrously humid air obviously itself must have the same general structure—a common structure of cirrus clouds. Through local convection, however, and abrupt changes in velocity at the upper surface of these clouds the air currents to which they are due presumably often are deflected into curves of changing radii. Hence, perhaps, the curved or plumed cirrus. Mare's tails are streaks of snow falling from and generally trailing behind small alto- or cirro-cumuli. They often are curved by changes with level of wind direction.

Cirro-stratus (Ci.-St.).—When cirrus clouds thicken, as they usually do on the approach of a cyclonic storm, they gradually merge into a broad cloud layer, having the appearance of a more or less continuous white veil of uneven and often fibrous texture (Fig. 70), to which the name cirro-status has been given. Its altitude is nearly that of the cirrus, of which indeed it is only a dense and extensive form, though its under surface is not so high. Like its forerunner, the thinner cirrus, it also consists

[81] Simpson, *Qr. Jr. Roy. Meteorol. Soc.*, 38 (1912), p. 291.

FIG. 70.

Cirro-stratus, and advection fog, seen from Mount Wilson, Cal. (F. Ellerman, photo.)

FIG. 71.

Cirro-cumuli. (F. Ellerman, photo.)

of ice crystals, as is evident from the various types of halos it forms about the sun and moon.

The origin of these clouds is substantially the same as that of the cirrus; that is, convection, which in turn may be caused by general expansion of the air below or by convergence of winds, such as occurs in the cyclone. Frequently, as explained above, the cirro-stratus is only the higher and swifter portion of the cyclonic cloud system, the result of forced convection to great altitudes.

Cirro-cumulus (Ci.-Cu.).—Cirro-cumuli are small, fleecy cumulus clouds, generally 6 to 7 kilometres above the surface; that is, in the lower cirrus region. They usually occur in large numbers, producing an effect sometimes described as "curdled sky"; frequently, also, in groups and rows that remind one of the patterns (not the scales) on the backs of mackerel. Hence the expression "mackerel-back sky," commonly abbreviated to "mackerel sky" (Fig. 71).

Their origin obviously is due chiefly to a single cause—local vertical convection, induced by an overrunning, cold layer of air, when the cumuli are in rows, or by unequal local heating of a layer of varied humidity. To each convective rise of the air there evidently must be an equivalent descent, and if the heating maxima are numerous the minima between must also be numerous, thus producing many rising currents, each with its small cumulus, surrounded by descending air and relatively clear sky. Through precipitation and turbulence the cirro-cumulus often develops into a cirro-stratus, or alto-stratus.

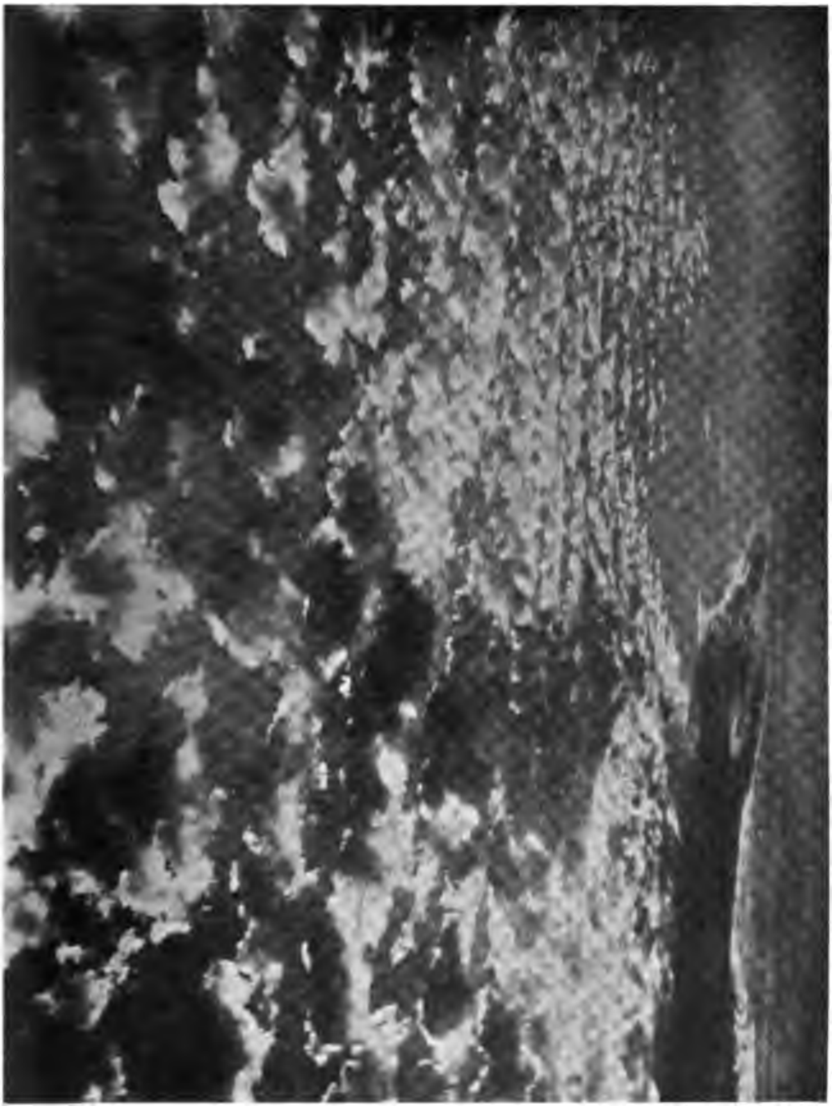
Alto-stratus (A.-St.).—The alto-stratus is a thick, grayish cloud veil (Fig. 72), at times compact and fibrous in structure, and again thinner, like a heavy cirro-stratus, through which the sun or moon may dimly be seen. Its average elevation (under surface) is about 4 kilometres. It may result from the forward running of air forced up by the convergence of winds in the storm area of a cyclone, from the spreading tops of cumuli, from the flow of warmer over colder air, from falling precipitation out of alto- and cirro-cumuli, or from the mere radiational cooling in place, of a layer of relatively humid air—humid from the evaporation of alto-cumuli, perhaps.

Alto-cumulus (A. Cu).—The name alto-cumulus has been given to those detached, fleecy clouds, with shaded portions

FIG. 72.

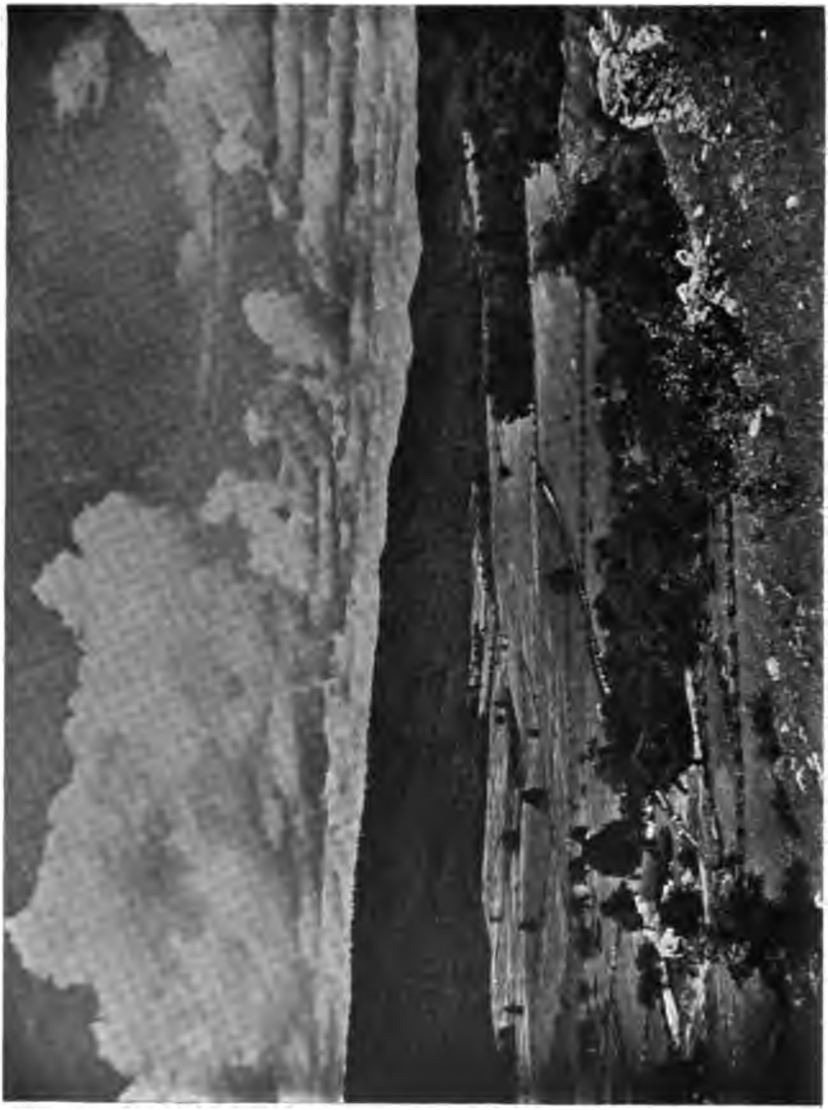
Alto-stratus, and advection fog, seen from Mount Wilson, Cal. (F. Ellerman, photo.)

FIG. 73.



Alto-cumulus. (A. J. Weed, photo.)

FIG. 74.



Cumulus and strato-cumulus, or roll cumulus, near Gap Mills, W. Va. (L. W. Humphreys, photo.)

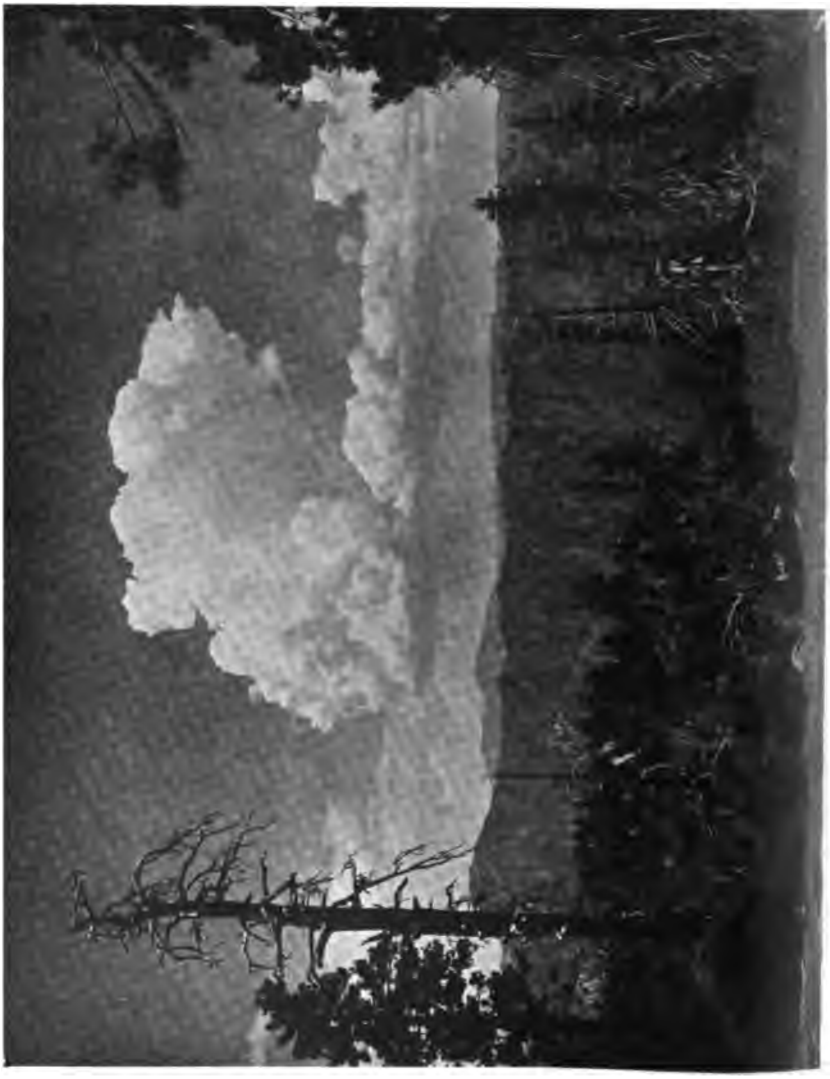
(Fig. 73), often occurring in closely packed groups and rows, that resemble enlarged cirro-cumuli, and doubtless are formed in much the same way. The moisture involved, especially during fair weather, seems often to be furnished by previously evaporated cumuli. Their average altitude is approximately that of the alto-stratus, that is, 4 kilometres. Indeed, detached portions of forming or evaporating alto-stratus also are generally called alto-cumuli, the more exact term, fracto-alto-stratus, not being in use.

Strato-cumulus (St.-Cu).—Strato-cumuli are large rolls of dark cloud more or less connected with thinner clouds which together cover nearly or quite the entire sky (Fig. 74). Their bases are flat and at about the same height, generally 1.5 to 2 kilometres. They are formed by vertical convection, as is obvious from their rounded tops and flat bases at approximately the same level—the common saturation level. Their shallow depth and broad expanse are due, presumably, to an overlying layer of small, or even inverted, temperature gradient through which rising air cannot easily penetrate. This name is also given to a stratus of irregular density, and thus to all that entire range of clouds between the uniform stratus and the discrete cumuli.

Nimbus (Nb.).—The nimbus is any thick, extensive layer of formless cloud from which rain or snow is falling. The average altitude of its under-surface is of the order of 1 kilometre. It is produced chiefly by some type of forced convection: the converging of wind currents as occurs especially in front of cyclonic centres, the upward deflection of winds by either land or cold atmospheric barriers, and the under-running of warmer by colder air. In part, however, the cooling and consequent condensation often is owing to the mixing of cold air with warm, and to the transfer of warm air to a colder region, where it is cooled by contact, by mixing with cooler air, and by excess of radiation loss over radiation gain.

Fracto-nimbus (Fr.-Nb.).—The fracto-nimbus, popularly known as *scud*, is that low, detached cloud fragment, too thin and fog-like to produce rain, that occasionally is seen drifting rapidly beneath a heavy nimbus at an average elevation of probably not more than 100 to 300 metres. It may rise, like steam, during or following rainfall on a warm surface, especially in valleys and on the sides of mountains (where it is often called

FIG. 75.



fog) which it ascends. It is also caused by forced convection over cliffs or other obstacles.

Cumulus (Cu).—The cumulus (Fig. 75), often called

Fig. 76.

Cumulus cloud formed by convection over fire on Sister Elsie Peak, Calif. September 13, 1913. (O. H. Lawrence, photo.)

"woolpack," is a dense, detached cloud with a rapidly changing cauliflower head and flat base at the saturation level of rising air. Its illuminated portions are snow white, while the shaded parts

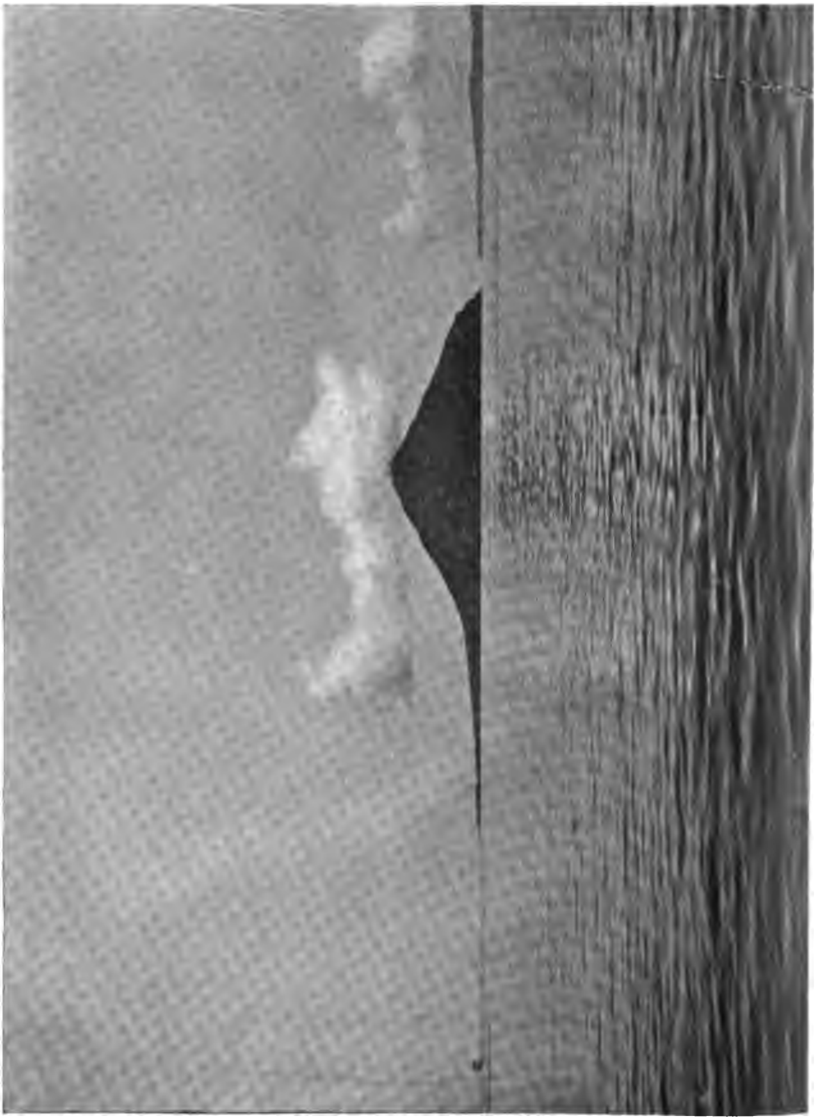


FIG. 77.

are unusually dark. Its border is sharply defined and, when near the sun, very bright. The average altitude of the base is about 1.5 kilometres, and of the top rather more than 2 kilometres.

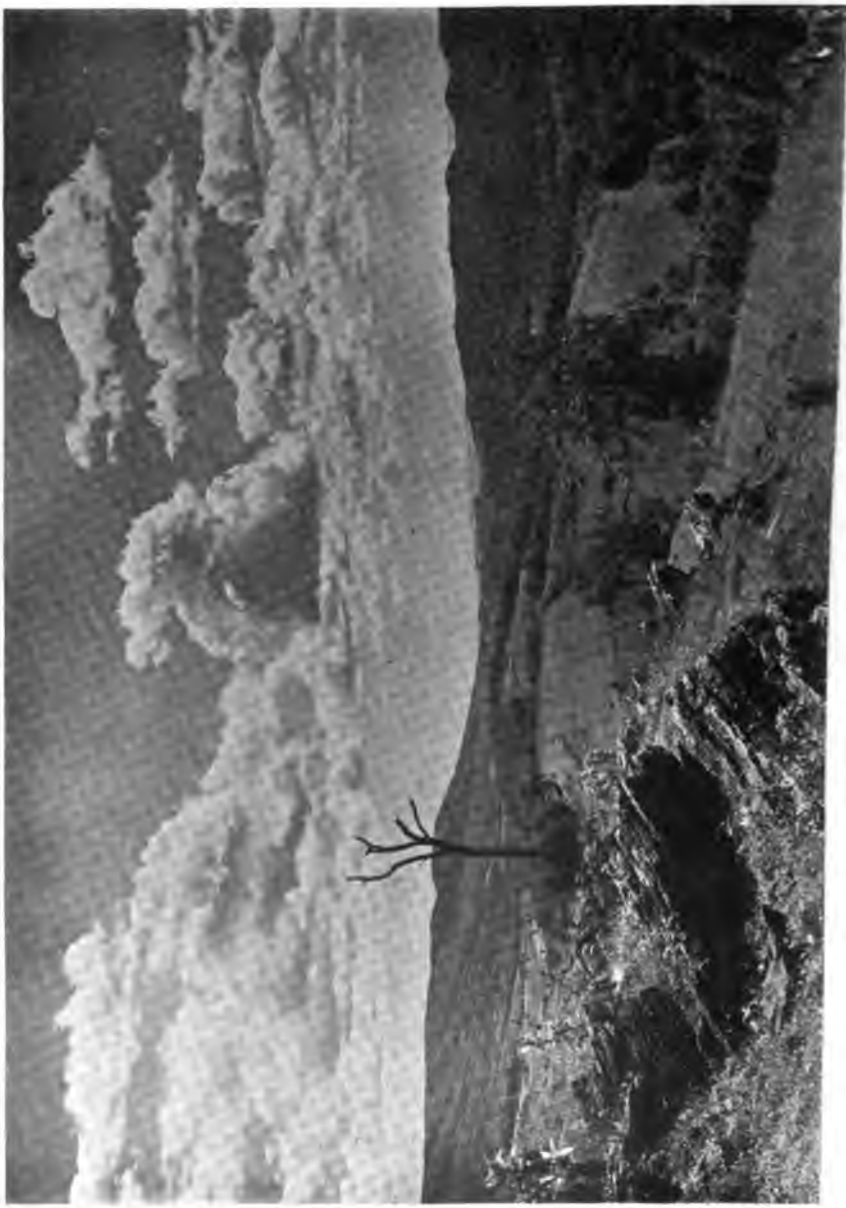
Cumuli are produced entirely by vertical convection induced by temperature differences—even fires sometimes cause them (Fig. 76). Hence they are always frequent in tropical regions, and also over continents at higher latitudes during summer. For the same reason, they occur over land most numerously of afternoons, and at sea late in the night. At times rather low cumuli form a sort of coastal fringe along the locus of upward convection—that is, a short way out over the sea at night, and a few miles inland during the day—that might, perhaps, be called coast cumuli—attendants of the land breeze and the sea breeze, respectively. They often occur over reefs and islands (Fig. 77), whose presence frequently is thus revealed while they themselves are still below the horizon. Occasionally they even parallel a large river on either side where there is rising air over the hills and bottoms and sinking over the cooler water. Further, since vertical convection depends only on the establishment of a proper vertical temperature gradient, it follows that cumuli may also form at high latitudes over the warmer portions of the ocean, or, indeed, wherever there is a sufficient temperature contrast between the surface and overlying air to induce marked upward currents.

Fracto-cumulus (Fr.-Cu.).—During the initial stages, especially, of their development cumuli often are small, and appear tattered and torn like detached and dissolving masses of fog (Fig. 78). While in this condition such clouds are often called fracto-cumuli.

Cumulo-nimbus (Cu.-Nb.).—The cumulo-nimbus (Fig. 79), a necessary accompaniment of every thunderstorm, is, as its name implies, a cumulus cloud from which rain is falling. It is very turbulent and much the deepest of all clouds, being usually anywhere from 1 to 4 or 5 kilometres thick—occasionally even 10 or more, especially in the tropics. Its times and places of occurrence and mode of formation are all the same as those of the cumulus.

Stratus (St.).—The stratus is a low, fog-like cloud of wide extent, often merging into a nimbus and again clearing away like lifted fog. Its average altitude is between 0.5 and 1 kilometre. It seems often to result from forced convection due to the under-

Fig. 78.



Cumulus and fracto-cumulus, in Monroe Co., W. Va., Peters Mountain to left. (L. W. Humphreys, photo.)

Fig. 79.

Cumulo-nimbus, over Loudoun Valley, Va. (A. J. Weed, photo.)

Fig. 80.

Billow cloud. (A. J. Henry, photo.)

FIG. 81.

Billow clouds, regular and irregular, over Washington, D. C. (A. J. Henry, photo.)

running of cold air, and also, perhaps, to the mixing of humid layers of different temperatures. In some cases, that of the "velo" cloud, for instance, in southern California, it is only sea fog drifting over relatively warm land.

SPECIAL CLOUD FORMS.

Although it might seem that the above cloud types, including their numerous gradations and transitions, are exhaustive, there nevertheless are several occasional forms sufficiently distinct to justify individual names and special descriptions.

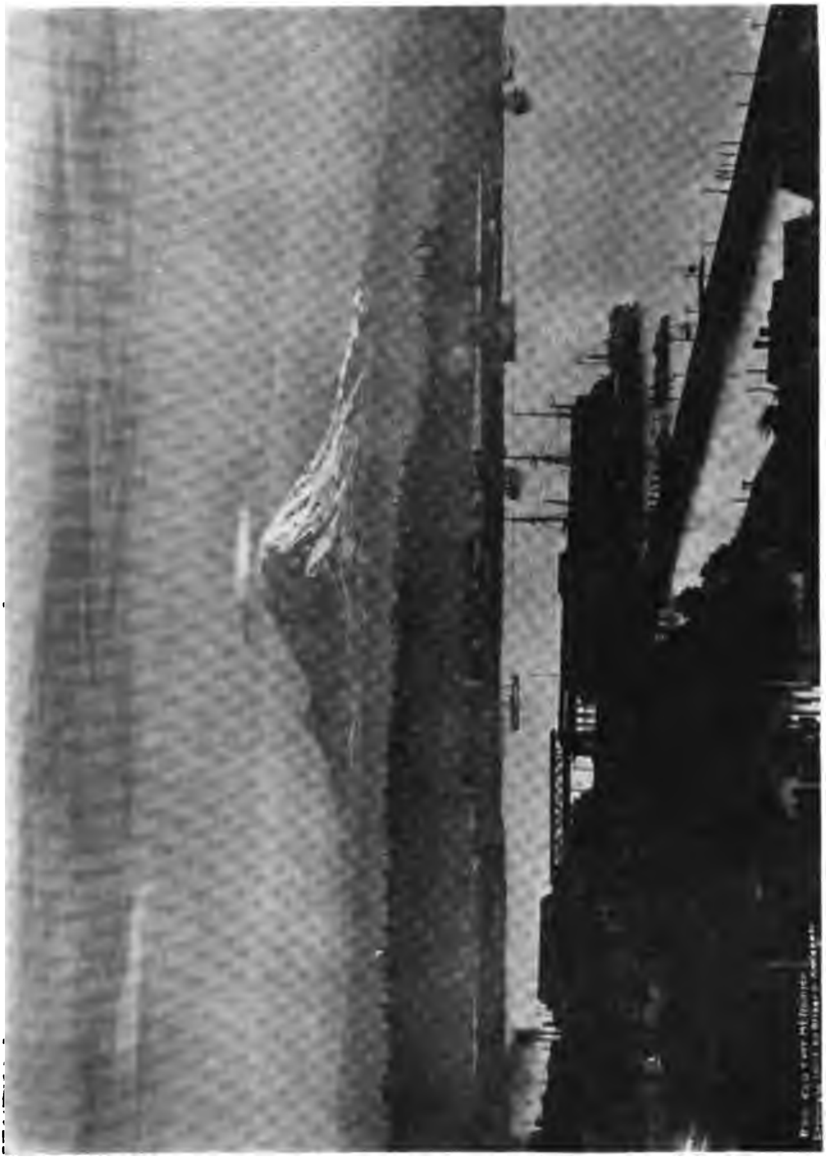
Billow Cloud.—Billow clouds (Figs. 80 and 81), also called windrow clouds and wave clouds, occur in series of approximately regularly spaced bands, generally with intervening strips of clear sky. They usually form in the lower cirrus region—that is, at elevations of 6 to 8 kilometres—but may occur at any level from the surface—fogs are occasionally billowed—up to that of the highest cirrus. They are caused by the flow of one air stratum over another of different temperature and density and usually of different humidity.

It has been shown [82] that when two strata of air of different densities or vapor content flow over each other billows of great wave-length and often of large amplitude are generated in the same manner that winds produce ocean billows. As the series of waves progress the atmosphere involved obviously rises and falls, and therefore is subjected to alternate dynamical heating and cooling, with the maxima and minima temperatures corresponding to the troughs and crests respectively. Hence when the under layer is wholly or nearly saturated the wave crests are cloudy and the troughs clear. If, however, the humidity is not high, it is obvious that wind billows may exist without the incidental clouds.

It is interesting to note that, although the billow cloud appears to consist continuously of the same mass, nevertheless, it is rapidly evaporating on the rear or descending portion of the wave and as speedily forming on the front or ascending portion.

[82] Helmholtz, *Sitz. d. Akad. d. Wiss.*, Berlin, 1888, i, p. 646; 1889, ii, p. 761. W. Wien, *Sitz. d. Akad. d. Wiss.*, Berlin, 1894, ii, p. 509; 1895, i, p. 361. A. Wegener, *Beiträge Phys. d. fr. Atmos.*, 2 (1906), p. 55; 4 (1911), p. 23.

Fig. 82.



Lenticular cloud, over Mount Rainier. (O. P. Anderson, photo.)

FIG. 83.

Lenticular clouds and fracto-cumuli over Rocky Mountains near Corona, Colo.

Lenticular Cloud.—The lenticular cloud (Figs. 82 and 83) is formed by the upward deflection of the wind over mountain peaks, the cloud often appearing some distance away at the crest of a billow; and, perhaps, by similar deflections and other disturbances due to rising air currents. In some cases, doubtless, the cooling is accentuated by the low temperature of the peak itself. In either case the cloud particles are rapidly evaporated as they are carried away, and the thickness of the whole mass reduced to zero at no great distance.

FIG. 84.

Crest cloud, windward side, seen from the Pali, near Honolulu. (A. M. Hamrick photo.)

Crest Cloud.—The crest cloud (Figs. 84 and 85) is formed by the upward deflection of the wind by a long mountain ridge. It usually covers the higher slopes as well as the top, and, though called cloud by people in the valleys below, is likely to be designated fog by any one actually in it. Occasionally condensation occurs only along the upper reaches of the deflected winds, in which case the cloud belt is above and to the leeward of the mountain ridge.

In either case the individual droplets are quickly evaporated and the cloud form preserved only through continuous condensation from renewed air. It is permanent in the same sense

that a cataract is permanent through the continuous supply of water by the stream above.

Banner Cloud.—The banner cloud (Fig. 86), as its name implies, resembles a great white flag floating from a high mountain peak. In strong winds the pressure to the immediate leeward of such a peak is more or less reduced, and the resulting low temperature, intensified, perhaps, by the mountain surface, appears to be the cause of this singular cloud that, though con-

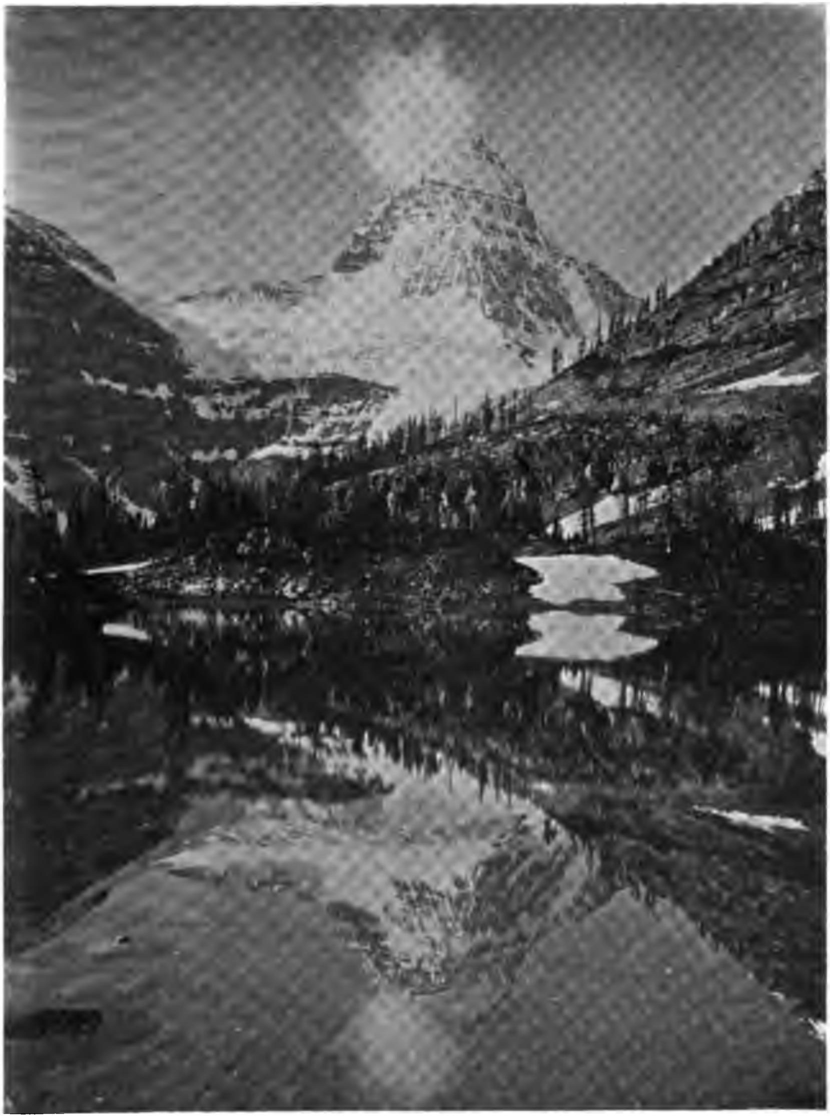
FIG. 85.

Crest cloud, lee side, seen from Honolulu. (A. M. Hamrick, photo.)

tinuously evaporating, as constantly re-forms in the turbulent wake.

Scarf Cloud.—It occasionally happens that as a cumulus rises rapidly and to great heights a thin, cirrus-like cloud, convex upward, forms above the cumulus head and, at first, entirely detached from it. As the cumulus continues to rise the flossy cloud becomes more extensive and rests on the thunder head or heads. A little later it mantles the shoulders, the heads being free (Fig. 87), and may even drape the sides of the cumulus. In all stages it resembles a great silken scarf, hence the above-suggested name. It often is called false cirrus, but that name

Fig. 86.



Banner cloud, Mount Assiniboine, near Banff, Canada. (C. D. Walcott, photo.)

Fig. 87.

Scarf cloud. (W. A. Bentley, photo.)

is now and better applied to a different formation. It has also been called cap cloud, but this is confusing, because the same term has long been applied loosely to any cloud that hovers above, or, especially, rests upon a mountain peak, and, besides, the cap analogy applies to only the early stages.

It is caused by the elevation and consequent expansion and cooling of the air immediately and to some distance above the rising mass of the cumulus. Generally this expansion of the superincumbent atmosphere produces no visible effect, but occasionally there exists a thin stratum of nearly saturated air in which an alto-stratus might form, or, indeed, later does form, and when this is lifted by the rising cumulus it immediately develops a local cirrus-like cloud. But if the saturated layer is thin, as it often is, the cumulus head may easily rise quite above it into drier air, leaving the filmy cloud at practically its original level, the level of the humid stratum, or completely absorbing it.

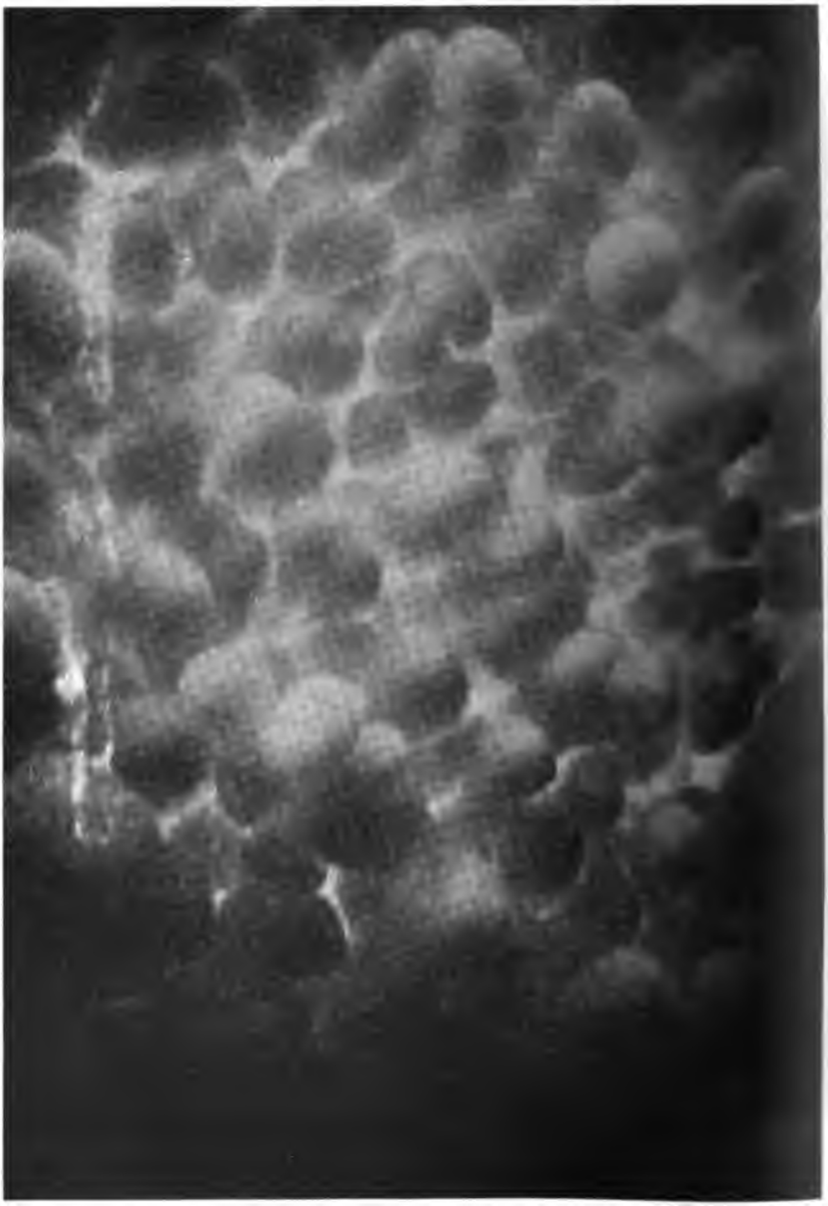
False Cirrus.—The name "false cirrus" formerly was applied indifferently to the scarf cloud, just described, and to those gray locks, to speak figuratively, combed out from old thunder heads by the upper winds. At present, however, the term usually is restricted to the latter phenomenon.

Although the lower atmosphere, up to at least 3 or 4 kilometres, generally is comparatively calm whenever cumuli are most conspicuous, it nevertheless occasionally happens that the highest thunder heads reach into a stratum of much greater velocity in which, therefore, the topmost portions of the cloud are drawn out into wispy bands and fibres of snow crystals—a truly cirrus cloud whose peculiar origin is, perhaps, its only claim to the special name "false cirrus." The same name is also applied to the thinner edges of the *anvil cloud,* or spreading top of a cumulo-nimbus.

Mammato-cumulus.—The mammato-cumulus, sometimes called pocky-cloud, festoon-cloud, "rain balls," sack-cloud, or other similar name, is essentially a reversed cumulus (Fig. 88). It generally occurs in an alto-stratus or cumulo-stratus cloud, and seldom except in connection with a severe thunderstorm.

As is well known, ice in the form of hail and snow often occurs in the upper portions of large cumuli. If, now, this snow, say, is drifted out just above a stratus cloud, it is obvious that the cooled layer will descend at many different places, and that at

FIG. 88.



Mammato-cumulus. (L. C. Twyford, photo.)

Fig. 89.

Tornado cloud, Antler, N. Dak. (W. H. Wegner, photo.)

each such place there will be produced a sag or pendulous bulge in the cloud base and, of course, a lift or rise wherever the counter-current obtains, thus producing the festooned appearance characteristic of this rather unusual cloud.

Tornado or Funnel Cloud.—The tornado cloud (Fig. 89) is only a funnel-shaped extension of, generally, if not quite always, a cumulo-nimbus. It is produced by the expansional cooling incident to the rapid rotation of the atmosphere in which it appears.

CLOUD HEIGHTS.

Relation to Humidity.—The heights of clouds have been measured by several obvious methods of triangulation, and from

Average Cloud Heights in Kilometres.

Station	Ci.	Ci.-St.	Ci.-Cu.	A.-St	A.-Cu.	St.-Cu.	Nb.	Cu.-Nb. top	Cu. Top	Cu. Base	Fr.-Cu.	St.
	1. SUMMER, CHIEFLY APRIL TO SEPTEMBER.											
Bossekop, 70° N	8.32	6.61	5.35	4.65	3.42	1.34	0.98	3.90	2.10	1.32	....	0.66
Pavlovsk, 60° N	8.81	8.09	4.60	....	3.05	1.85	....	4.68	2.41	1.64	2.15	0.84
Upsala, 60° N	8.18	6.36	6.45	2.77	3.95	1.77	1.20	3.97	2.00	1.45	1.83	....
Potsdam, 52½° N	9.05	8.08	5.89	3.29	3.63	2.16	1.79	3.99	2.10	1.44	1.71	0.68
Trappes, 49° N	8.94	7.85	5.83	3.79	3.68	1.82	1.08	5.48	2.16	....	1.40	0.94
Toronto, 43½° N	10.90	8.94	8.88	4.24	3.52	2.06	....	....	1.70		....	....
Blue Hill, 42° N	9.52	10.10	6.67	6.25	3.76	1.16	1.19	9.03	2.90	1.78	....	0.51
Washington, 39° N	10.36	10.62	8.83	5.77	5.03	2.87	1.93	4.96	(2.45)	1.18	....	0.84
Allahabad, 25½° N	10.76	.....	11.28	....	4.50	....	0.84	....	1.76		....	....
Manila, 14½° N	11.13	12.97	6.82	4.30	5.71	1.90	1.38	6.45	1.84		....	1.06
Batavia, (year), 6° S.	11.49	10.59	6.30	....	5.40	....	....	....	1.74		....	0.70
	2. WINTER, CHIEFLY OCTOBER TO MARCH.											
Pavlovsk	8.74	7.09	5.98	....	3.17	1.50	....	....	1.60	1.12	....	1.00
Upsala	6.98	5.46	6.13	4.09	4.15	1.96	0.99	5.18	1.52	0.71	1.22	0.51
Potsdam	8.07	7.65	5.41	2.99	3.35	1.42	1.28	4.74	1.74	0.99	1.02	0.61
Trappes	8.51	5.85	5.63	3.82	4.27	1.61	1.05	3.85	2.37	....	1.43	....
Toronto	9.98	8.53	8.25	4.18	2.50	1.54	....	....	1.33		....	....
Blue Hill	8.61	8.89	6.16	4.57	3.66	1.60	0.65	....	1.62	1.54	....	0.61
Washington	9.51	9.53	7.41	4.80	3.82	2.40	1.80	3.73	2.28	1.20	....	1.13
Manila	10.63	11.64	6.42	3.90	4.64	2.32	1.49	3.14	1.82		....	....

the data thus obtained it appears, as one might infer *a priori*, that whatever condition tends to increase the relative humidity tends also to lower the cloud levels, since the greater this humidity the less the amount of convectional cooling essential to condensation. Hence, in general, each type of cloud is lower in winter than summer, lower over humid than over desert regions, lower over oceans than continents, and lower with increase of latitude.

The table on this page, copied from Hann's "Lehrbuch der Meteorologie," gives the average summer and winter heights of clouds at places of widely different latitudes.

Levels of Maximum Cloudiness.—When the frequency of clouds is tabulated with reference to elevation, maxima and minima are found with the layers to which they obtain growing thicker with decrease of latitude. This phenomenon, as a whole, is interesting, but it will be necessary, in discussing it, to consider the different levels separately, since each has its own explanation.

Fog Level.—As already explained, fogs, whether caused by radiation or advection, are surface phenomena, seldom more than 100 to 200 metres thick. Hence the surface of the earth, because of the fogs that form upon it, is itself a level of maximum condensation or maximum " cloudiness."

Cumulus Level.—Since the cumulus and the cyclone nimbus both are due to vertical convection—the first thermal, the second forced—it is obvious that the base of each occurs approximately at the saturation level; that is, the level at which a mass of air rising from the surface will have cooled to its dew-point. Clearly, too, clouds cannot form at a lower level, the air there being unsaturated, and even if drifted in they would evaporate. Further, ordinary thermal convection usually does not extend to much higher altitudes, because the cooling of the rising mass through expansion and evaporation (the outer portions, at least, of the cloud evaporate) quickly brings it to or below the temperature of the surrounding air at the same level, except in the case of the largest cumuli, in which the amount of evaporation is very small in comparison to the total condensation. Hence, foul weather cumuli and the lower cyclone clouds mark a second level of maximum cloudiness, commonly 1 to 2 kilometres above the surface.

Alto-cumulus Level.—During fair, calm, summer weather vertical convection is very strong, but, as the relative humidity is low, the resulting clouds are sufficiently high to furnish, on evaporation, much of the moisture that later often condenses into the alto-stratus or alto-cumulus. Hence the alto-cumulus, 3.5 to 4 kilometres above the surface, marks another level of decided maximum cloudiness.

Cirro-stratus Level.—Since the different types of cirrus formed in the region of a cyclone (the cirro-stratus being, perhaps, the most frequent) are spread far in advance of the storm itself by the swift upper winds, it follows that they also mark a level of maximum cloud frequency.

Cirrus Level.—During fair weather thin cirri often occur, as already explained, at or near the top of the troposphere, due, probably, to that marked cooling of the upper atmosphere characteristic of " highs," and, as these are the highest of all clouds, it is obvious that they denote a final level of maximum cloudiness, one whose average elevation in middle latitudes is about 10 kilometres.

Regions of Minimum Cloudiness.—Between the levels of maximum cloudiness there obviously must be regions of minimum condensation. These are:

Scud Region.—Since one level of maximum cloud formation, including fog, is at the surface of the earth and the next at an elevation of approximately 1.5 kilometres, the average base height of the cumulus, it follows that the intervening region is one of minimum cloudiness, the absolute minimum being just above the highest fog. The name " scud region " might be appropriate to this space, since " scud " is, perhaps, the only cloud that occurs in it.

Intercumulus Region.—The intercumulus region of minimum condensation lies, as the name suggests, between the cumulus and alto-cumulus levels of maximum cloudiness. Its elevation is, roughly, 2.5 to 3.5 kilometres.

Alto-stratus Region.—As the alto-cumulus and the cirro-stratus mark adjacent levels of maximum cloudiness at the heights of about 4 and 8 kilometres respectively, it follows, as above, that the region between, at the heights of 4.5 to 6 kilometres, especially the higher alto-stratus region, must be one of minimum cloudiness. And this it is, because (*a*) it is above the level of diurnal convection and therefore of most cumulus clouds; (*b*) the clouds of intermediate level formed in cyclonic areas are not blown forward so rapidly nor, therefore, over such wide areas as are the cirri; and (*c*) the atmosphere at this level in anticyclones is nearly always dry, apparently dynamically warmed, and therefore non-cloud-forming.

Intercirrus Region.—Since the cirrus region furnishes two adjacent levels of maximum cloudiness, a foul (cyclonic) and a fair weather type, whose elevations are about 8 and 10 kilometres respectively, it follows that an intercirrus region of minimum cloudiness must lie between them at any elevation of, say, 8.5 to 9.5 kilometres.

Isothermal Region.—Obviously water vapor is not carried by convection above the troposphere. Neither can it accumulate by diffusion in the lowest portion of the stratosphere to a density greater than that of saturation at the minimum temperatures of this level incident to the passage of anticyclones; and even this low density must rapidly decrease, under the influence of gravity, with increase of elevation. Hence, there being but little water vapor present, and the relative humidity being very low, clouds cannot form in the stratosphere; that is, beyond an elevation of about 11 kilometres in middle latitudes.

There are, then, five principal levels of maximum cloudiness:

1. Fog level, surface of the earth or water.
2. Cumulus level, height above surface about 1.5 kilometres.
3. Alto-cumulus level, height above surface about 4 kilometres.
4. Cirro-stratus level, height above surface about 8 kilometres.
5. Cirrus level, height above surface about 10 kilometres.

There also are five regions of minimum condensation:

1. Scud region, 100 to 300 metres elevation.
2. Intercumulus region, 2.5 to 3.5 kilometres elevation, roughly.
3. Alto-stratus region, 4.5 to 6 kilometres elevation, roughly.
4. Intercirrus region, 8.5 to 9.5 kilometres elevation, roughly.
5. Isothermal region, beyond 11 kilometres elevation.

Cloud Depth or Thickness.—It is known that the thickness of clouds varies from the 8 to 10 kilometres of the most towering cumulus, usually associated with a violent hailstorm, down to that of a vanishingly thin cirrus. Systematic measurements of cloud thickness, however, have not been numerous. The best, perhaps, were made at Potsdam and are given in the following table copied from Hann's "Lehrbuch der Meteorologie":

Cloud Thickness.

Cloud	A.-St.	A.-Cu.	St.-Cu.	Nb.	Cu.-Nb.	Cu.	Fr.-Cu.
Depth in metres: Average....	510	194	353	(590)	2070	669	214
Depth in metres: Maximum..	1310	370	1265	1240	>4600	2230	430
Depth in metres: Minimum..	105	50	50	160	340	90	70
Number of observations............	6	18	18	16	21	22	26

Cloud Velocities.—The velocity of a cloud is usually the velocity of the air in which it floats, except in the case of a stationary type—crest cloud, banner cloud, et cetera—or a billow cloud. With these exceptions, it therefore is approximately the gradient velocity at the cloud level, which varies with altitude, latitude, temperature, and pressure distribution.

Average values, observed at certain places, are given in the following table, also copied from Hann's "Lehrbuch der Meteorologie":

Average Wind Velocity in Metres per Second.

	Ci.	Ci.-St.	Ci.-Cu.	A.-St.	A.-Cu.	St.-Cu.	Nb.	Cu.-Nb. Top	Cu. Top	Cu. Base	Pr.-Cu.	St.
	1. SUMMER, CHIEFLY APRIL TO SEPTEMBER.											
Bossekop, 70° N.	18	18	11	13	11	5	6	..	7	7	..	7
Upsala, 60° N.	20	(39)	17	5	12	7	7	..	7	6	8	..
Potsdam, 52½° N.	22	24	13	11	10	9	11	9	8	6	7	7
Trappes, 49° N.	23	23	23	15	13	9	10	14	10	9	8	10
Blue Hill, 42° N.	30	30	18	25	13	10	14	22	13	9	..	6
Washington, 39° N.	30	27	23	18	16	10	8	15	7	..	..	6
Manila, 14½° N.	13	16	3	..	11	4	..	..	..	..	..	..
Batavia (year), 6° S.	12	19	3	..	6	6	..	..	4	..	..	1
	2. WINTER, CHIEFLY OCTOBER TO MARCH.											
Upsala	23	13	18	..	13	12	6	18	12	..	12	..
Potsdam	28	20	24	16	16	12	13	28	10	(14)	12	10
Trappes	23	19	27	18	14	11	16	..	12	12	11	10
Blue Hill	37	41	36	25	24	13	13	..	..	15	..	10
Washington	35	30	33	21	21	15	12	21	11	..	..	10
Manila	13	16	3	19	4	8	6	..	..	..	..	..

CHAPTER XV

THE THUNDERSTORM.

Introduction.—A thunderstorm, as its name implies, is a storm characterized by thunder and lightning, just as a dust storm is characterized by a great quantity of flying dust. But the dust is never in any sense the cause of the storm that carries it along, nor, so far as is known, does either thunder or lightning have any influence on the course—genesis, development, and termination—of even those storms of which they form, in some respects, the most important features. No matter how impressive or how terrifying these phenomena may be, they never are anything more than mere incidents to, or products of, the peculiar storms they accompany, as will be made clear by what follows. In short, they are never in any sense either storm-originating or storm-controlling factors.

Origin of Thunderstorm Electricity.—Many have supposed that, whatever the genesis of the thunderstorm, the lightning, at least, is a product or manifestation of the free electricity always present in the atmosphere—normal atmospheric electricity. Observations, however (discussed later), seem definitely to exclude this assumption. Thus, while the difference in electrical potential between the surface of the earth and a point at constant elevation is, roughly, the same at all parts of the world, the number and intensity of thunderstorms vary greatly from place to place. Further, while the potential gradient at any given place is greatest in winter, the number of thunderstorms is most frequent in summer, and while the gradient in the lower layer of the atmosphere, at many places, usually is greatest from 8 to 10 o'clock, both morning and evening, and least at 2 to 3 o'clock P.M. and 3 to 4 o'clock A.M., no closely analogous relations hold for the thunderstorm.

But how, then, is the great amount of electricity incident to a thunderstorm generated? Fortunately an answer to this question based on careful experiments and numerous observations, and that greatly aids our understanding of the interrelations between the various thunderstorm phenomena, has been given by Dr. G. C. Simpson,[83] of the Indian Meteorological Department.

[83] Memoirs, Indian Meteorl. Dept., Simla, 1910, 20, pt. 8; *Phil. Mag.*, 30, 1, 1915.

Doctor Simpson's observations, just referred to, were made at Simla, India, at an elevation of about 7000 feet above sea-level, and covered all of the monsoon seasons, roughly, April 15 to September 15, of 1908 and 1909. He also made observations of the electrical conditions of the snow at Simla during the winter of 1908–9.

A tipping bucket rain gauge gave an automatic, continuous record of the rate and time of rainfall, while a Benndorf [84] self-registering electrometer marked the sign and potential of the charge acquired during each two-minute interval. A second Benndorf electrometer recorded the potential gradients near the earth, and a coherer of the type used in radiotelegraphy registered the occurrence of each lightning discharge.

All obvious sources of error were examined and carefully guarded against. Hence it would seem that the conclusions drawn from the thousands of observations given in the memoir are fully justified; and especially so since several independent series of similar observations made at different times, by different people, and at places widely separated, have given confirmatory results in every case. Simpson's records show that—

(1) The electricity brought down by the rain was sometimes positive and sometimes negative.
(2) The total quantity of positive electricity brought down by the rain was 3.2 times greater than the total quantity of negative electricity.
(3) The period during which positively charged rain fell was 2.5 times longer than the period during which negatively charged rain fell.
(4) Treating charged rain as equivalent to a vertical current of electricity, the current densities were generally smaller than 4×10^{-15} amperes per square centimetre; but on a few occasions greater current densities, both positive and negative, were recorded.
(5) Negative currents occurred less frequently than positive currents, and the greater the current density the greater the preponderance of the positive currents.

[84] *Physikal. Ztschr.*, Leipzig, 1906, 7, 98.

(6) The charge carried by the rain was generally less than 6 electrostatic units per cubic centimetre of water, but larger charges were occasionally recorded, and in one exceptional storm (May 13, 1908) the negative charge exceeded 19 electrostatic units per cubic centimetre.

(7) As stated in paragraph (3) above, positive electricity was recorded more frequently than negative, but the excess was the less marked the higher the charge on the rain.

(8) With all rates of rainfall positively charged rain occurred more frequently than negatively charged rain, and the relative frequency of positively charged rain increased rapidly with increased rate of rainfall. With rainfall of less than about 1 millimetre in two minutes, positively charged rain occurred twice as often as negatively charged rain, while with greater intensities it occurred 14 times as often.

(9) When the rain was falling at a less rate than about 0.6 millimetre in two minutes, the charge per cubic centimetre of water decreased as the intensity of the rain increased.

(10) With rainfall of greater intensity than about 0.6 millimetre in two minutes the positive charge carried per cubic centimetre of water was independent of the rate of rainfall, while the negative charge carried decreased as the rate of rainfall increased.

(11) During periods of rainfall the potential gradient was more often negative than positive, but there were no clear indications of a relationship between the sign of the charge and the sign of the potential gradient.

(12) The data do not suggest that the negative electricity occurs more frequently during any particular period of a storm than during any other.

Concerning his observations on the electrification of snow Doctor Simpson says:

So far as can be judged from the few measurements made during the winter of 1908–9, it would appear that—

(1) More positive than negative electricity is brought down by snow in the proportion of about 3.6 to 1.
(2) Positively charged snow falls more often than negatively charged.
(3) The vertical electric currents during snowstorms are, on the average, larger than during rainfall.

While these observations were being secured a number of well-devised experiments were made to determine the electrical effects of each obvious process that takes place in the thunderstorm.

Freezing and thawing, air friction, and other things were tried, but none produced any electrification. Finally, on allowing drops of distilled water to fall through a vertical blast of air of sufficient strength to produce some spray, positive and important results were found, showing:

(1) That breaking of drops of water is accompanied by the production of both positive and negative ions.
(2) That three times as many negative ions as positive ions are released [thus leaving the drops charged positively].

Now, a strong upward current of air is one of the most conspicuous features of the thunderstorm. It is always evident in the turbulent cauliflower heads of the cumulus cloud—the parent, presumably, of all thunderstorms. Besides, its inference is compelled by the occurrence of hail, a frequent thunderstorm phenomenon, whose formation requires the carrying of raindrops and the growing hailstones repeatedly to cold and therefore high altitudes. And from the existence of hail it is further inferred that an updraft of at least 8 metres per second must often occur within the body of the storm, since, as experiment shows,[85] air of normal density must have approximately this upward velocity to support the larger drops, those of 4 mm. or more in diameter,

[85] P. Lenard, *Met. Zeit.*, vol. 21, p. 249, 1904.

and, because of its greater weight, even a stronger updraft to support the average hailstone.

Experiment also shows [86] that raindrops of whatever size can not fall through air of normal density whose upward velocity is greater than about 8 metres per second, nor themselves fall with greater velocity through still air; that drops large enough, 4.5 mm. in diameter and up, if kept intact, to attain through the action of gravity a greater velocity than 8 metres per second with reference to the air, whether still or in motion, are so blown to pieces that the increased ratio of supporting area to total mass causes the resulting spray to be carried aloft or, at least, left behind, together with, of course, all original smaller drops. Above sea-level this limiting velocity is greater. It is given approximately by equating the weight of the drop, $4/3\ \pi\ r^3 g\ (\rho - \sigma)$, to the supporting force, $\pi\ r^2\ \sigma\ v^2$, roughly, in which r is the radius of the drop, ρ its density, σ the density of the air, v the velocity in question, and g the gravitational acceleration. Hence the limiting velocity increases in practically the same ratio that the square-root of the density decreases. Thus at an elevation of 3 kilometres above sea-level, where the barometric pressure is about 520 mm. and the temperature, say, 15° C. lower than at the surface, the limiting velocity is approximately 9.4 metres per second, instead of 8, the value for normal density, or density at 0° C. and 760 mm. pressure. Clearly, then, the updrafts within a cumulus cloud frequently must be strong and therefore break up at about the same level, that of maximum rain accumulation, innumerable drops which, through coalescence, have grown beyond the critical size, and thereby, according to Simpson's experiments, produce electrical separation within the cloud itself. Obviously, under the turmoil of a thunderstorm, such drops may be forced through the cycle of union (facilitated by any charges they may carry) and division, of coalescence and disruption, from one to many times, with the formation on each at every disruption, again *according to experiment*, of a correspondingly increased electrical charge. The turmoil compels mechanical contact between the drops, whereupon the disruptive equalization of their electrical potential breaks down their surface tensions and insures coalescence. Hence, once started, the electricity of a thunderstorm rapidly grows to a considerable maximum.

[86] P. Lenard, *Met Zeit.*, vol. 21, p. 249, 1904.

After a time the larger drops reach, here and there, places below which the updraft is small—the air cannot be rushing up everywhere—and then fall as positively charged rain, because of the processes just explained. The negative electrons, in the meantime, are carried up into the higher portions of the cumulus, where they unite with the cloud particles and thereby facilitate their coalescence into negatively charged drops. Hence the heavy rain of a thunderstorm should be positively charged, as it almost always is, and the gentler portions negatively charged, which also very frequently is the case.

Such in brief is Doctor Simpson's theory of the origin of the electricity in thunderstorms, a theory that fully accounts for the facts of observation and in turn is itself abundantly supported by laboratory tests and imitative experiments.

If this theory is correct—and it seems well founded—it must follow that the one essential to the formation of the giant cumulus cloud—namely, the rapid uprush of moist air—is also the one essential to the generation of the electricity of thunderstorms. Hence the reason why lightning seldom occurs except in connection with a cumulus cloud is understandable and obvious. It is simply because the electrifying process of splashing is vigorously active in this cloud and nearly absent in others.

The occasional lightning in connection with snowstorms, dust storms, and volcanic eruptions may in each case be due to the fact that the collision of solid particles produces electrification.[87]

The Violent Motions of Cumulus Clouds.—From observations, and from the graphic descriptions of the few balloonists and aviators who have experienced the trying ordeal of passing through the heart of a thunderstorm, it is known that there is violent vertical motion and much turbulence in the middle of a large cumulus cloud, a fact which, so far as it relates to the theory alone of the thunderstorm, it would be sufficient to accept without inquiring into its cause. However, to render the discussion more nearly complete, it perhaps is worth while, since it is a mooted question, to inquire what the probable cause of the violent motions in large cumulus clouds really is—motions which, in the magnitude of their vertical components and degree of turmoil, are never exhibited by clouds of any other type nor met

[87] Rudge, *Proc. Roy. Soc., A*, 90, 256, 1914.

with elsewhere by either kites or balloons of any kind, manned, sounding, or pilot.

It has been shown by von Bezold [88] that sudden condensation from a state of supersaturation, and also sudden congelation of undercooled cloud droplets, would, as a result of the heat thus liberated, cause an equally sudden expansion of the atmosphere, and thereby turbulent motions analogous to those observed in large cumuli. However, as von Bezold himself points out, it is not evident how either the condensation or the freezing could suddenly take place throughout a cloud volume great enough to produce the observed effects. Besides, these eruptive turmoils, whatever their genesis, undoubtedly originate and run their course in regions already filled with cloud particles in the presence of which no appreciable degree of supersaturation can occur. Hence the rapid uprush and the violent turbulence in question obviously must have some other cause, which indeed is provided by the difference between the actual temperature gradient of the surrounding atmosphere and the adiabatic temperature gradient of the saturated air within the cloud itself.

Consider a warm summer afternoon, temperature 30° C., and assume the dew-point to be 18° C. Now, the adiabatic decrease of temperature of non-saturated air is about 1° C. per 100 metres increase in elevation, and therefore, under the assumed conditions, vertical convection of the surface air causes condensation to begin at an elevation of approximately 1.5 kilometres—allowing for the increase of volume per unit mass of vapor. From this level, however, so long as the cloud particles are carried up with the rising air, the rate of temperature decrease for at least a couple of kilometres is much less—at first about one-half the previous rate, and appreciably less than that of the surrounding clear air. After a considerable rise above the level of initial condensation, half a kilometre, say, the raindrops have so increased in size as to lag behind the upward current and even to drop out, while at the same time the amount of moisture condensed per degree fall of temperature grows rapidly less, as shown by the saturation adiabats of Fig. 65. Hence for both reasons—because the heat of the water is no longer available to the air from which it was condensed, the drops having been left behind, and because a decreasing amount of latent heat is to be

[88] *Sitzber. K. Preuss. Akad. d. Wiss.*, Berlin, 1892, 8, 279–309.

had from further condensation, there being less and less precipitation per degree cooling—the rate of temperature decrease again approaches the adiabatic gradient of dry air, or 1° C. per 100 metres change of elevation.

Obviously, then, for some distance above the level at which condensation begins to set free its latent heat the temperature of the rising mass of moist air departs farther and farther from the temperature of the surrounding atmosphere at the same level, and therefore its buoyancy for a time as steadily increases. But, of course, as explained above, this increase of buoyancy does not continue to any great altitude.

In the lower atmosphere continuous and progressive convection builds up the adiabatic gradient so gradually that no great difference between the temperature of the rising column and that of the adjacent atmosphere is anywhere possible. Hence, under ordinary conditions, the uprush in this region is never violent. But whenever the vertical movement of the air brings about a considerable condensation it follows, as above explained, that there is likely to be an increase in its buoyancy, and hence a more or less rapid upward movement of the central portion, like air up a heated chimney, and for the same reason, together with, because of viscosity, a rolling and turbulent motion of the sides, of the type so often seen in towering cumulus clouds. Obviously, too, the uprushing column of air must continue to gain in velocity so long as its temperature is greater, or density less, than that of the surrounding atmosphere, except as modified by viscosity, and therefore have its greatest velocity near the level at which these two temperatures are the same. Hence the rising column must ascend somewhat beyond its point of equilibrium, and then, because slightly undercooled, correspondingly drop back.

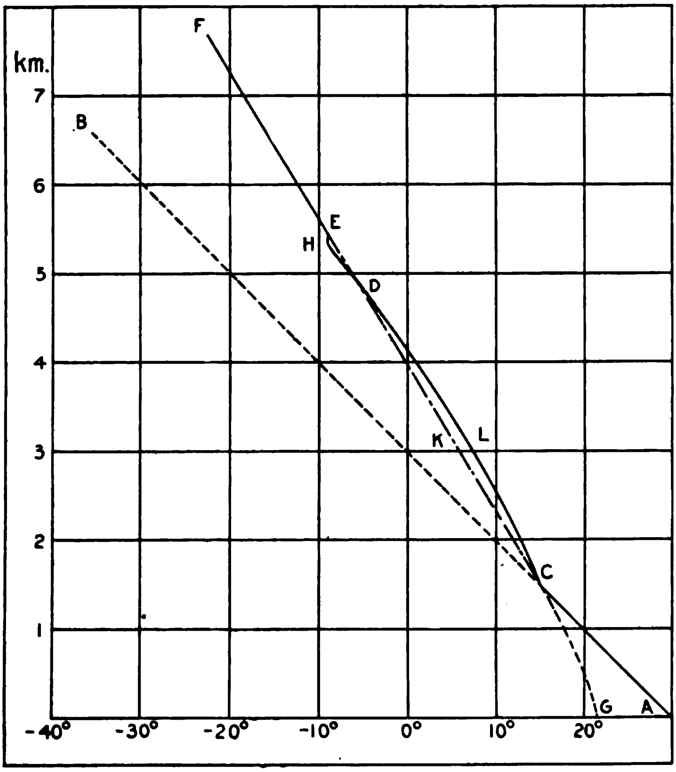
Fig. 90, based upon approximately average conditions, illustrates the points just explained. The elevation is in kilometres and the temperature in degrees Centigrade.

AB is the adiabatic temperature gradient for non-saturated air, about 1° C. per 100 metres change in elevation. *GCKDEF* is the supposed temperature gradient before convection begins, or a decrease, in accordance with observations, of 6° C., about, per kilometre increase in elevation, except near the surface, where the temperature decrease, before convection has begun, ordinarily

is less rapid, and at elevations between 5 kilometres, roughly, and the isothermal level, where it is more rapid.

As convection sets in, the temperature decrease near the surface soon approximates the adiabatic gradient for dry air, and this condition extends gradually to greater altitudes, till, in the given case, condensation begins at the level *C*, or where the tem-

FIG. 90.



Temperature gradients within (CLD) and without (CKD) cumulus clouds.

perature is 15° C. Here the temperature decrease, under the assumed conditions, suddenly changes from 10° C. per kilometre increase of elevation to rather less than half that amount, but slowly increases with increase of altitude and consequent decrease of temperature. At some level, as *L*, the temperature difference between the rising and the adjacent air is a maximum. At *D* the temperature of the rising air is the same as that of the air

adjacent, but its momentum presumably carries it on to some such level as *H*. Within the rising column, then, the temperature gradient is given approximately by the curve *ACLDHE*, while that of the surrounding air is substantially as shown by the curve *ACKDEF*.

The cause, therefore, of the violent uprush and turbulent condition within large cumulus clouds is the difference between the temperature of the inner or warmer portions of the cloud itself and that of the surrounding atmosphere at the same level as indicated by their respective temperature gradients *CLD* and *CKD*. Clearly, too, while some air must flow into the condensation column all along its length, the greatest pressure difference, and therefore the greatest inflow, obviously is at its base. After the rain has set in, however, this basal inflow is from immediately in front of the storm, and necessarily so, as will be explained later.

The approximate difference in level between *D* and *H*, or the height to which the momentum of the ascending column will force it to rise after it has cooled to the temperature of the surrounding air, may easily be computed in terms of the vertical velocity at *D* and the difference between the temperature gradients of the rising and the non-rising masses of air. Let the vertical velocity of the rising column be *V* centimetres per second at *D*; let the average absolute temperature between *D* and *H* be *T*; and let the difference between the temperature of the rising air and the surrounding air change uniformly at the rate δT per centimetre change of elevation.

Obviously the kinetic energy of the rising air at *D* will be used up in lifting it to some greater elevation. But the weight of a mass m of this air is not mg (g being gravity acceleration), which it would be *in vacuo*, but $mg\left(\frac{\rho-\sigma}{\rho}\right)$, in which ρ is the density of the air in question and σ the density of the surrounding air. Thus at h centimetres above *D*, $\rho-\sigma=\frac{h\,\delta\,T\rho}{T}$, approximately, and therefore the weight of the mass m at this level is $mg\,\frac{h\,\delta\,T}{T}$, approximately. Hence the work in lifting the mass m through this altitude h is its average weight, $\frac{mg\,h\,\delta\,T}{2\,T}$, multiplied by the distance h.

Hence $\frac{1}{2} m V^2 = m g x^2 \frac{\delta T}{2 T}$ or $x^2 = \frac{V^2 T}{g \delta T}$
in which x, directly proportional to V, is the height in centimetres to which the air will rise above D.

Let T = 265° C. absolute, corresponding to the conditions shown in Fig. 90; let the temperature gradient be 6° C. per kilometre in the free air and 8° C. per kilometre in the rising air, and T the temperature change between the two per centimetre change of elevation, therefore 1/50000; and let V = 12 metres per second, then

$$x = 1200 \sqrt{\frac{5 \times 10^4 \times 265}{980}} = 1.395 \text{ kilometres.}$$

Since the height of the barometer at an elevation of 5 kilometres above sea-level is, roughly, 400 mm., and since the supporting force of an updraft is proportional to the product of its density by the square of its velocity, it follows that the vertical velocity in metres per second necessary to support the largest drops at this elevation is given by the equation,

$$V = v \sqrt{\frac{760}{400}},$$

in which v is the required velocity at normal density. But, as above explained, v = 8 metres per second, about, and therefore V = 11 metres per second, approximately.

Hence the above assumption that the vertical velocity at D is 12 metres per second appears to be conservative.

Viscosity between the rising and the adjacent air prevents the actual height attained from being quite equal to the above theoretical value; nevertheless, the maximum elevation, or what might be called the "momentum level" often, especially in the case of the largest and most active cumulus clouds, is much greater than the equilibrium level.

Convectional Instability.—Rapid vertical convection of humid air, as we have seen, is essential to the production of the cumulus cloud and, therefore, to the generation of the thunderstorm. Hence it is essential to consider the conditions under which the vertical temperature gradient necessary to this convection can be established. These are:

1. Strong surface heating, especially in regions of light winds; a frequent occurrence.

The condition that the winds be light is not essential,

or perhaps even favorable, to the genesis of all thunderstorms, but only to the local or heat variety, and favorable to these simply because winds, by thoroughly mixing the air, prevent the formation of isolated rising columns, the progenitors of this particular type of storm.

2. The overrunning of one layer of air by another at a temperature sufficiently lower to induce convection. This apparently is the cause of practically all ocean thunderstorms. It seems also to be the chief cause of those that so frequently occur on land in connection with cyclones.

3. The underrunning and consequent uplift of a saturated layer of air by a denser layer; a frequent occurrence to a greater or less extent and presumably, therefore, occasionally, at least, one of sufficient magnitude to produce a thunderstorm.

Here the underrunning air lifts both the saturated layer and the superincumbent unsaturated layer, and thereby forces each to cool adiabatically. But as both layers are lifted equally, while, because of the latent heat of condensation, the saturated layer cools much slower than the dry, it follows that a sufficient mechanical lift of a saturated layer of air would establish between it and the non-saturated layer above a superadiabatic temperature gradient and thereby produce local convection, cumulus clouds, and perhaps a thunderstorm.

Periodic Recurrence of Thunderstorms.—While thunderstorms may develop at any hour of any day, they nevertheless have three distinct periods of maximum occurrence: (*a*) Daily, (*b*) yearly, and (*c*) irregularly cyclic. Each maximum depends upon the simple facts that the more humid the air and the more rapid the local vertical convections the more frequent and also the more intense the thunderstorms, for the obvious reason that it is rapid vertical convection of humid air that produces them.

Daily Land Period.—Vertical convection of the atmosphere over land areas reaches its greatest altitudes and thereby produces the heaviest condensation and largest cumulus clouds when the surface is most heated; that is, during afternoons. Hence the hours of maximum frequency of inland or continental thunderstorms are, in most places, 2 to 4 P.M.

Daily Ocean Period.—Because of the great amount of heat rendered latent by evaporation, because of the considerable depth to which the sea is penetrated by solar radiation, and because of the high specific heat of water, the surface temperature of the ocean increases but little during the day, and because of convection or the sinking of any surface water that has appreciably cooled and the bringing of the warmest water always to the top, it decreases but slightly at night. Indeed, the diurnal temperature range of the ocean surface usually is but a small fraction of one degree C., while that of the atmosphere at from 500 to 1000 metres elevation is several fold as great.[89] Hence those temperature gradients over the ocean that are favorable to rapid vertical convection are most frequent during the early morning hours, and therefore the maximum of ocean thunderstorms usually occurs between midnight and 4 A.M.

Yearly Land Period.—Just as inland thunderstorms are most frequent during the hottest hours of the day, so, too, and for the same reason, they are, in general, most frequent over the land during the hottest months of the year, or rather during those months when the amount of surface heating, and therefore the vertical temperature gradient, is a maximum. This will be better understood by reference to the winter and summer temperatures of Fig. 16, determined, as previously explained, by averaging 185 and 231 records, respectively, obtained by sounding balloons sent up from Munich, Strassburg, Trappes, and Uccle, places of about the same latitude and having generally similar climates. It will be seen that the temperature of the air not only is much higher at all levels during summer than during winter, but also decreases through the first three kilometres much more rapidly.

That this important difference between the temperature gradients of winter and summer is general and not peculiar to the above localities is obvious from the fact that during summer the surface of the earth gradually grows warmer and therefore induces correspondingly frequent and vigorous convection, while during winter it as steadily becomes colder and therefore only occasionally, at the times of temporary warming, induces convections strong enough to form large and well-defined cumulus clouds.

From these several considerations it is evident that:

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[89] Braak, *Beitr. z. Physik d. fr. Atmosph.*, Leipzig, 1914, **6**, 141.

a. Winter convections cannot, in general, rise to nearly so great altitudes nor with such velocity as those of summer.

b. The absolute humidity of summer air may at times be greatly in excess of that of winter.

c. The winter snow level usually is much below that of summer.

Hence thunderstorms, since they depend, as explained, upon the action of strong vertical convection on an abundance of *rain* drops, necessarily occur most frequently during the warmer seasons, and only occasionally during the colder months. In middle latitudes, where there are no late spring snows to hold back the temperatures, the month of maximum frequency is June. In higher latitudes, where strong surface heating is more or less delayed, the maximum occurs in July or even August.

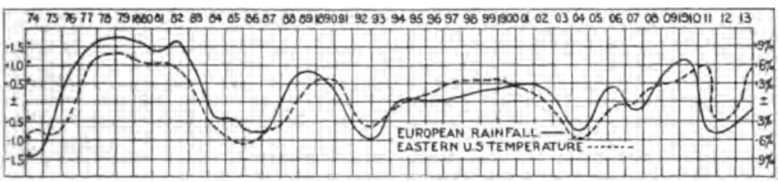
Yearly Ocean Period.—Over the oceans, on the other hand, temperature gradients favorable to the genesis of thunderstorms, and therefore the storms themselves, occur most frequently during winter and least frequently during summer. This is because the temperature of the air at some distance above the surface, being largely what it was when it left the windward continent, greatly changes from season to season, while that of the water, and, of course, the air in contact with it, changes relatively but little through the year. That is, over the oceans the average decrease of temperature with increase of elevation obviously is least and, therefore, thunderstorms fewest in summer, and greatest, with such storms most numerous, in winter.

Cyclic Land Period.—Since thunderstorms are accompanied by rain and since over land they are most numerous during summer, it would appear that they must occur most frequently either in warm or in wet years and least frequently in cold or in dry years. Further, if it should happen, as it actually does, that, for the earth as a whole, warm years are also wet years and dry years cold years, it would appear logically certain that, for the entire world, the maxima numbers of thunderstorms must belong to the years that are wet and warm and the minima to those that are cold and dry.

A complete statistical examination of these statements is not possible, owing to the fact that meteorological data are available for only portions of the earth's surface and not for the whole of it. Nevertheless, well-nigh conclusive data do exist. The annual

rainfall, for instance, to the leeward of a large body of water obviously must bear the same relation to the annual average windward temperature that the total annual precipitation over the entire world does to the annual average world temperature. In each case the amount of evaporation or amount of water vapor taken into the atmosphere, and therefore the amount of subsequent precipitation, clearly must increase and decrease with the temperature. An excellent test and complete support of this deduction is furnished by Fig. 91, in which the full line represents the smoothed annual European precipitation,[90] and the dotted line smoothed annual average temperatures over the eastern United States. Obviously, as supported by the data graphically represented in Fig. 91, the warmer the air as it leaves America

FIG. 91.



Relation of European rainfall to eastern United States temperature.

the greater the moisture it must take up in its passage across the Atlantic, and therefore the greater its supply of humidity on reaching Europe and the heavier the subsequent precipitation. Clearly, too, the same relations must apply to the entire earth that so obviously should and so demonstrably do hold for the North Atlantic and its adjacent continents.

Beyond a reasonable doubt, therefore, for the world as a whole, warm years are wet and cold ones are dry. Hence, as above stated, it is practically certain that the maxima of thunderstorms occur during years that are wet, or warm—for the two are identical—and the minima during years that are dry or cold. A partial and, so far as it goes, a confirmatory statistical test of this conclusion is given by Fig. 92. The lower group of curves is based on an exhaustive study by Dr. van Gulik[91] of thunder-

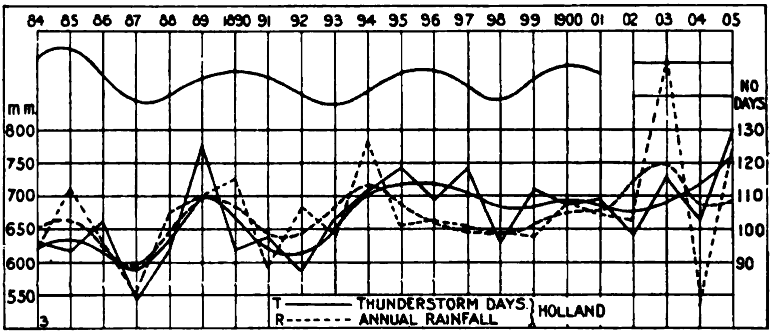
[90] Hellmann, "Die Niederschläge in den Norddeutschen Stromgebieten," Berlin, 1906, vol. 1, pp. 336–337, and elsewhere.

[91] *Meteorologische Zeitschrift,* Braunschweig, 1908, 25th Jhrg., 108.

storms and lightning injuries in Holland. The continuous zigzag line gives the actual number of thunderstorm days, and the continuous curved line the same numbers smoothed. The broken lines give, respectively, the actual and the smoothed values of the annual average precipitation. The upper curve represents the variations in the smoothed number of destructive thunderstorms [92] (number of thunderstorm days not readily available) in Germany.

The original data on which this last curve is based indicate a continuous and rapid increase of thunderstorm destructiveness throughout the period studied, 1854–1901. Presumably, however, this increase is real only to the extent that the country has

FIG. 92.



Relation of annual number of thunderstorm days to total annual precipitation—Holland. The uppermost wavy curve shows the variation in the smoothed number of destructive thunderstorms in Germany.

become more densely populated and more thickly studded with destructible property. Since thunderstorms are caused by rapid vertical convection and heavy condensation, and since the temperature of the air upon which these in turn depend has not, on the decade average, measurably changed since reliable records began, at least a hundred years ago, there clearly is no logical reason for believing that the decade average either of the frequency or the intensity of the storms themselves has materially changed during that time. At any rate, this element—that is, the rapid increase suggested by insurance data—has been omitted from the curve and only the fluctuation factor retained.

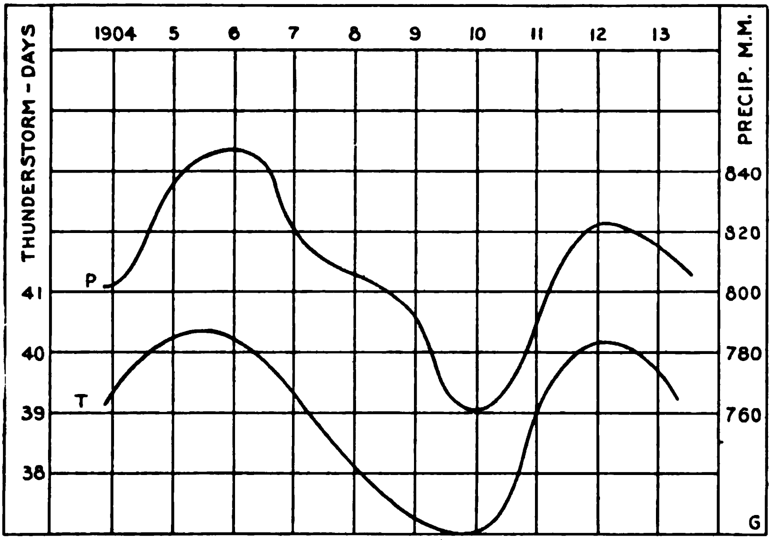
It will be noticed that the curve of thunderstorm frequency

[92] Otto Steffens, *Ztschr. f. d. gesamte Versicherungswiss.*, Berlin, 1904, 4, pt. 4. (Also Diss.-Berlin, 1904.)

for all Holland closely parallels the curve of thunderstorm injury in all Germany. Hence it seems safe to infer that the frequency of thunderstorm varies pretty much the same way over both countries, and presumably also over many other portions of Europe; that is, roughly as the rainfall varies, or, considering the world as a whole, roughly as the temperature varies.

Additional statistical evidence of the relation between the annual number of thunderstorm days and the total annual precipitation, kindly furnished by P. C. Day, in charge of the

FIG. 93.



Relation of annual number of thunderstorm days, T, to total precipitation, P—United States

Climatological Division of the Weather Bureau, is shown by Fig. 93, in which the upper line gives, in millimetres, the smoothed annual precipitation of 127 stations scattered over the whole of the United States, and the lower line the smoothed average annual number of thunderstorm days at these same stations. It was thought at first that this relation might differ greatly for those portions of the United States whose climates are radically dissimilar, and for this reason the stations east of the one hundredth meridian provisionally were classed separately from those west of it; but the results for the two sections, being substantially alike, show that for this purpose their division is entirely unnecessary.

As will be seen from the figure, the earliest statistics used are those of 1904. This is because the annual number of such days reported rapidly decreases from 1904 back to about 1890. Indeed, the annual number of thunderstorm days reported per station since 1903 is almost double the annual number per station (practically the same stations) from 1880 to 1890. The transition from the smaller to the larger number was due in great measure, doubtless, to an alteration in station regulations equivalent to changing the official definition of a thunderstorm from "thunder *with* rain" to "thunder *with* or *without* rain." This, however, does not account for the fact that from 1890 to 1904 the average annual number of thunderstorm days reported per station increased, at a nearly constant rate, almost 100 per cent. Either the storms did so increase, which from the fact that there have been no corresponding temperature changes seems incredible, or else there was, on the average, an increase of attention given to this particular phenomenon. At any rate, so continuous and so great an increase in the average number of thunderstorm days cannot be accepted without abundant confirmation, and for this reason the earlier thunderstorm records provisionally have been rejected.

Obviously a much closer relation between the number of thunderstorm days and total precipitation would hold for some months and seasons than for others, but no such sub-grouping of the data has been made, though, presumably, it would give interesting results. The whole purpose of this portion of the study was to arrive at some definite idea in regard to the cyclic change of thunderstorm frequency, to see with what other meteorological phenomena this change is associated, and, if possible, to determine its cause.

Now, it is well known that the average temperature of the world as a whole follows in general the sun-spot changes, in the sense that the greater the number of spots the lower the temperature, and the smaller the number of spots the higher the temperature. This regular relation, however, often is greatly modified[93] by the presence in the high atmosphere of volcanic dust, one invariable effect of which is a lower average temperature. Hence the warm and the cold periods are irregularly cyclic, and

[93] Humphreys, W. J., *Bull. Mount Weather Observatory*, Washington, 1913, **6**, 1. Also in Part IV of this book.

also irregular in intensity. Hence, also, the annual amount of precipitation, the frequency of thunderstorms, and many other phenomena must perforce undergo exactly the same irregular cyclic variation.

As already stated, the statistical evidence bearing on these conclusions neither is nor can be complete, but the deductions are so obvious and the statistical data already examined so confirmatory that but little doubt can exist of their general accuracy.

Cyclic Ocean Period.—The record of thunderstorms over the ocean is not sufficiently full to justify any conclusions in regard to their cyclic changes. Possibly, as in the yearly and the daily periods, the ocean cyclic period may be just the reverse of that of the land, but this is not certain.

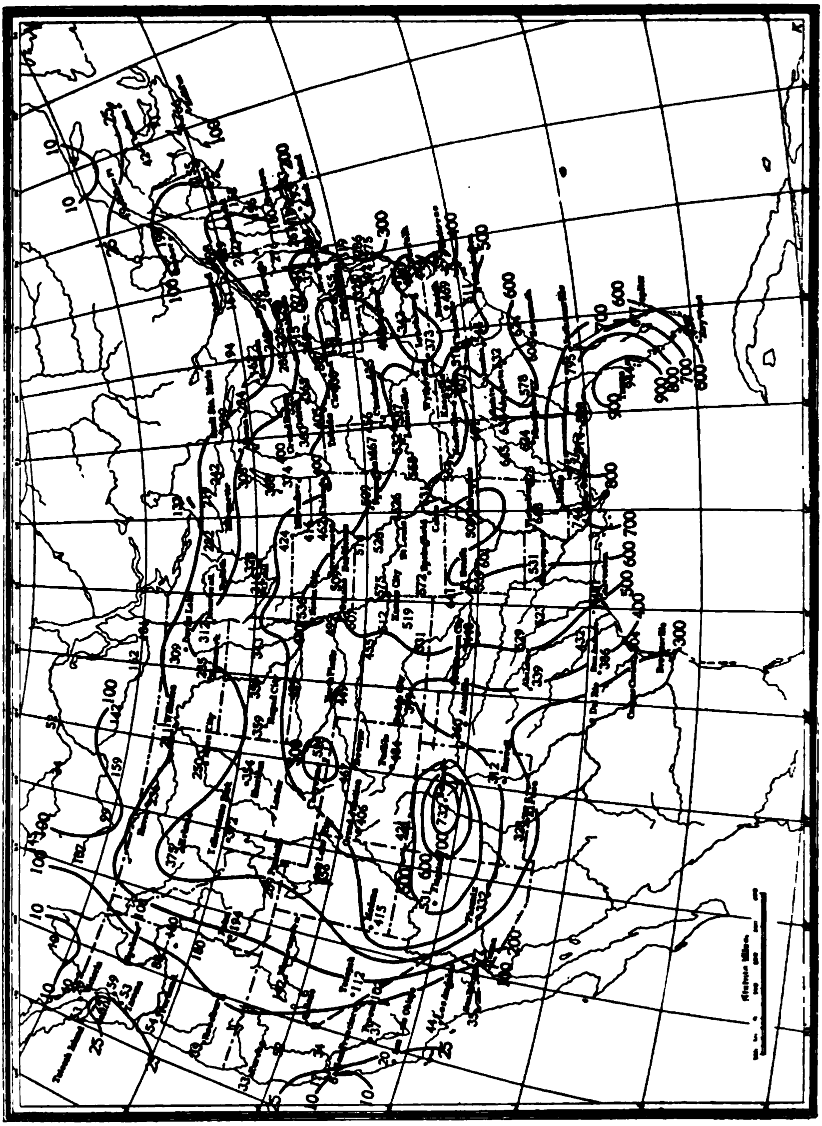
Geographic Distribution.—The geographic distribution of the thunderstorm may safely be inferred from the fact that it is caused by the strong vertical convection of humid air. From the nature of its formation one would assume—and the assumption is supported by observation—that the thunderstorm must be rare beyond either polar circle, especially over Greenland and over the Antarctic continent, rare over great desert regions wherever situated, rare over the trade belts of the oceans, and, on the other hand, increasingly abundant with increase of temperature and humidity, and therefore, in general, most abundant in the more rainy portions of the equatorial regions. The east coast of South America, from Pernambuco to Bahia, is said to be an exception.

An interesting and instructive example of the annual geographic distribution of thunderstorms is given by Fig. 94, copied from a statistical study of this subject by W. H. Alexander.[94] Although this example, based on a ten-year (1904–1913) average, refers to only the United States and southern Canada, it nevertheless shows the great influence of humidity, latitude, and topography on thunderstorm frequency.

One of the most striking facts shown by this map is the relatively unusual occurrence of this phenomenon along the Pacific Coast. This exceptional condition is explained by the fact that during the summer time, or season of strong vertical convection, the temperature of the on-shore winds of that region is too low and their humidity too small to permit of the ready formation

[94] *Monthly Weather Review*, 43, p. 322, 1915.

FIG. 94.



Total number of thunderstorm days in the United States during the ten years, 1904–1913, inclusive. (Alexander.)

over the heated interior of abundant cumuli, without which, as already explained, thunderstorms do not occur.

Pressure and Temperature Distribution.—In illustrating the occurrence of thunderstorms with reference to the disposition of isobars and isotherms, or the distribution of atmospheric pressure and temperature, typical weather maps of the United States,[95] Figs. 95 to 109, have been used, not because the thunderstorms of this country are different in any essential particular from those of other countries, but chiefly as a matter of convenience in making the drawings. To facilitate their study, each of the several types discussed is illustrated with three consecutive maps. The first shows the 12-hour antecedent conditions, the second the particular pressure-temperature distribution in question, and the third the 12-hour subsequent conditions.

In these figures the isobars, in corrected inches of mercury as read on the barometer and reduced to sea-level, and the isotherms in Fahrenheit degrees, are marked by full and dotted lines respectively. The legend "LOW" is written over a region from which, for some distance in every horizontal direction, the pressure increases. Similarly, the legend "HIGH" applies to a region from which, in every horizontal direction, the pressure decreases. The arrows, as is customary on such maps, fly with the wind, while the state of the weather is indicated by the usual U. S. Weather Bureau symbols. All refer to the time of observation, except that of the thunderstorm, which covers the previous 12 hours.

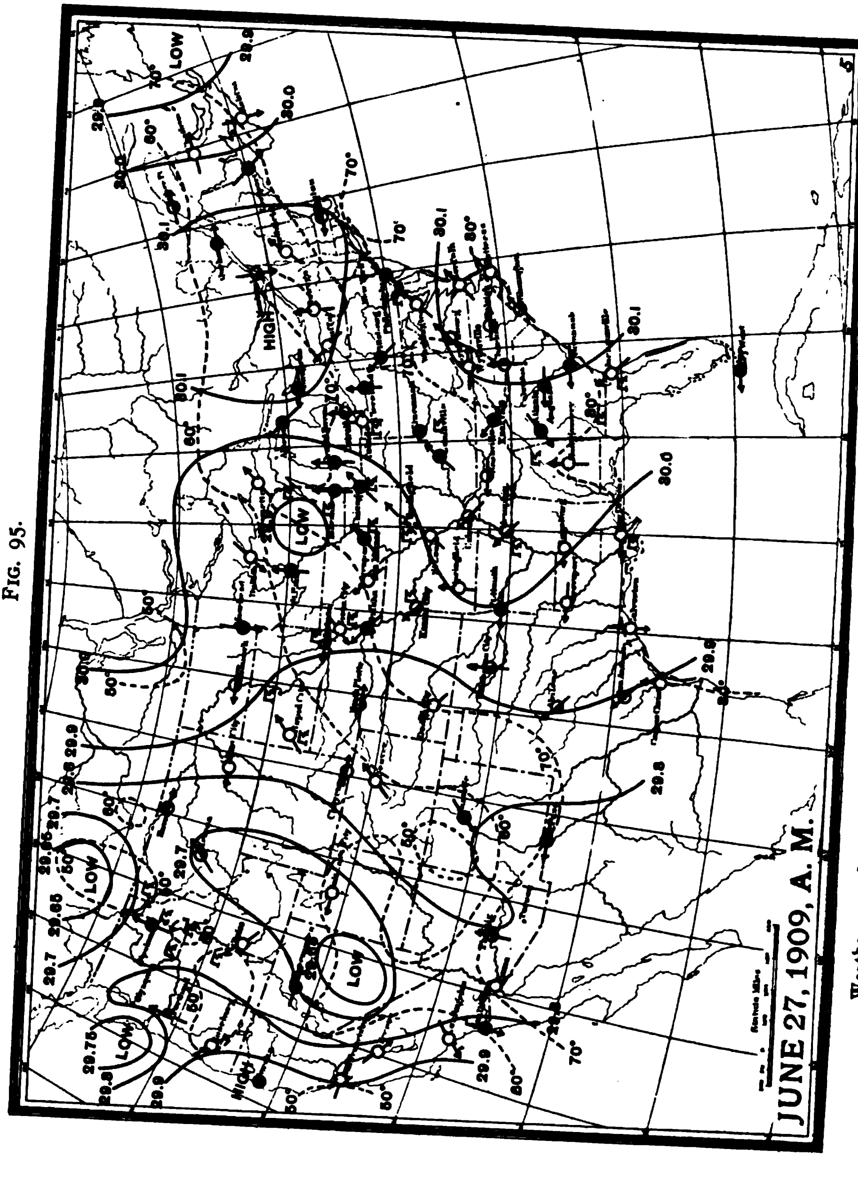
Obviously, the key to the geographic distribution of thunderstorms—that is, the distribution of conditions likely to induce strong vertical convection of humid air—is also the key to their probable location with reference to any given system of isotherms and isobars, or distribution of atmospheric temperature and pressure. From this standpoint the places of their most frequent occurrence are:

a. Regions of high temperature and widely extended nearly uniform pressure (see Figs. 95, 96, and 97).

The conditions are still more favorable to the genesis of thunderstorms when the air is humid and the pressure, partly

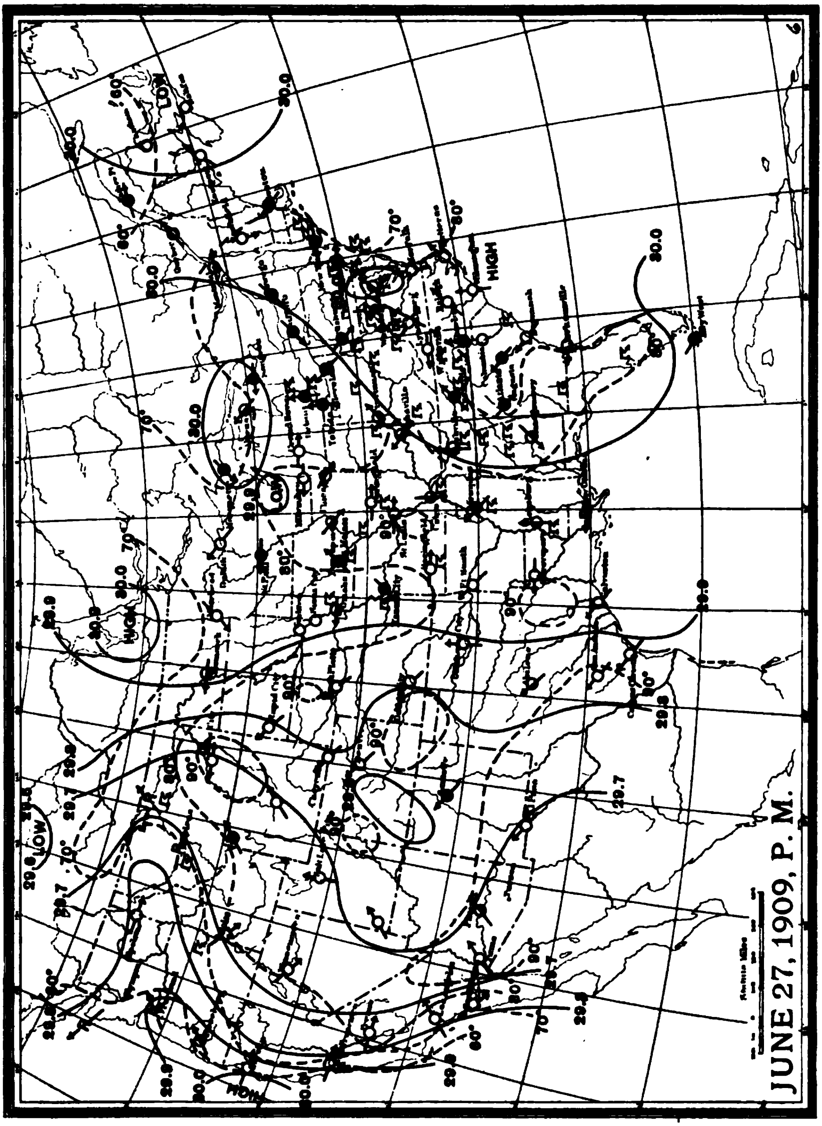
[95] The author wishes to acknowledge the kind coöperation of the official forecasters of the U. S. Weather Bureau in selecting maps typical of thunderstorm conditions in the United States.

FIG. 95.



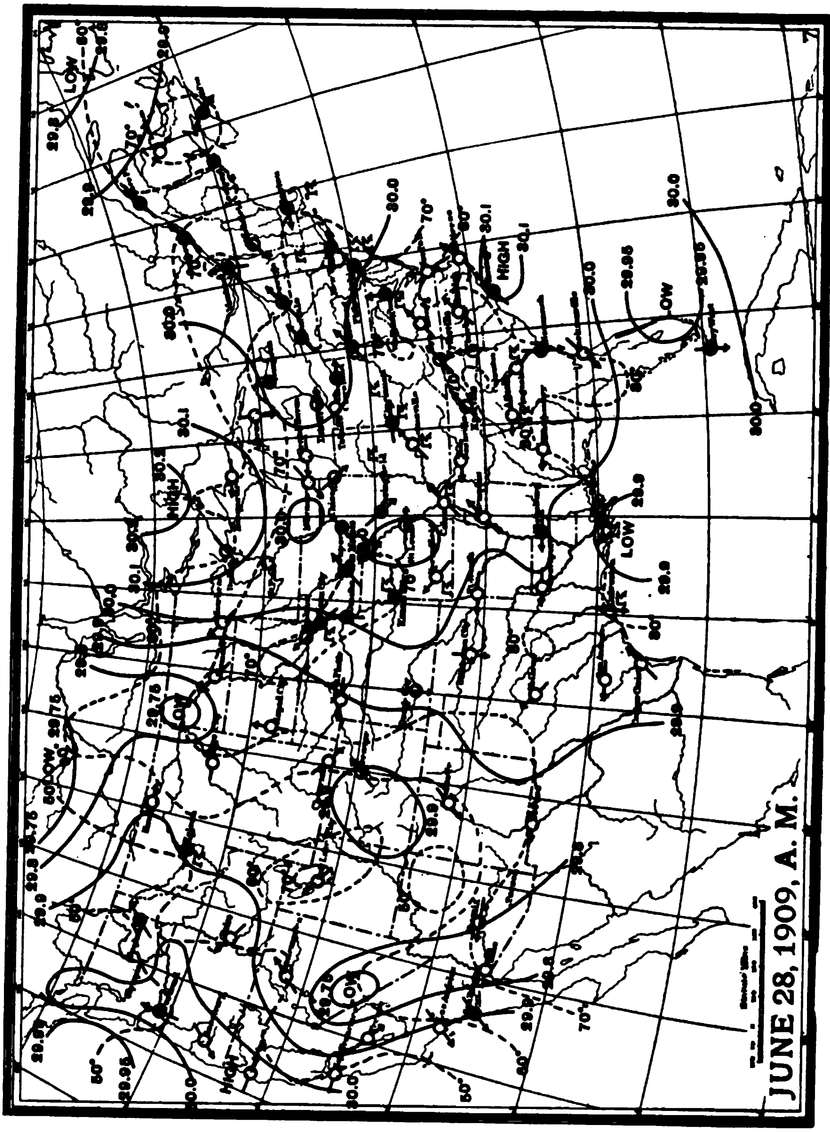
Weather map, 8 A.M., June 27, 1909, typical conditions at beginning of "heat" thunderstorms: O, clear; ◐, partly cloudy; ●, cloudy; R, rain; K, thunderstorm.

FIG. 96.



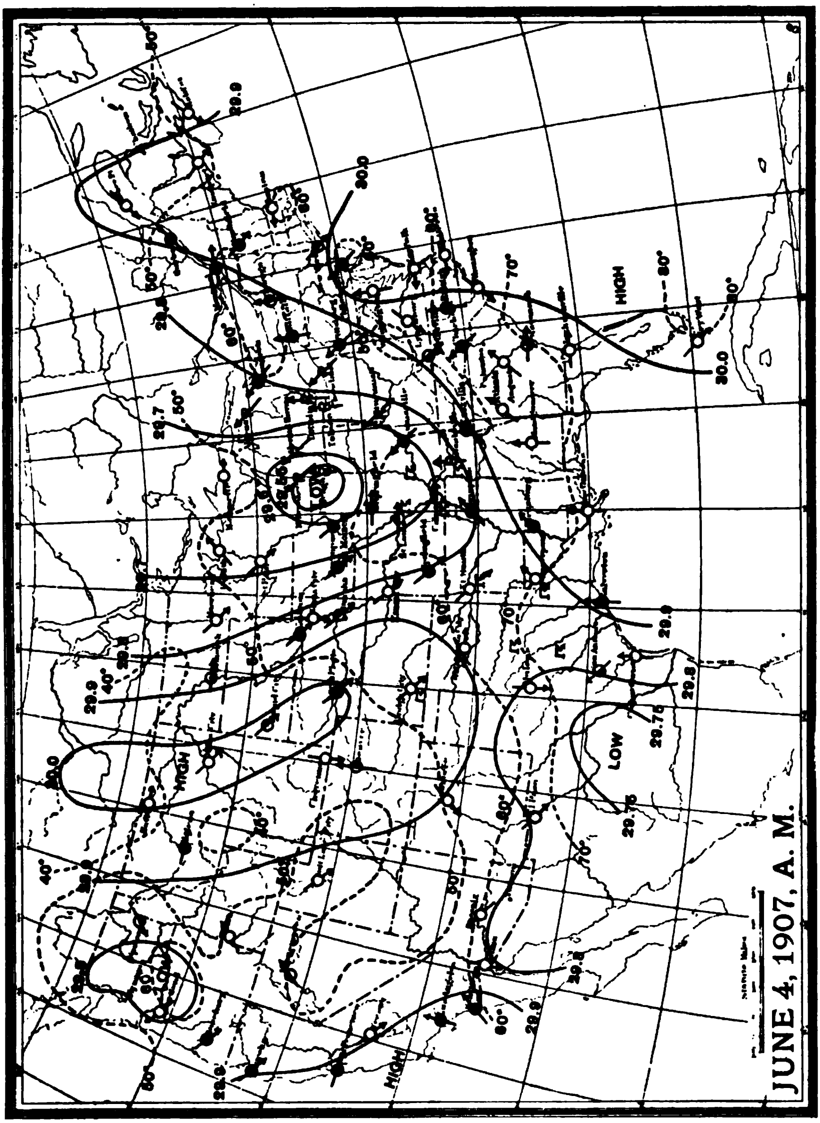
Weather map, 8 P.M., June 27, 1909, typical of "heat" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R, rain; R̄, thunderstorm.

FIG. 97.



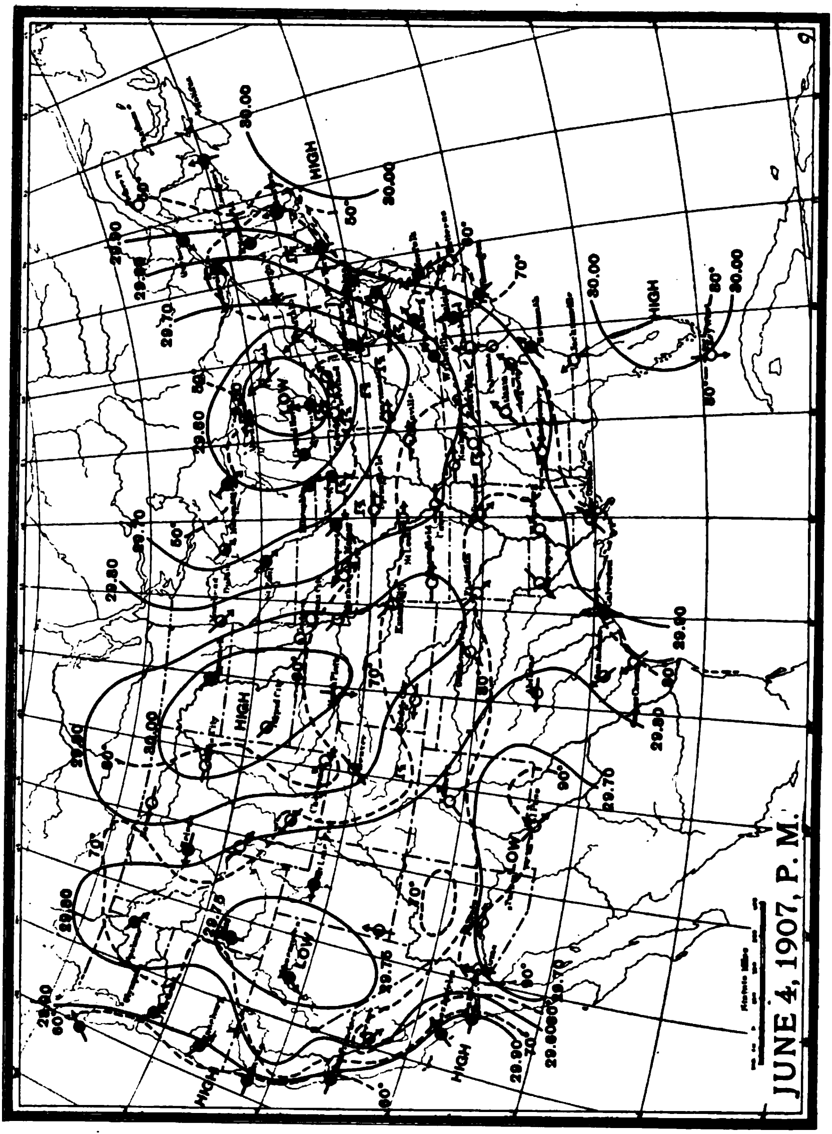
Weather map, 8 A.M., June 28, 1909, typical of conditions at decline of "heat" thunderstorms: O, clear; ◑, partly cloudy; ●, cloudy; R, rain; R̸, thunderstorm.

Fig. 98.



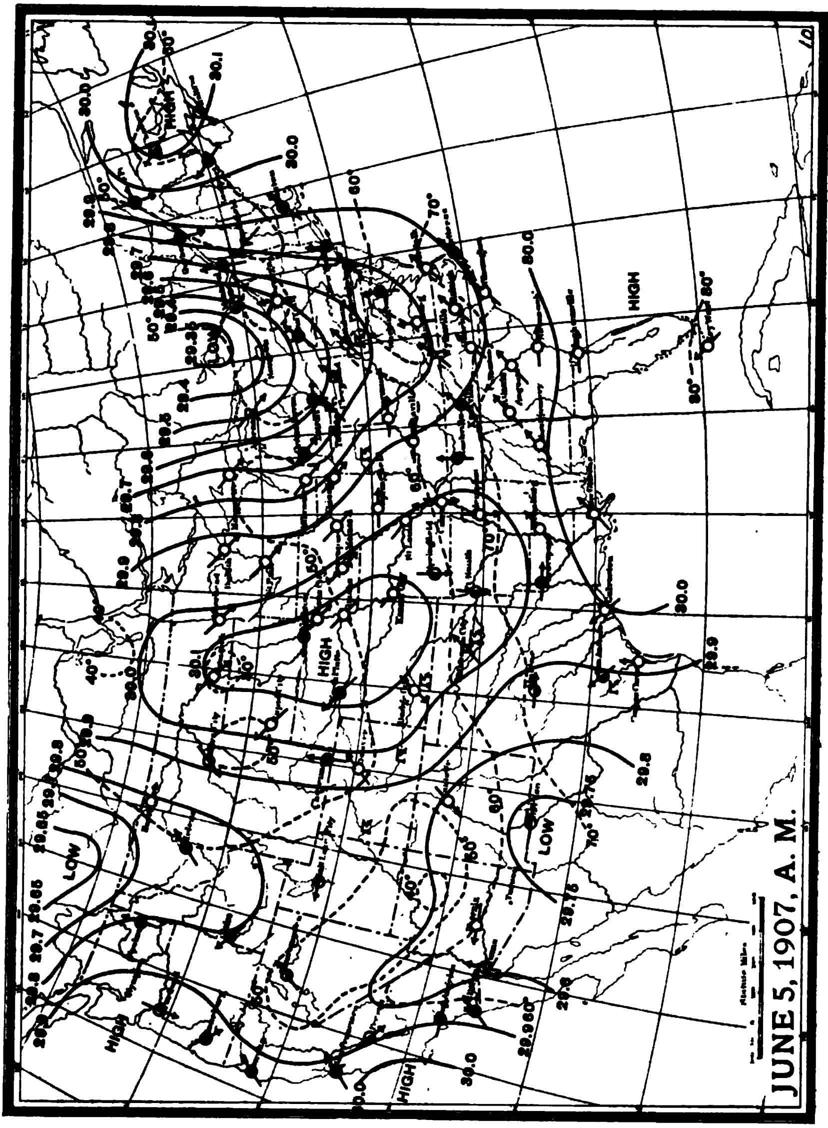
Weather map, 8 A.M., June 4, 1907, typical of conditions at beginning of "cyclonic" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R, rain; R̄, thunderstorm.

FIG. 99.



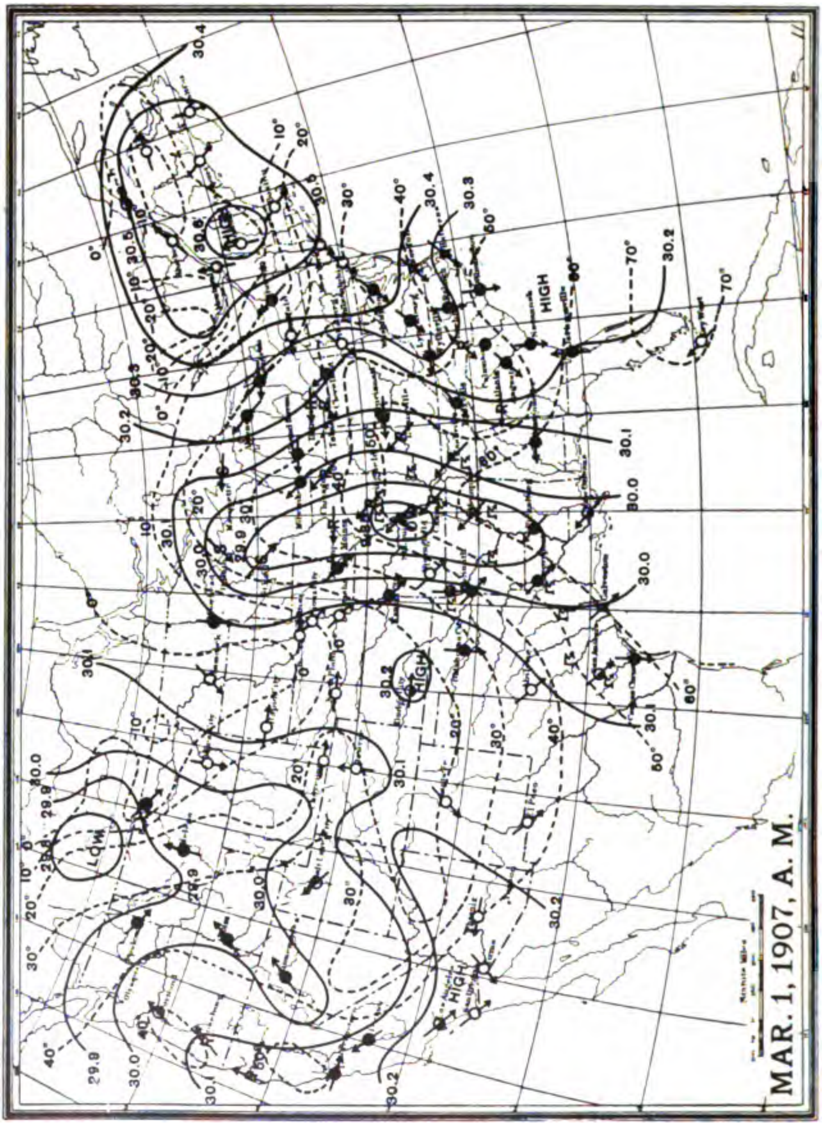
Weather map, 8 P.M., June 4, 1907, typical of "cyclonic" thunderstorms: ○. clear; ◐. partly cloudy; ●. cloudy; R. rain; R̄. thunderstorm.

FIG. 100.



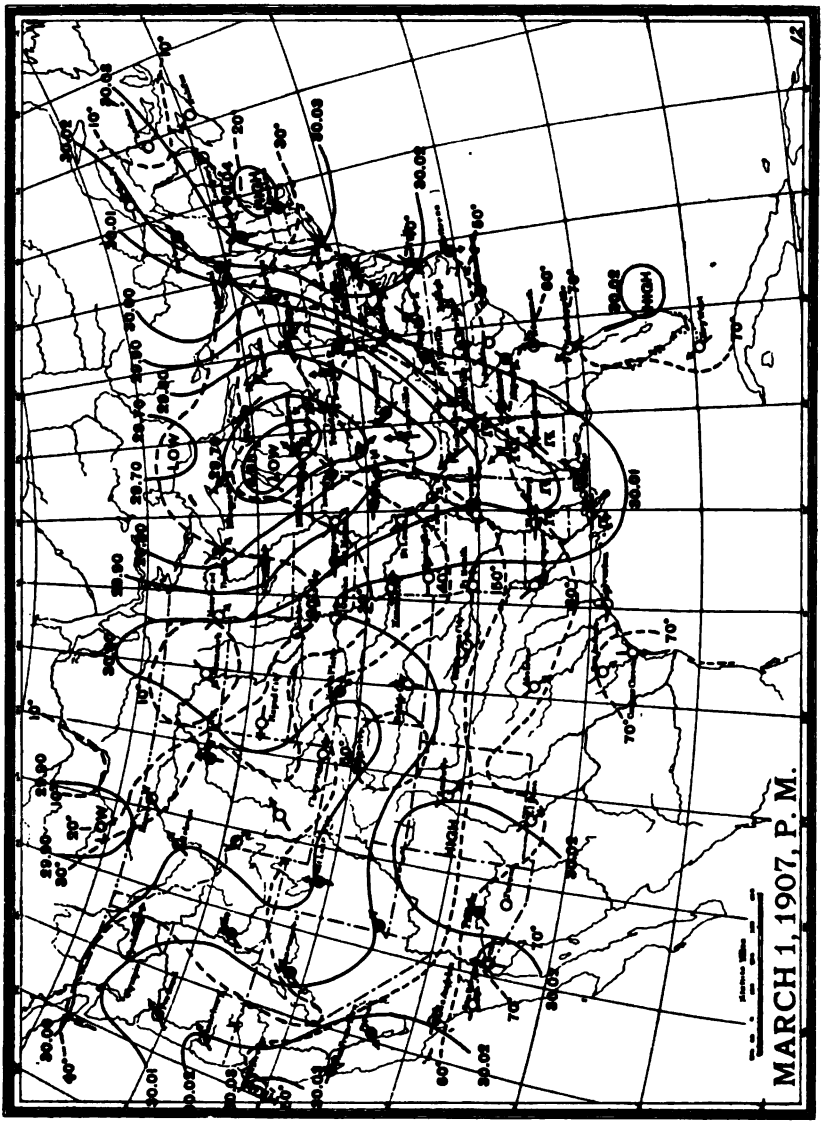
Weather map, 8 A.M., June 5, 1907, typical of conditions at decline of "cyclonic" thunderstorms: ○ clear; ◐, partly cloudy; ●, cloudy; R, rain; K, thunderstorm.

FIG. 101.



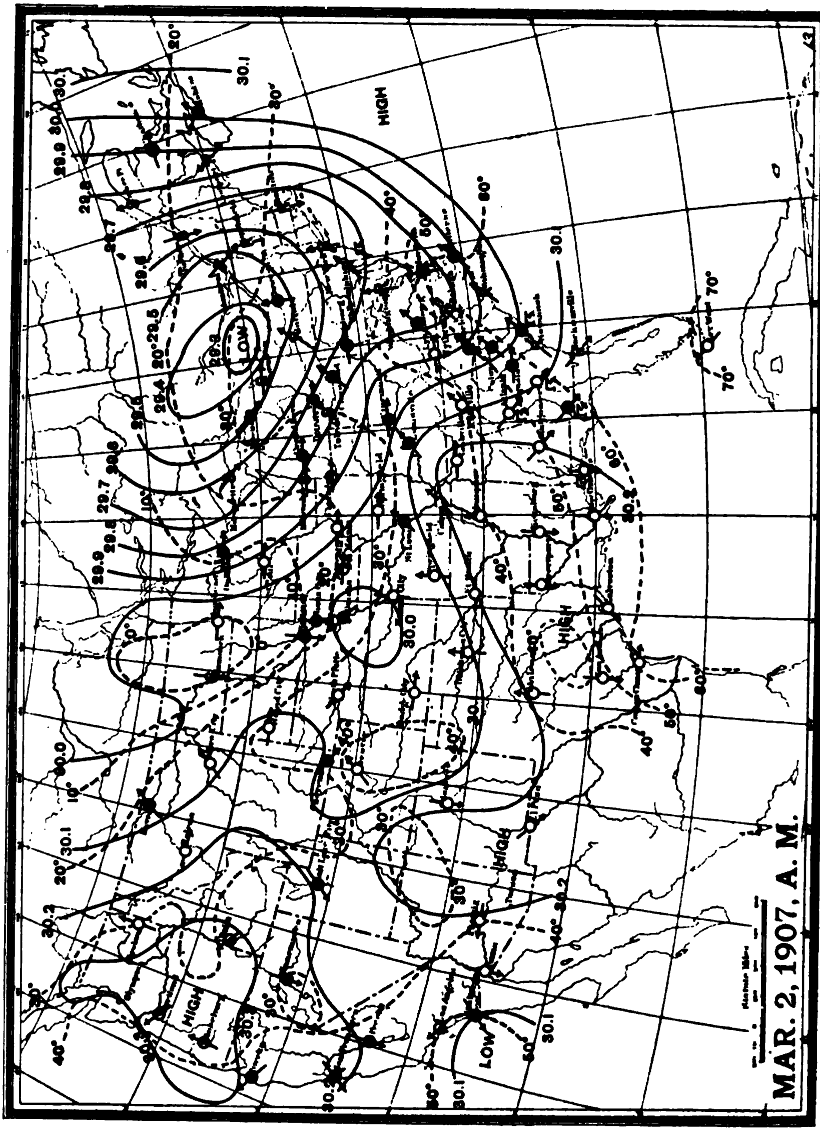
Weather map, 8 A.M., March 1, 1907, typical of conditions at beginning of "tornadic" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R, rain; K, thunderstorm.

FIG. 102.



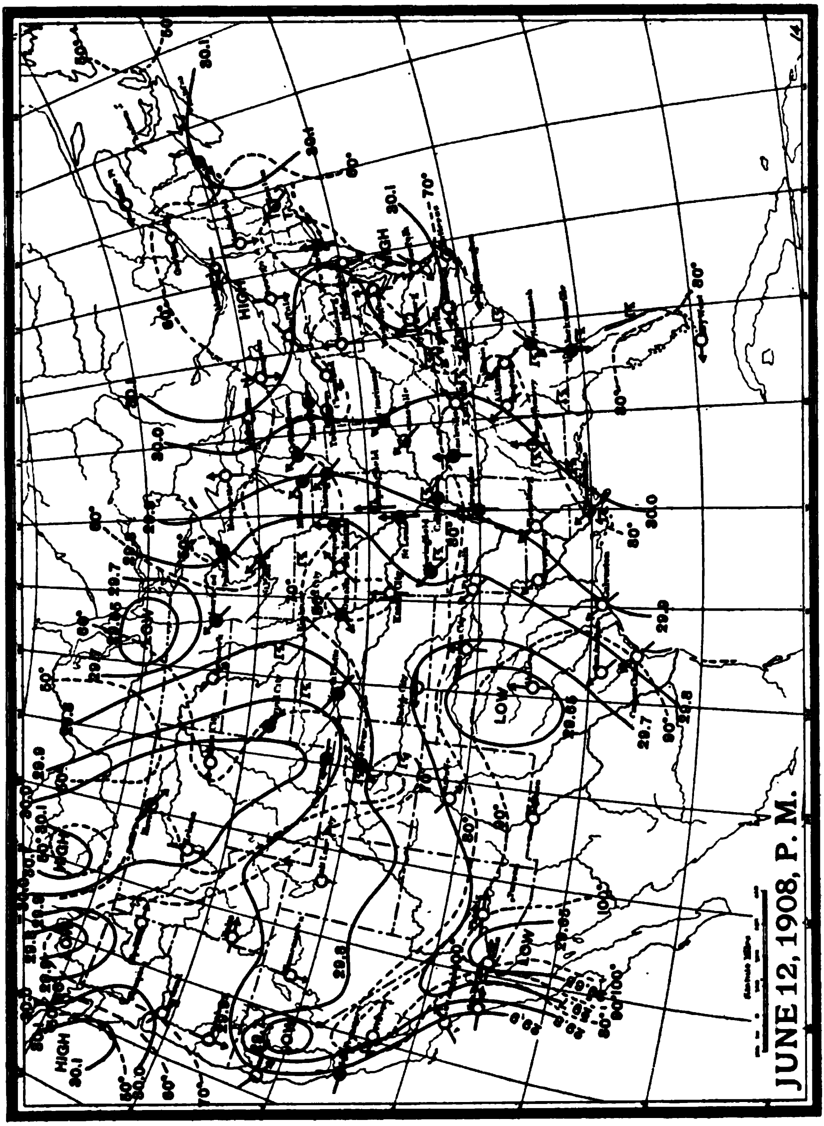
Weather map, 8 P.M., March 1, 1907, typical of "tornadic" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R, rain; K, thunderstorm.

Fig. 103.



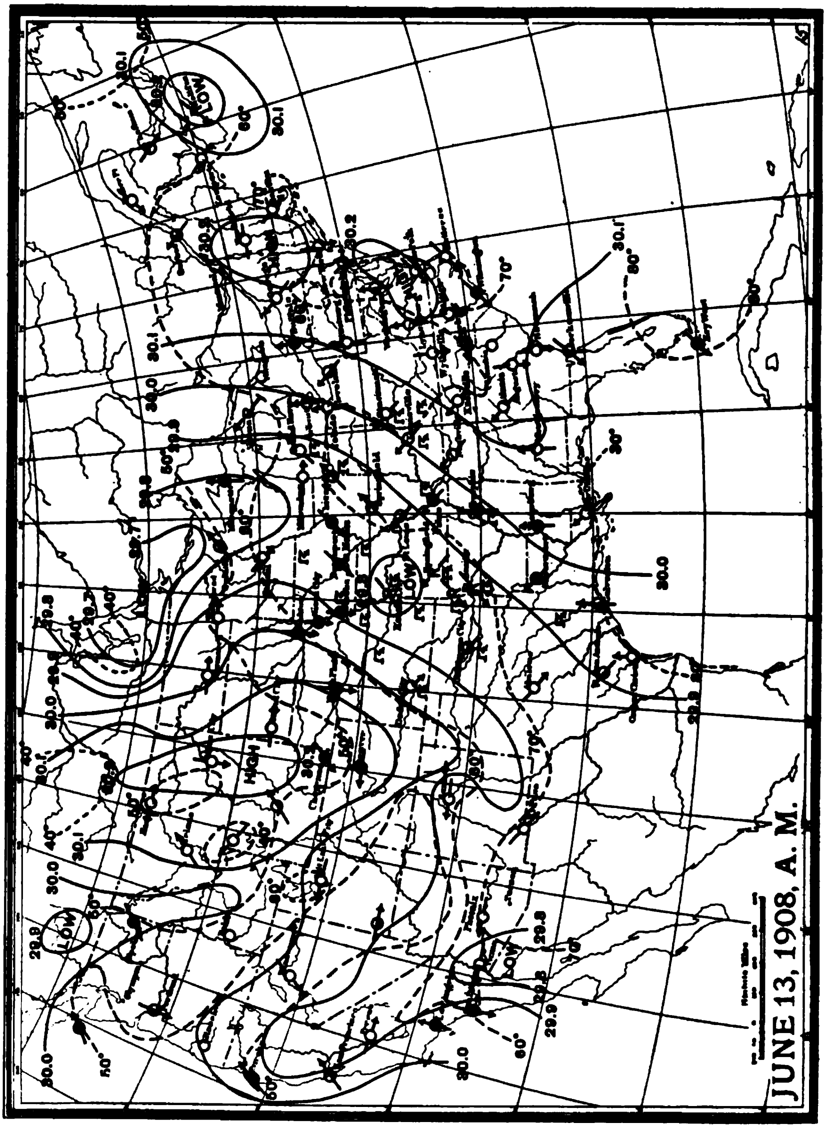
Weather map, 8 A.M., March 2, 1907, typical of conditions at decline of "tornadic" thunderstorms: O, clear; ◐, partly cloudy; ●, cloudy; R, rain; K, thunderstorm.

FIG. 104



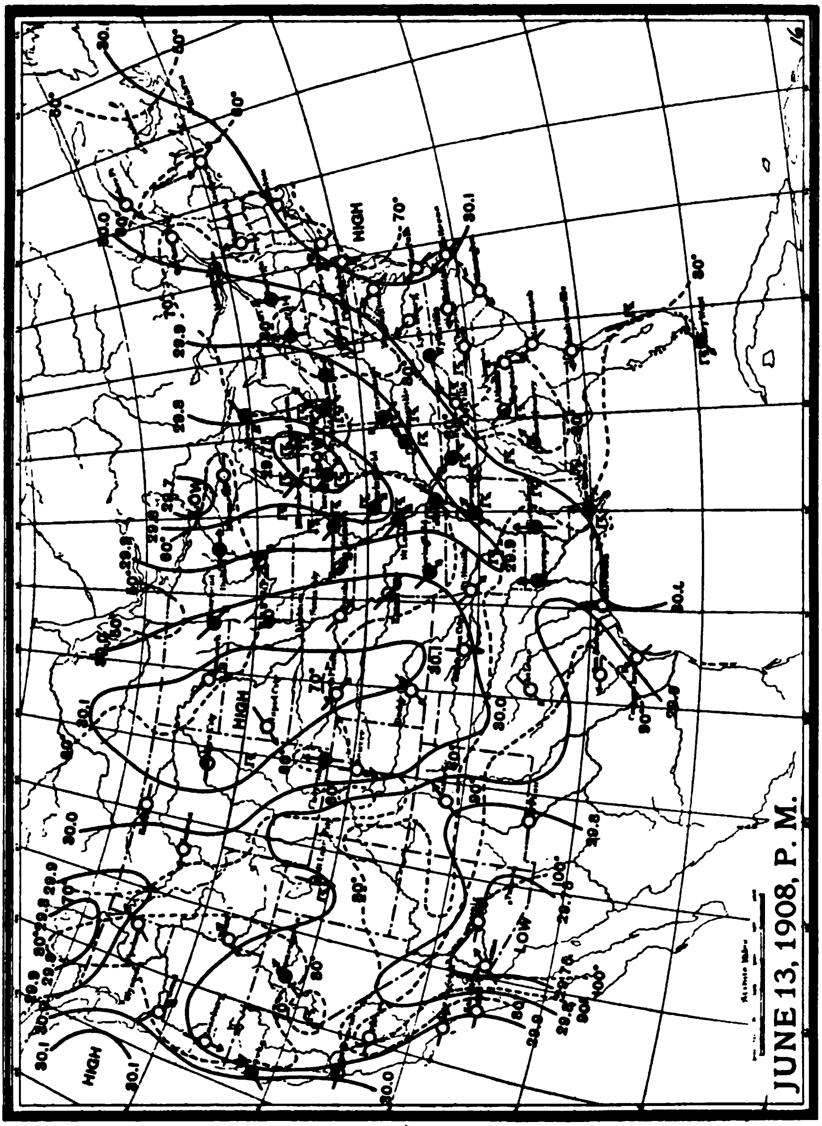
Weather map, 8 P.M., June 12, 1908, typical of conditions at beginning of "trough" thunderstorms: O, clear; ◐, partly cloudy; ●, cloudy; R, rain; K̅, thunderstorm.

Fig. 105.



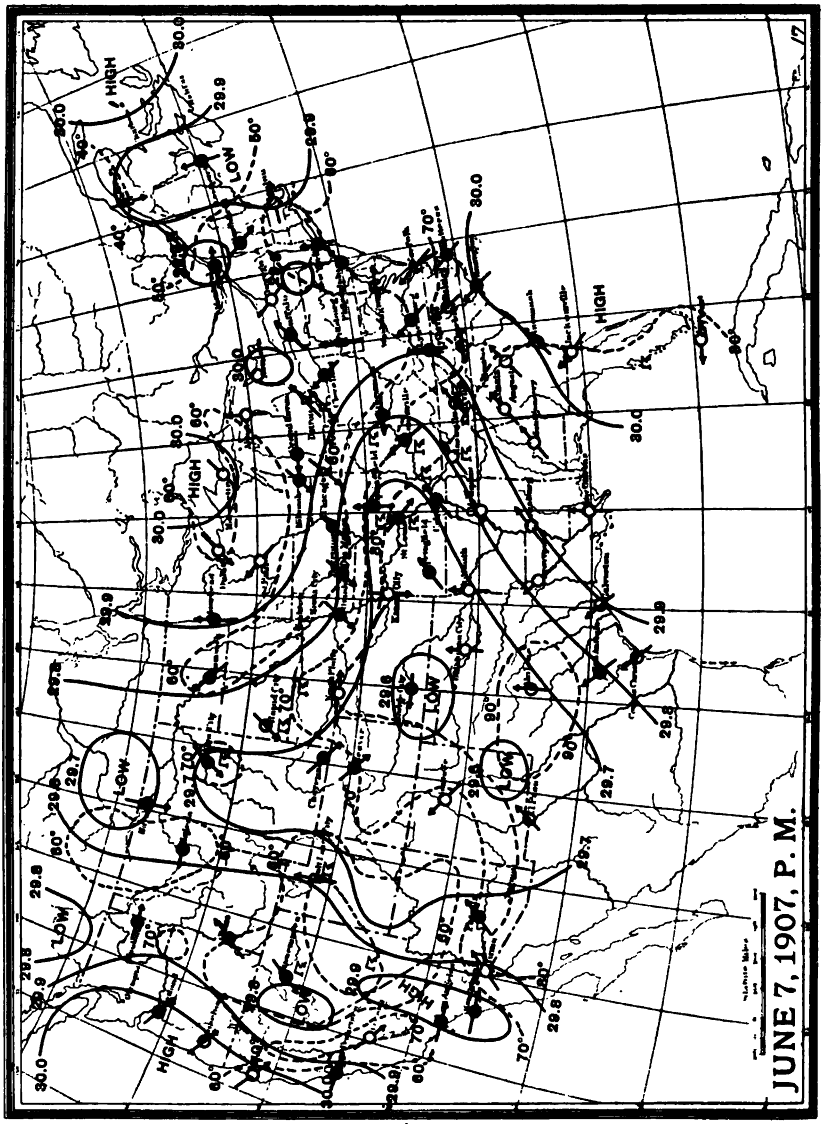
Weather map, 8 A.M., June 13, 1908, typical of "trough" thunderstorms: ○, clear; ◑, partly cloudy; ●, cloudy; R, rain; Ʀ, thunderstorm.

FIG. 106.



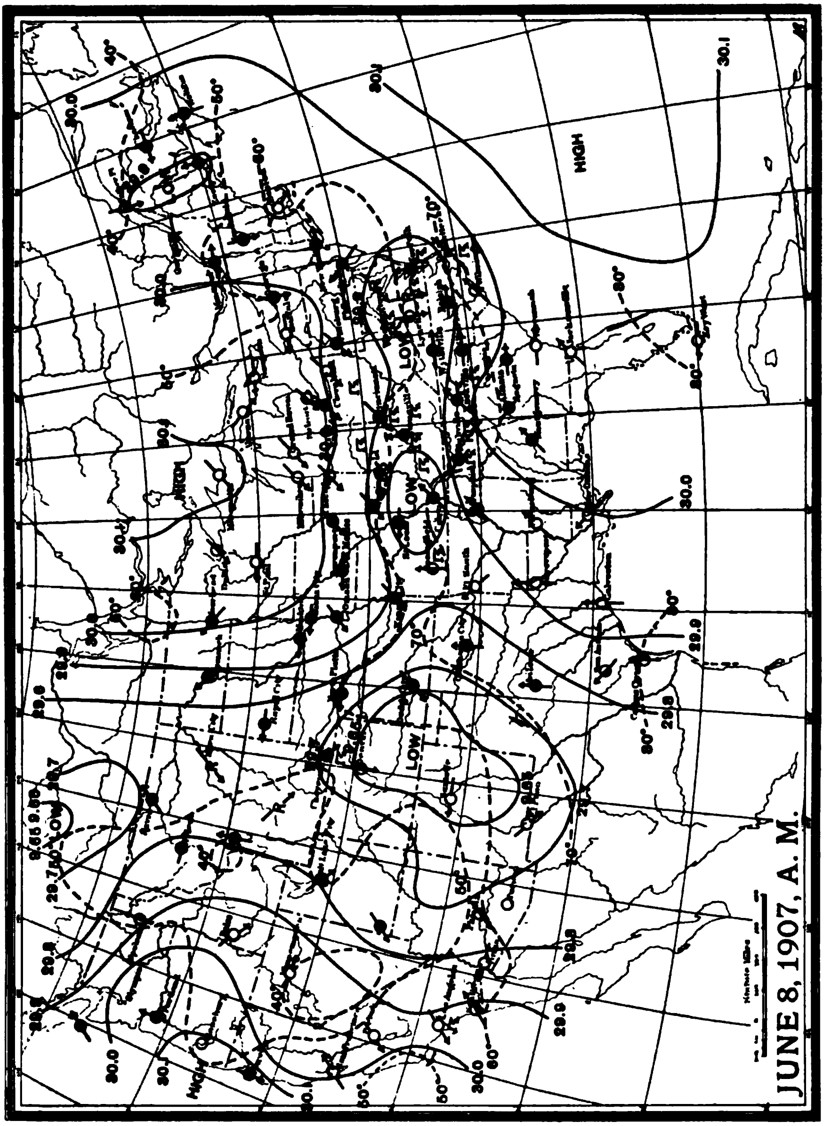
Weather map, 8 P.M., June 13, 1908, typical of conditions at decline of "trough" thunderstorms: O, clear; ◐, partly cloudy; ●, cloudy; R, rain; R̅, thunderstorm.

FIG. 107.



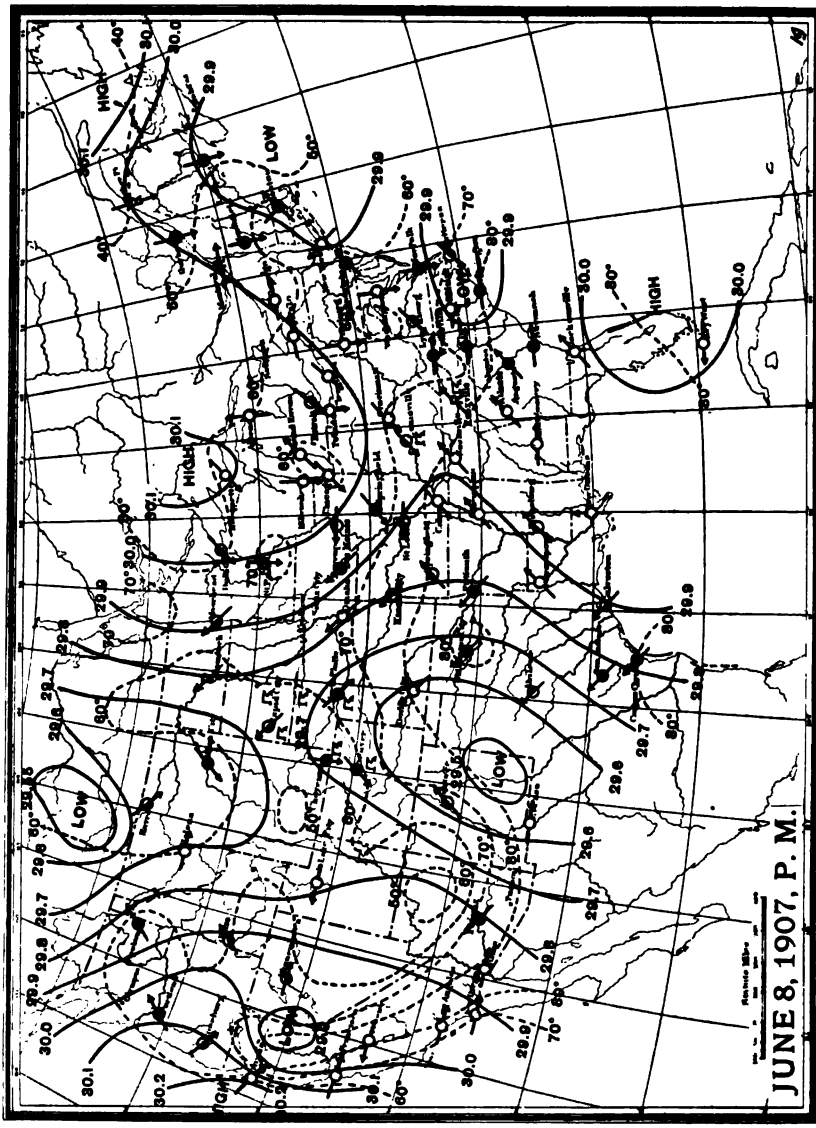
Weather map, 8 P.M., June 7, 1907, typical of conditions at beginning of "border" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R, rain; R̸, thunderstorm.

Fig. 108.



Weather map, 8 A.M., June 8, 1907, typical of "border" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R, rain; K, thunderstorm.

FIG. 109.



Weather map, 8 P.M., June 8, 1907, typical of conditions at decline of "border" thunderstorms: ○, clear; ◐, partly cloudy; ●, cloudy; R rain; K, thunderstorm.

because of the humidity, slightly below normal or, at most, but little above normal.

When the pressure is approximately uniform the winds are light, and therefore the turbulence and general mixing of the lower air practically negligible, hence every opportunity is given for the surface air to become strongly heated and thereby, finally, to establish vigorous local convections, with their consequent thunderstorms. Such storms, always favored by the drafts up the sides of mountain ranges, and particularly by those up steep mountain peaks and strongly heated valleys, are, of course, most frequent of summer afternoons, and are especially liable to occur at the end of two or three days of unusually warm weather, when the lower air has become so heated that convection extends to relatively great altitudes. They develop here and there sporadically, hence the name "*local*" thunderstorms; last, as a rule, only an hour or two, and travel but a short distance—those that form over mountain peaks often do not travel at all. They also frequently are referred to as "*heat*" thunderstorms, from the fact that, under the given conditions, the necessary initial convection is essentially, if not wholly, due to surface heating.

Local or heat thunderstorms seldom are especially violent and dangerous, and fortunately so, since they are exceedingly numerous, constituting, as they do, well-nigh the only type of thunderstorm in the tropics, and also, perhaps, the most common type in the warmer portions of the temperate zones.

b. The southeast quadrant (northeast, in the Southern Hemisphere), or less frequently, the southwest (northwest, in the Southern Hemisphere), of a regularly formed low, or typical cyclonic storm (see Figs. 98, 99, and 100).

In this case, the temperature gradient essential to a rapid vertical convection is not produced chiefly by surface heating, as it is during the genesis of "*heat*" thunderstorms, but, in great measure, results from the more or less crossed directions of the under- and over-currents of air, the under being directed spirally inward toward the region of lowest pressure and the over tending to follow the isobars. The surface air of the quadrant in question, therefore, normally flows from lower and warmer latitudes, while with increasing altitude the winds come more and more nearly from the west, or even northwest. This crossing of the air currents, then, the lower coming from warmer sections and

the upper from regions not so much warmer—possibly even colder—progressively increases the vertical temperature gradient, or rate of temperature decrease with increase of altitude, and therefore may frequently be, doubtless often finally is, the determining cause of rapid vertical convection and the formation of a thunderstorm.

This particular type of thunderstorm, commonly known as the "*cyclonic*" thunderstorm, is almost wholly confined to the temperate and higher zones, for the simple reason that tropical cyclones, themselves of infrequent occurrence, seldom can have the necessary temperature contrast between its winds of different directions, since all come from equally warm regions. Nevertheless, thunderstorms do occur in connection with many, perhaps nearly all, tropical cyclones. They occur, however, either incidentally during the development of such disturbances or else, later, along their borders and not, or but rarely, within the body of strong winds, and belong, therefore, to the "*heat*" variety rather than to the "*cyclonic*."

c. The barometric valley between the branches of a distorted or V-shaped cyclonic isobar (see Figs. 101, 102, and 103).

This region is also favorable to the formation of secondary lows, which, though often of small area, sometimes are very intense, even to the genesis of the tornado, the severest of all storms.

Just how specific examples of this type of pressure distribution originated may not always be clear, but, however established, such distribution necessarily leads to opposing surface winds, since these always blow inward at an angle to the isobars, and also to more or less oppositely directed neighboring upper currents, a kilometre or more above the surface, where they tend rather closely to follow the isobars. The winds of each level, the lower and the upper, tend to produce independent effects. Thus the opposing or conflicting surface winds cause such an irregular mixing of the air and such over- and under-running of currents as is likely to establish, here and there, a convection of thunderstorm magnitude. Hence, presumably, the frequency of thunderstorms along the valleys of low-pressure basins. On the other hand, the oppositely directed adjacent, not conflicting, but swift upper currents, on being deflected or drawn together by the convection of the rising current, have their radii of curvature so changed as mechanically to produce, in the middle atmosphere,

violent vortices of limited cross-section, in conformity with the well-known law of the conservation of areas; that is, in conformity with the fact that the angular velocity, or rate of spin, varies inversely with the square of the radius of curvature. The more violent of these vertical atmospheric whirls, accompanied by thunder and rain and often extending down to the surface of the earth, where they become destructive, are known as tornadoes. Hence thunderstorms generated in the barometric region under discussion, the region in which tornadoes most frequently originate and develop, might properly be called "*tornadic*" thunderstorms.

Atmospheric conflicts and turmoil of the nature just described obviously may occur at any place along the protrusion, or valley, of the low-pressure basin, and therefore often do occur, even simultaneously, here and there, along its entire length, and together form the well-known "line squall." Besides, as the whole cyclonic condition moves forward in general from west to east, maintaining, in a measure, for many hours its identity of form and nature, it follows that its valley of low pressure, and therefore its line of thunderstorms, must also travel with it in the same general direction and with approximately the same velocity.

A line or row of thunderstorms—a "line squall"—as observations show, always moves across its own axis, not necessarily at right angles, but nevertheless across and not parallel to it, nor even approximately so. The chief reason for this is not the axial direction of the low-pressure valley, which, indeed, though usually running south, may have any orientation from the parent basin, but rather the fact that the valley itself, together with its accompanying thunderstorm conditions, travels across and not along its own direction.

In this connection it is also worth noting that the temperature distribution in the wake of a thunderstorm renders the occurrence of an immediate successor improbable, as will be explained later. Hence, while a considerable number of thunderstorms may and often do travel abreast, they can never follow each other closely in file.

d. The region covered by a low-pressure trough between adjacent high-pressure areas (Figs. 104, 105, and 106).

Along the adjacent borders of two neighboring anticyclones—that is, along the barometric trough between them—the surface

winds from one side are more or less directly opposed to those of the other, each flowing spirally outward from the region of higher pressure. Hence, because of the overrunning, as explained under *c,* and the resulting temperature gradients, this also is a region of frequent thunderstorms. Here, too, a number of more or less independent storms may exist simultaneously along the same line, and advance abreast for considerable distances across the country.

There does not appear to be any independent or distinctive name for the thunderstorm generated under this type of pressure distribution. Perhaps it might, with some justification, be called the "*anticyclonic*" thunderstorm, or even the "trough" storm.

e. The boundary between warm and cold waves (see Figs. 107, 108, and 109).

Along such a boundary the direction of flow of the warm, humid layers of air is more or less opposite, as shown on the maps, to that of the colder ones. Therefore it must frequently happen that at irregular intervals along such a boundary upper air coming from the cold area overruns a section of surface air belonging to the warm region. Now, wherever this overrunning on the part of the cold air does occur the vertical temperature gradient is likely to be abruptly and greatly increased, and wherever, as a result of either the initial overflow or of further movement, the new gradient exceeds the adiabatic rate of temperature change, as, analogous to case *b,* it often must, under the given conditions, vertical convection, with rain, thunder, and lightning, is apt to occur. Hence, as stated, the boundary between warm and cold waves is another place favorable to the genesis of the thunderstorm. Here, too, as in cases *c* and *d* above, a characteristic name is lacking. Possibly with reference to its place of occurrence—that is, along the boundary between warm and cold waves—one might call it the "*border*" storm.

The above five types of weather conditions, together with their innumerable variations and combinations, probably include all that are distinctly favorable to the development of thunderstorms. Each tends to establish an adiabatic, or even superadiabatic, temperature gradient up to the cloud level, the one thing essential to the production of strong vertical convection, the progenitor, as we have seen, of the thunderstorm.

Thunderstorm Winds.—Shortly, say 20 minutes or so, before

the rain of a thunderstorm reaches a given locality the wind at that place, generally light, begins to die down to an approximate calm and to change its direction. At first it usually is from the south or southwest in the northern hemisphere; from the north or northwest in the southern, and in both more or less directly across the path of the storm itself. When the change is complete, it blows for a few minutes, rather gently, directly toward the nearest portion of the storm front, and finally, as the rain is almost at hand, abruptly and in rather violent gusts, away from the storm and, because it has come from above, as will be explained later, in about the direction that the storm is traveling, a direction that, in most cases, differs appreciably from that of the original surface wind. Usually this violent gusty wind lasts through only the earlier portion of the disturbance and then is gradually but rather quickly succeeded by a comparatively gentle wind, that though following the storm at first, frequently, after an hour or so, blows in the same general direction as the original surface wind.

The cause of the thunderstorm winds needs to be carefully considered if one would understand at all clearly the mechanism of the storm itself.

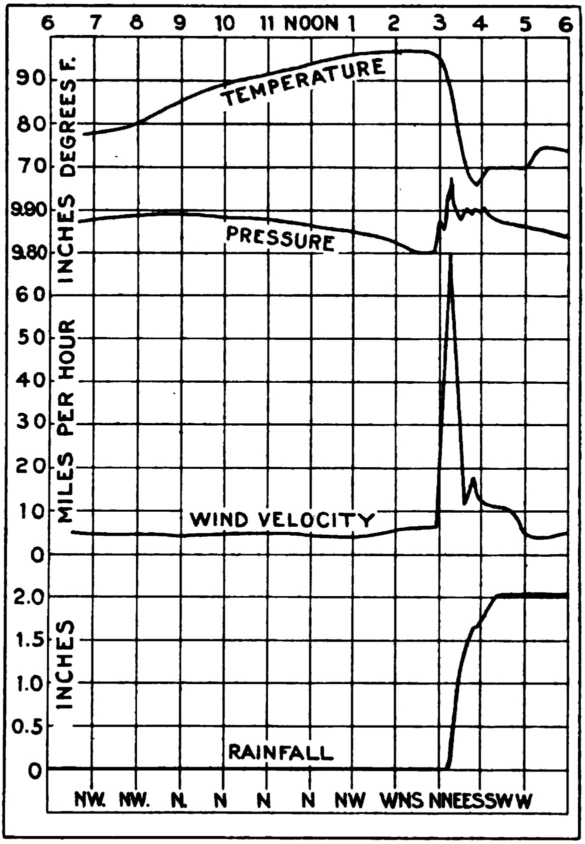
As already explained, this type of storm owes its origin to that vertical convection which results from a more or less super-adiabatic temperature gradient. It is this gradient, no matter how established, whether by simple surface heating or by the over and under running of layers of air of widely different temperatures, that permits, or rather forces, the production of the cumulus cloud in which and by the motions of which the electricity that characterizes the thunderstorm is generated.

Nevertheless, as everyone knows, the passage of a cumulus cloud overhead, however large, so long as no rain is falling from it, does not greatly affect the direction and magnitude of the surface wind—does not bring on any of the familiar gusts and other thunderstorm phenomena. Hence, somehow or other, the rain is an important factor, both in starting and in maintaining the winds in question, for they do not exist before the rain begins nor continue after it has ceased. On the other hand, it cannot be assumed that the rain is the whole cause of these winds, for they do not accompany other and ordinary showers, however heavy the downpour may be.

The actual course of events, illustrated by Fig. 110, taken from the records obtained at Washington, D. C., during the passage of the notable thunder squall of July 30, 1913, seems to be about as follows:

First.—An approximately adiabatic temperature gradient pre-

FIG. 110.



Course of meteorological elements on a thunderstorm day. (Washington, D. C., July 30, 1913.)

sumably is established over a wide area, roughly up to the base level of the cumulus clouds, all of which, because of a practically common temperature and common humidity over the whole region, must have substantially the same base level and therefore often appear *en échelon*, as shown in Fig. 111. But while the uprising branches of the existing convection currents, due to super-

FIG. III.

Cumuli *en échelon*. (Loudon Valley, Va., E. B. Calvert, photo.)

adiabatic gradients, may be localized and here and there rather rapid, the return or compensating down-flow is relatively wide-spread and correspondingly gentle. The condition essential to a local and rapid down-flow, that is, a local decided cooling at a high altitude, does not exist, and therefore the counterpart to the upward current is nowhere conspicuous.

Second.—The convections in the cumuli are accelerated by virtue of the latent heat of vaporization set free in them, and thus

FIG. 112.

Towering cumulus. (West end of Java, E. E. Barnard, photo.)

one or more of them rapidly developed. In some cases great size and remarkable altitudes are attained, as illustrated by Fig. 112.

Third.—After a time, as a result of the abundant condensation induced by the convectional cooling, rain is formed at a considerable altitude where, of course, the air is quite cold, in fact so cold that often hail is produced. Now this cold rain, or rain and hail, as it falls, and as long as it falls, chills the air from the level of its formation all the way to the earth, partly as a result of its

initial low temperature and partly because of the evaporation that takes place during its fall. Hence this continuously chilled column of air, because, partly, of the frictional drag of the rain, but mainly because of the increase, due to this chilling, of its own density, immediately and necessarily becomes a concentrated and vigorous return branch of the vertical circulation. In fact, it (or gravity acting through it) becomes the sustaining cause of the storm's circulation. At the same time, because of the downward blow and because of the retardation of the winds by surface friction, the barometric pressure is abruptly increased, as will be explained later.

It will be worth while to consider some of these statements a little more closely, and to test them with possible numerical values.

Omitting, as one may, the effects of radiation, there seem to be but three possible ways by which the cooling of a thunderstorm may be obtained: (*a*) by the descent of originally potentially cold air; (*b*) by chilling the air with the cold rain; (*c*) by evaporation. Each of these will be considered separately.

(*a*) Obviously no portion of the upper air could maintain its position if potentially even slightly colder than that near the surface, that is, so cold that even after warming up adiabatically in a fall to the surface it still would be colder than the air displaced. If at all potentially colder it would fall until it itself became the surface air. Hence the great decrease in temperature that comes with a thunderstorm is not the result of the descent of a layer of air originally potentially cold for, as explained, an upper layer sufficiently cold to give, after its descent, the actual cooling could not exist. Again, any descending air must come from either below the under surface of the cloud or from above this level. If from below, then, because of adiabatic heating during its descent through air which, as above explained, has practically the adiabatic temperature gradient, it must reach the earth at substantially the original surface temperature. If from above it would, as is obvious from Fig. 90, reach the earth even warmer than the original surface temperature. Hence, looked at in any way, case *a* clearly is inadmissible.

Possibly the above statements may seem to contravene the explanation that many thunderstorms originate in the establishment by cross currents of superadiabatic temperature gradients.

In reality, however, they are in harmony with that explanation which is based on the fact that such gradients can not be maintained, but must at once cause vertical convection. Besides, such mechanically established gradients merely initiate but do not, as we shall see, maintain the storm.

(*b*) Let the under surface of the thunderstorm cloud be 1500 metres above the earth, and the column of air cooled by the cold rain and its evaporation, 2000 metres high. Let the surface temperature be 30° C., and the temperature gradient before the storm begins adiabatic up to the under-cloud level, and let there be a 2-centimetre rainfall.

Now at the temperature assumed, a column of air 2000 metres high whose cross section is 1 square centimetre, and whose base is at sea level, weighs, roughly, 210 grams, and its heat capacity, therefore, is approximately that of 50 grams of water. At the top of this column the temperature can be, at most, only about 20° C. lower than at the bottom, corresponding to the adiabatic or maximum temperature gradient, and if the rain leaves the top at this temperature but reaches the earth 7° C. colder than the surface air before the storm (temperatures that seem at least to be of the correct order), it will have been warmed 13° C. during its fall and the air column, at the expense of whose heat this warming was produced, cooled, on the average, about 0.5° C. But, as a matter of fact, the air usually is cooled 5° C. to 10° C. Hence, while the temperature of the air necessarily is reduced to some extent by mere heat conduction to the cold rain, much the greater portion of the cooling clearly must have some other origin. Further, since *a* is inadmissible and *b* only a minor contributing factor, it follows by exclusion that of the three obvious causes only evaporation is left to account for much the greater portion of the cooling. Consider, then, whether evaporation really can produce the effects observed.

(*c*) It is a common thing in semiarid regions to see a heavy shower, even a thunder shower, leave the base of a cloud and yet fail utterly to reach the surface of the earth. Hence, it appears quite certain that in the average thunderstorm a considerable portion of the rain that leaves the cloud may evaporate before it reaches the ground, and therefore that the temperature decrease of the atmosphere may largely be owing to this fact. But if so,

why then, one properly might ask, does not an equally great temperature drop accompany all heavy rains?

The answer is obvious: It is because, as a rule, the temperature is higher, the relative humidity lower, and the temperature gradient more nearly adiabatic during a thunderstorm than at the time of an ordinary rain. Other rains, those that are accompanied by long horizontal and slow, rather than rapid upward movements of the air, begin only when the humidity is so high that but little evaporation and therefore but little cooling from this source can take place. In such rains there is nothing that can greatly increase the density of the air and consequently there is no rapidly descending current or wind. Thunderstorms, on the other hand, are developed by strong vertical convection which establishes a nearly adiabatic gradient and when the relative humidity, in the case of the heat thunderstorm, at least, is low, 50 per cent., say. Evaporation into this air, as soon as the rain has begun, obviously must be rapid, with the consequent cooling and increase of density correspondingly great. Hence, since the temperature gradient was already nearly adiabatic, a strong downward current necessarily is established in the midst of the falling and evaporating rain. Further, whatever the type of thunderstorm, the descending air, which can be no more than saturated at the base of the cloud, dynamically warms so rapidly that evaporation into it can not keep pace with its water capacity. That is, evaporation which takes place all the way from cloud to earth, by rendering the air locally cool and dense, causes it to fall, while this fall, in turn, through dynamical heating, maintains the evaporation. Hence the downrush of the air must continue so long as there is an abundant supply of local rain, and cease when the rain becomes light.

It will be instructive now to return to the numerical values and compute a probable magnitude of the cooling due to evaporation. As before, let a 2-centimetre rain leave the cloud, but let one-fourth of the rain that started, or half a centimetre, be evaporated. This would consume 303 heat units from an air column 2000 metres high whose heat capacity is that of only 50 cubic centimetres of water. Hence, as a result of evaporation alone, the temperature of the air column would be lowered on the average by about 6° C. Evaporation, therefore, appears to be both necessary and sufficient to produce all or nearly all the cooling of a thunderstorm.

But what is the effect of this evaporation on the density of the atmosphere? Since the molecular weight of water is 18 while the average molecular weight of air is approximately 28.9, it follows that the amount of evaporation above assumed would decrease the density of the atmosphere by, roughly, one part in a thousand. On the other hand, a decrease in temperature of 6° C., that would be produced by the evaporation assumed, would increase it by about one part in fifty. Hence the resultant of these two opposing effects is substantially that of the second alone; that is, a distinct increase in density, and a consequent downrush of cold air.

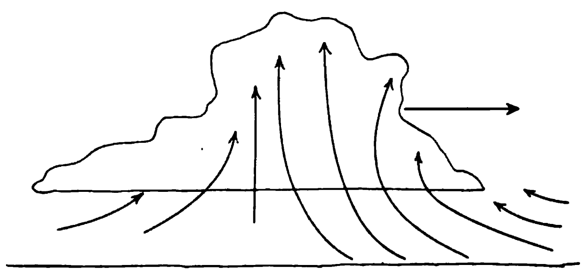
Doubtless, as already implied, the evaporation of thunderstorm rain, and therefore the drop in temperature and the consequent fractional gain in density, all increase with decrease of elevation. In some measure, however, this effect is counteracted by the higher temperatures of the lower layers—the higher the absolute temperature the less, proportionately, the change of density per degree change of temperature. But no matter how nor to what extent the details may vary, it seems quite certain that the cold rain of a thunderstorm and its evaporation together must establish a local downrush of cold air, an observed important and characteristic phenomenon, really the immediate cause of the vigorous circulation, whose rational explanation has been attempted in the past few paragraphs.

As the column or sheet of cold air flows down it maintains in great measure its original horizontal velocity and, therefore, on reaching the earth rushes forward in the direction of the storm movement, underrunning and buoying up the adjacent warm air. And this condition, largely due, as explained, to condensation and evaporation, once established necessarily is self-perpetuating, so long as the general temperature gradient, humidity, and wind direction are favorable. It must be remembered, however, that thunderstorm convection, rising air just in front of, and descending air with, the rain, does not occur in a closed circuit, for the air that goes up does not return nor does the air that comes down immediately go up again, there simply is an interchange between the surface air in front of the storm and the upper air in its rear. The travel of the storm, by keeping up with the under-running cold current, just as effectually maintains the temperature contrasts essential to this open-circuit convection

as does continuous heating on one side and cooling on the other maintain the temperature contrast essential to a closed circuit convection.

The movements of the warm air in front of the rain, the lull, the inflow, and the updraft, resemble somewhat those of a horizontal cylinder resting on the earth where the air is quiet and rolling forward with the speed of the storm. Similarly, the cold air in its descent and forward rush, together with the updraft of warm air, also resembles a horizontal cylinder, but one sliding on the earth and turning in the opposite direction from that of the forward-rolling or all-warm cylinder. In neither case, however, is the analogy complete, for, as above explained, the air that goes up

FIG. 113.



Principal air movements in the development of a cumulus cloud.

remains aloft, while the cold air that comes down is kept by its greater density to the lower levels. The condition of flow persists, as do cataracts and crestclouds (clouds along mountain crests), but here, too, as in their case, the material involved is ever renewed.

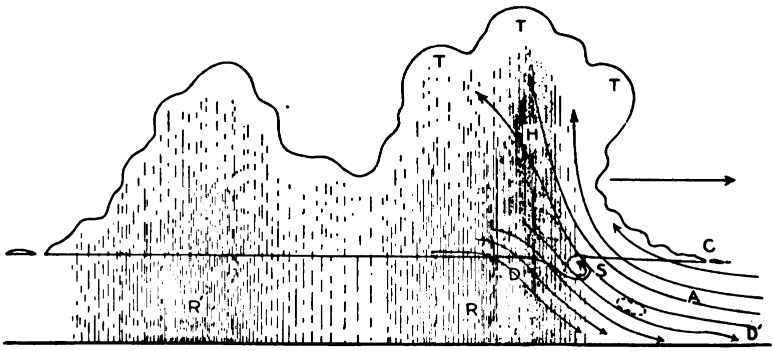
The Squall Cloud.—Between the uprising sheet of warm air and the adjacent descending sheet of cold air horizontal vortices are sure to be formed in which the two currents are more or less mixed. The lower of these vortices can only be *inferred* as a necessary consequence of the opposite directions of flow of the adjacent sheets of warm and cold air, for there is nothing to render them visible. Neither can any vortices that may exist within the cloud be seen. Near the front lower edge of the cumulo-nimbus system, however, and immediately in front of the sheet of rain, or rain and hail, the rising air has so nearly reached its dew point that the somewhat lower temperature pro-

duced by the admixture of the descending cold air is sufficient to produce in it a light fog-like condensation which, of course, renders any detached vortex at this position quite visible.

This squall cloud, in which the direction of motion on top is against the storm, may be regarded as a third horizontal thunderstorm cylinder much smaller but more complete than either of the others.

Schematic Illustrations.—The above conceptions of the mechanism of a thunderstorm can, perhaps, be made a little clearer with the aid of illustrations. Fig. 113, a schematic representation of a thunderstorm in the making, gives the boundary of a large

Fig. 114.



Ideal cross-section of a typical thunderstorm. *A*, ascending air; *D*, descending air; *C*. storm collar; *S*, roll scud; *D'*, wind gust; *H*, hail; *T*, thunderheads; *R*, primary rain; *R'*, secondary rain.

cumulus cloud from which rain has not yet begun to fall, and the stream lines of atmospheric flow into it. When the cloud is stationary and there is no surface wind the updraft obviously will be more or less symmetrical about a vertical through its centre, but when it has an appreciable velocity, as indicated in the figure, it is equally obvious that most, often nearly all, of the air entering the cloud will do so through its front under-surface. At this stage there will be no concentrated or local down current, only counter settling of the air round about, because, as previously explained, the air cataract requires strong local cooling, and this, in turn, calls for local rain.

Fig. 114 schematically represents a well-developed thunderstorm in progress. The rain, often mixed with hail, cools the air through which it falls, both by conduction and evaporation, the

hail also by fusion, and as the temperature gradient over a considerable area already was closely adiabatic it follows that the actual temperatures within the rain column must be lower than those of the surrounding air at corresponding levels all the way from the surface of the earth to within the cloud, that is, throughout and a little beyond the nonsaturated or evaporating region. As soon, then, as this column or sheet of air is sufficiently cooled it flows down and forward and all the atmospheric movements peculiar to the thunderstorm are established substantially as represented.

Referring to the figure: The warm ascending air is in the region *A;* the cold descending air at *D;* the dust cloud (in dry weather) at *D′;* the squall cloud at *S;* the storm collar at *C;* the thunder heads at *T;* the hail at *H;* the primary rain, due to initial convection, at *R;* and the secondary rain at *R′*. This latter phenomenon, the secondary rain, is a thing of frequent occurrence and often is due, as indicated in the figure, to the coalescence and quiet settling of drops from an abandoned portion of the cumulus in which and below which winds and convection are no longer active.

The thunderstorm is also frequently accompanied by false cirri, occasionally by scarf clouds and even, though rarely, by mamato-cumuli; but, as none of these is essential to it, all, therefore, are omitted from the above schematic illustration.

Thunderstorm Pressures.—Before the onset of a thunderstorm there usually if not always is a distinct fall in the barometer. At times this fall is extended over several hours, but whether the period be long or short the rate of fall usually is greatest at the near approach of the storm. Just as the storm breaks, however, the pressure rises very rapidly, usually from 1 to 2 millimetres, fluctuates irregularly, and finally, as the storm passes, again becomes rather steady but usually at a somewhat higher pressure than prevailed before the rain began.

The cause of these pressure changes is rather complex. The decrease in the absolute water vapor of the air as a whole, measured by the condensation, and the decrease in the temperature of the lower air—perhaps more than offset by the latent heat set free in the upper—both tend to increase the atmospheric pressure, and each contributes its share to the final result. Both these effects, however, are comparatively permanent, and while they

may be mainly responsible for the increase of pressure after the storm has gone by, they probably are not the chief factors in the production of the initial and quickly produced pressure maximum. Here at least two factors, one obvious, the other inconspicuous, are involved. These are: (*a*) the rapid downrush of air, and (*b*) the interference to horizontal flow caused by the vertical circulation.

The downrush of air clearly produces a vertically directed pressure on the surface of the earth, in the same manner that a horizontal flow produces a horizontally directed pressure against the side of a house. But a pressure equal to that given by 2 mm. of mercury, a pressure increase frequently reached in a thunderstorm, would mean about 2.72 grams per square centimetre, or 27.2 kilograms per square metre, and require a wind velocity of, roughly, 50 kilometres per hour or 14 metres per second. Now, the velocity of the downrush of air in a thunderstorm is not at all accurately known, but while at times probably very considerable, the above value of 14 metres per second seems to be excessive; in fact, its average value may not be even half so great. If in reality it is not, then, since the pressure of a wind varies as the square of its velocity, it follows that less than one-fourth of the actual pressure increase can be caused in this way. Hence it would seem that there probably is at least one other pressure factor, and, indeed, such a factor obviously exists in the check to the horizontal flow caused by vertical convection.

To make this point clear: Assume two layers of air, an upper and a lower, flowing parallel to each other. Let their respective masses per unit length in the direction of their horizontal movement be M and m, and their velocities V and v. Now, if, through convection, say, the whole or any portion of the lower layer is carried aloft, it must be replaced below by an equal amount of the upper air.

Let the whole of the lower layer be carried up. To produce the rainfall above assumed, 2 centimetres, this layer would have to be at least 1 kilometre deep; but no matter what its depth if it should merely change places with the upper air, there obviously could be no effect on the flow nor on the height of the barometer. Even if the different layers should mingle and assume a common velocity V', the rate of flow would still remain unchanged, in ac-

cordance with the law of the conservation of linear momentum, and the barometer reading unaltered.

In symbols we would have the equation

$$MV + mv = (M + m)\ V'.$$

Hence, neither interchange nor mingling of the two air currents, upper and lower, can change the vertical mass of the atmosphere, nor, therefore, the surface pressure. But, then, in the case of atmospheric convection there is something more than simple mingling of two air currents, and the linear momentum does not, in general, remain constant. The increased surface velocity following convection, a phenomenon very marked in the case of a thunderstorm, causes an increased frictional drag and therefore a greater or less decrease in the total flow. Suppose this amounts to the equivalent of reducing the velocity of a layer of air only 25 metres thick from V to v, and let $V = 5v$. That is, the equivalent of the one-three-hundred-and-twentieth part of the atmosphere having its flow reduced to one-fifth its former value. This would reduce the total flow of the atmosphere by about 1 part in 400, and thereby increase the barometric reading by nearly 2 millimetres.

It would seem, then, that the friction of the thunderstorm gust on the surface of the earth, through the consequent decrease in the total linear momentum of the atmosphere and, therefore, its total flow, must be an important contributing cause of the rapid and marked increase of the barometric pressure that accompanies the onset of a heavy thunderstorm.

To sum up: The chief factors contributing to the increase of the barometric pressure during a thunderstorm appear to be, possibly in the order of their magnitude: (*a*) decrease of horizontal flow, due to surface friction; (*b*) vertical wind pressure, due to descending air; (*c*) decrease in total humidity, due to precipitation; (*d*) lower temperature, due largely to evaporation—probably more than offset by the heat of condensation.

Thunderstorm Temperatures.—Before the onset of the storm the temperature commonly is high, but it begins rapidly to fall with the first outward gust and soon drops often as much as 5° C. to 10° C., because, as already explained, this gust is a portion of the descending air cooled by the cold rain and by its evaporation.

As the storm passes the temperature generally recovers somewhat, though it seldom regains its original value.

Thunderstorm Humidity.—As previously explained, heavy rain, at least up in the clouds, and therefore much humidity, and a temperature contrast sufficient to produce rapid vertical convection, are essential to the genesis of a thunderstorm. Hence during the early forenoon of a day favorable to the development of heat thunderstorms both the absolute and relative humidity are likely to be high. Just before the storm, however, when the temperature has greatly increased, though the absolute humidity still is high, the relative humidity is likely to be rather low. On the other hand, during and immediately after the storm, the relative humidity is high, owing to both evaporation and decrease of temperature, and a little later, at least, the absolute humidity, because of the removal of a large amount of moisture from the atmosphere, often, presumably, comparatively low.

"Rain-gush."—It has frequently been noted that the rainfall is greatest after heavy claps of thunder, a fact that appears to have given much comfort and great encouragement to those who maintain the efficacy of mere noise to produce precipitation—to jostle cloud particles together into raindrops. The correct explanation, however, of this phenomenon seems obvious: The violent turmoil and spasmodic movements within a large cumulus or thunderstorm cloud cause similar irregularities in the condensation and resulting number of raindrops at any given level. These in turn, as broken by the air currents, give local excess of electrification and of electric discharge or lightning flash. We have, then, starting toward the earth at the same time and from practically the same level, mass, sound, and light. The light travels with the greatest velocity, about 300,000 kilometres per second, and therefore the lightning flash is seen before the thunder is heard—its velocity being, roughly, only 330 metres per second—while the rain, with a maximum velocity of 8 to 10 metres per second with reference to the air, reaches the earth still later. In fact, it is the excessive condensation or rain formation up in the cumulus cloud that causes the vivid lightning and the heavy thunder. According only to the order in which their several velocities cause them to reach the surface of the earth it might appear, and has often been so interpreted, that lightning, the first perceived, is the cause of thunder, which, indeed, it is, and that heavy thunder,

the next in order, is the cause of excessive rain, which most certainly it is not.

Thunderstorm Velocity.—The velocity of the thunderstorm is nearly the velocity of the atmosphere in which the bulk of the cumulus cloud happens to be located. Hence, as the wind at this level is faster by night than by day and faster over the ocean than over land, it follows that exactly the same relations hold for the thunderstorm, namely, that it travels faster over water than over land and faster by night than by day. The actual velocity of the thunderstorm, of course, varies greatly, but its average velocity in Europe is 30 to 50 kilometres per hour; in the United States, 50 to 65 kilometres per hour.

Hail.—Hail, consisting of lumps of roughly concentric layers of compact snow and solid ice, is a conspicuous and well-known phenomenon that occurs during the early portion of most severe extratropical thunderstorms. But in what portion of the cloud it is formed and by what process the layers of ice and snow are built up are facts that, far from being obvious, become clear only when the mechanism of the storm itself is understood.

As before, let the surface temperature be 30° C. and the relative humidity 50 per cent., or the dew-point 18° C., nearly. Under these conditions saturation will obtain, and, therefore, cloud formation begin when the surface air has risen to an elevation of approximately 1.5 kilometres. Immediately above this level the latent heat of condensation reduces the rate of temperature decrease with elevation to about half its former value, nor does this rate rapidly increase with further gain of height. Hence in mid-latitudes, where the above assumptions correspond in general to average thunderstorm conditions, it is only beyond the 4-kilometre level that freezing temperatures are reached, and where hail, therefore, can form. In the tropics and, after mid-summer, in the warmer portions of the temperate regions, where the freezing level is very high, hail seldom occurs. Generally, either it is not formed at all, owing to insufficient cloud height, or, if formed, is melted before reaching the ground from its initial great altitude.

The process by which the nucleus of the hailstone is formed and its layer upon layer of snow and ice built up seems to be as follows: Such drops of rain as the strong updraft within the cloud blows into the region of freezing temperatures quickly con-

geal and also gather coatings of snow and frost. After a time each incipient hailstone gets into a weaker updraft, for this is always irregular and puffy, or else tumbles to the edge of the ascending column. In either case it then falls back into the region of liquid drops where it gathers a layer of water, a portion of which is at once frozen by the low temperature of the kernel. But again it meets an upward gust, or falls back where the ascending draft is stronger, and again the cyclic journey from realm of rain to region of snow is begun; and each time—there may be several—the journey is completed a new layer of ice and fresh layer of snow are added. In general the size of the hailstones will be roughly proportional to the strength of the convection current, but since their weights vary approximately (they are not homogeneous) as the cubes of their diameters, while the supporting force of the upward air current varies, also approximately, as only the square of their diameters, it follows that a limiting size is quickly reached. It is also evident, from the fact that a strong convection current is essential to the formation of hail, that it can occur only where this convection exists; that is, in the *front* portion of a heavy to violent thunderstorm.

Some meteorologists hold that the roll scud between the ascending warm and descending cold air is the seat of hail formation, but this is a mistaken assumption. Centrifugal force would throw a solid object, like a hailstone, out of this roll probably before a single turn had been completed. Besides, and this objection is, perhaps, more obviously fatal than the one just given, the temperature of the roll scud, because of its position, the lowest of the whole storm cloud, clearly must be many degrees above the freezing point. Indeed, as the above calculation shows, temperatures low enough for the formation of hail can not often obtain at levels much less than three times that of the scud, and therefore it evidently is in the higher levels of the cumulus and not in the low scud that hail must have its genesis and make its growth.

Chapter XVI

LIGHTNING.

INTRODUCTION.

About the middle of the eighteenth century Franklin and others clearly demonstrated that the lightning of a thunderstorm and the discharge of an ordinary electric machine are identical in nature, and thereby established the fact that many of the properties of the former may logically be inferred from laboratory experiments with the latter. There is, however, one important difference between the two phenomena that does not seem always to be kept in mind, namely, the distribution of the charge. In the one case, that of the laboratory experiment, the charge commonly exists almost wholly on the surface of the apparatus used, while in the other, that of the thunderstorm, it is irregularly distributed throughout the great cloud volume. Hence the two discharges, lightning and laboratory sparks, necessarily differ from each other in important details. Nevertheless, in each case the atmosphere must be ionized before the discharge can take place freely, and this condition seems, at times at least, to establish itself progresso-spasmodically. That is, a small initial discharge, losing itself in a terminal brush, is rapidly followed by another and another, each losing itself in a manner similar to the first, until a path from pole to pole is sufficiently ionized to permit of a free electric flow and quick exhaustion of the remaining charge. Fig. 115, copied from a photograph by Walker,[96] taken on a rapidly moving plate, shows how a laboratory spark spasmodically (doubtless influenced by the period of electrical oscillation) ionizes the air from either pole and thus progressively extends and finally closes the conducting path of complete discharge. There appears also to be good evidence that the lightning discharge often behaves in a manner generally similiar, though, perhaps, radically different, in certain details. Thus the free period of electrical oscillation that belongs to ordinary laboratory apparatus presumably affects the process of discharge building as well as the nature of the discharge after it is fully established; while, on the other hand, if, as seems practically certain, lightning is not oscil-

[96] *Annalen der Physik u. Chemie, Leipzig*, 1899, **68**, 776.

latory, it follows that its growth into a full flash must be acquired by some process independent of a periodic surge.

Lightning, however, usually is pulsatory, as is obvious from the flicker of sheet lightning, described below, discharge after discharge taking place in the same direction and along the same path. Occasionally these sequent discharges extend to unequal distances, the latter especially becoming feebler and shorter, as shown in Fig. 116, thereby in their decay inversely simulating the growth or progressive development of a freely oscillating laboratory discharge. However, being pulsatory, or consisting of a group of unidirectional discharges, is an entirely different thing from being oscillatory, that is, consisting of an equally spaced series the units of which are alternately in opposite directions.

FIG. 115.

Growth of an electric spark discharge. (Walter.)

It will be convenient, in discussing the facts about lightning, to classify the discharges according to their general appearance.

Streak Lightning.—When the storm is close by, the lightning discharge invariably appears to the unaided eye as one or more sinuous lines or streaks of vivid white or pink—invariably sinuous, because electrically the atmosphere is always heterogeneous or unequally ionized and the directive force constantly changing during, and because of, the discharge itself. Often there is one main trunk with a number of branches, all occurring at the same time and apparently instantaneously, while at other times there are two or more simultaneous though locally disconnected streaks. Frequently the discharge continues flickeringly (on rare occasions even steady, like a white-hot wire) during a perceptible time—occasionally a full second.

But all these phenomena are best studied by means of the camera, and have been so studied by several persons, among whom Walter, of Hamburg; Larsen, of Chicago; and Stead-

worthy, of Toronto, are among the most persistent and successful. Stationary cameras, revolving cameras, stereoscopic cameras, cameras with revolving plates, and cameras with spectrographic attachments have all been used, separately and jointly, and the results have abundantly justified the time and the labor devoted to the work.

Fig. 117, copied by permission from one of Walter's negatives, shows the ordinary tracery of a lightning discharge when photographed with a stationary camera. It is only a permanent

FIG. 116.

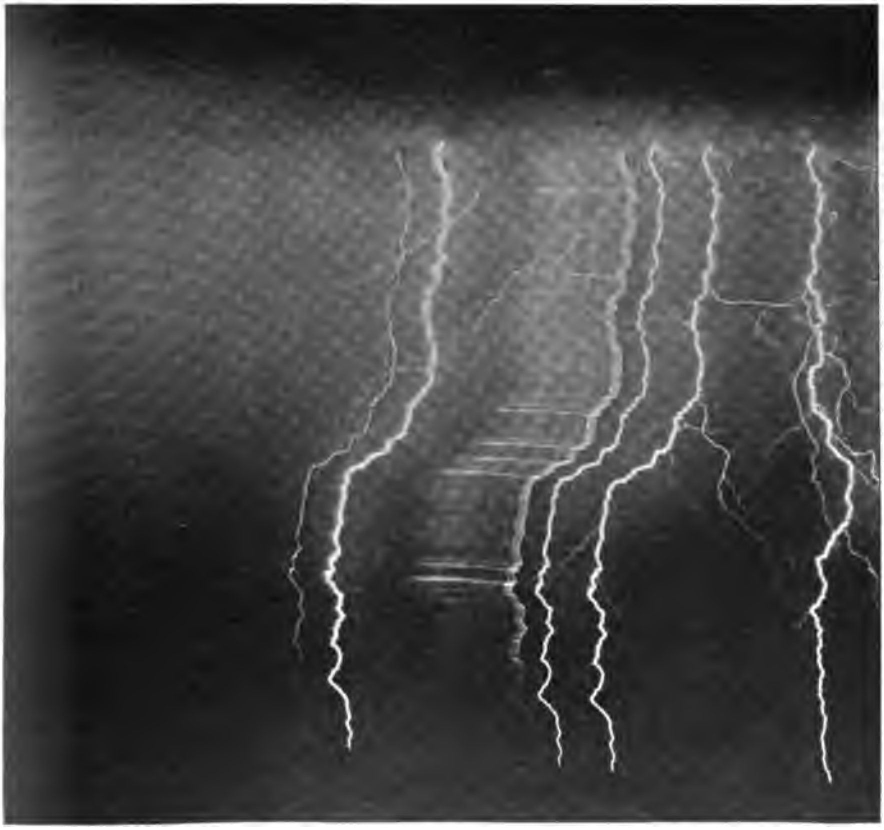
Streak lightning (sequent discharges), rotating camera. (Larsen.)

record of the appearance of the lightning to the unaided eye. Fig. 118, however, also copied by Walter's kind permission from one of his photographs, is a record of the same discharge obtained with a rotating camera. It will be noted that the more nearly vertical discharge occurred but once or was single; that this discharge was quickly followed by a second along the same path to about one-fourth of the way to the earth, where it branched off on a new course; that the second discharge was followed in turn at short but irregular intervals by a whole series of sequent discharges; that most of the discharges appeared as narrow intensely luminous streaks, and that one of the

Fig. 117.

sequent discharges appeared, not to the eye, but on the plate of the rotating camera, as a broad band or ribbon. On close inspection it will be obvious that the plaid-like ribbon effect is due, the warp to irregularities in the more or less continuous discharge,

FIG. 118.



Streak lightning (sequent discharges), rotating camera; companion to Fig. 117. (Walter.)

and the woof to roughly end-on and therefore brighter portions of the streak. Another point particularly worthy of attention is the fact that while the first and second discharges have several side branches the following ones remain entire from end to end and are nowhere subdivided.

Fig. 116, taken from a photograph obtained by Mr. Larsen, of Chicago, and kindly lent for use here by the Smithsonian

Institution, shows another series of sequent discharges similar to those of Fig. 118, except that in this case there was no ribbon discharge. The time of the whole discharge, as calculated by Mr. Larsen, was 0.315 second. Here, too, side branches occur with the first but only the first discharge. This, however, is not an invariable rule for occasionally, as illustrated by Fig. 119,

FIG. 119.

Streak lightning (sequent discharges), rotating camera. (Walter.)

copied from a published photograph by Walter, the side branches persist through two or three of the first successive discharges, but not through all. In such case each tributary when repeated follows, as does the main stream, its own original channel.

The phenomenon of sequent discharges, all along the same path, and the disappearance of the side branches with or quickly after the first discharge both seem reasonably clear. The first discharge, however produced, obviously takes place against very great resistance, and therefore under conditions the most favorable for the occurrence of side branches, or ramifications. But the

discharge itself leaves the air along its path temporarily highly ionized, puts a temporary line conductor with here and there a poorer conducting branch, in the atmosphere. This conductor is not only temporary (half the ions are reunited in about 0.15 second, the air being dusty[97]) but also so extremely fragile as to be liable to rupture by the violent disturbances, both explosive and of other types to be discussed later, it itself creates in the atmosphere. Because partly, perhaps, of just such interruptions, and because also of the volume distribution of the electricity which prevents a sudden and complete discharge, the actual discharge is divided into a number of partials that occur sequently. Obviously the breaks in the conducting (ionized) path, if they exist, are only here and there and but little more than sufficient to interrupt the flow. Hence the next discharge, if it occurs quickly, must follow the conducting and, therefore, original discharge path. Besides, in the subsequent discharges the original side branches will be quickly abandoned because of their greater resistance, or, what comes to the same thing, because of the more abundant ionization and consequent higher conductivity of the path of heaviest discharge.

This leaves to be explained the genesis of the initial discharge, the least understood perhaps of all the many thunderstorm phenomena. Judging from the voltages required to produce laboratory sparks, roughly 30,000 volts per centimetre, it is not obvious how such tremendous potential differences can be established between clouds or between a cloud and the earth as would seem to be necessary to produce a discharge kilometres in length, as often occurs. Indeed, a fatal objection to the assumption of such high voltage is the effect it would have on the velocity of fall and consequent size of the electrified raindrops. According to Simpson[98], thunderstorm rain often carries as much as 6 electrostatic units of electricity per c.c., and occasionally even more. Hence 30,000 volts per centimetre would produce an electric force on such rain roughly six-tenths that due to gravity and therefore either retard its fall, if directed upward, or, if directed downward, give it a velocity that would quickly break it into smaller drops. But thunderstorm rain does not consist essentially of smaller drops. On the contrary, as casual observation leads one to believe

[97] Rutherford, *Philosophical Magazine* **44**, p. 430, 1897.

[98] *Loc. cit.*, pp. 149–150.

and as measurements have shown[99], raindrops average larger (1 to 6 mm. in diameter) during a thunderstorm than at any other time. Their velocity of fall therefore can not be excessive, nor indeed does it ever appear to be greatly different from that of ordinary rain. Hence electrical gradients of the order above assumed do not exist between clouds and the earth.

Obviously the potential of individual drops may grow in either of two ways: (*a*) by the union of similarly charged smaller drops into larger ones. In this case, since capacity is directly proportioned to the radius, and the charge, after coalescence, to the volume (if droplets had equal size and charges), the potentials of the resultant drops, that is, their charges divided by their capacities, must be proportional to the squares of their radii, and therefore rapidly increase with coalescence and growth of size; (*b*) by evaporation of however charged drops. Here the charges remain constant and therefore the potential of each individual drop, being inversely proportional to its radius, obviously must become larger as the drop itself evaporates and gets smaller. In each case the tendency of the separate drops to discharge is increased, and the general ionization perhaps somewhat correspondingly increased, but the potential difference between the earth and the cloud as a whole unchanged. At present, therefore, one can do but little more than speculate on the subject of the primary lightning discharge, but even that much may be worth while since it helps one to remember the facts.

As already explained the electrical separation within a thunderstorm cloud is such as to place a heavily charged positive layer (lower portion of the cloud) between the earth and a much higher, also heavily charged, negative layer (upper portion of the cloud). Hence the discharges, or lightning, from the intermediate or positively charged layer may be to either the negative portion above, in some cases even an entirely different cloud, or the earth below. Further, through the sustaining influence and turbulence of the uprushing air there must be formed at times and places practically continuous sheets and streams of water, of course heavily charged and at high potential, and also layers and streaks of highly ionized air. That is, electrically speaking, heavily charged conducting sheets and rods, whether of coalesced drops or of ionized air, are over and over, so long as the storm

[99] Bentley, *Monthly Weather Review*, 32, p. 453, 1904.

lasts, momentarily placed here and there within the positively charged mass of the storm cloud.

Consider, then, what might be expected as the result of this peculiar disposition of charges and conductors, the result, namely, of the existence of a heavily surface-charged vertical conductor in a strongly volume-charged horizontal layer or region above and below which there are steep potential gradients to negatively charged parallel surfaces.

The conductor will be at the same potential throughout, and therefore the maxima of potential gradients normal to it will be at its ends, where, if these gradients are steep enough, and the longer the conductor the steeper the gradients, brush discharges will take place. Assume, then, that a brush discharge does take place and that there is a supply of electricity flowing into the conductor to make good the loss. The brush and the line of its most vigorous ionization, other things being equal, necessarily will be directed along the steepest potential gradient or directly toward the surface of opposite charge. But this very ionization automatically increases the length of the conductor, for a path of highly ionized air is a conductor, and as the length of the conductor grows so, too, does the steepness of the potential gradient at its forward or terminal end, and as the steepness of this gradient increases the more vigorous the discharge, always assuming an abundant electrical supply. Hence, an electric spark once started within a thunderstorm cloud has a good chance, by making its own conductor as it goes, of geometrically growing into a lightning flash of large dimensions. Of course, when the electrical supply is small the lightning is feeble and soon dissipated.

Whether the discharge actually does burrow its way through the atmosphere in some such manner as that indicated probably would be difficult, though not necessarily impossible, of observation. The gradual lengthening of the streak, if the discharge takes place in this manner, might be detected by photographing it on oppositely directed rapidly moving films. A phenomenon roughly analogous to the burrowing progress suggested[100] can indeed be produced on a photographic plate by bringing in contact with the film, some distance apart, two conducting points attached to the opposite poles of an influence machine. Brush discharges

[100] Leduc, *Comptes rendus*, Paris, 1899, 129, 37.

develop about each point, but the glow at the negative pole detaches itself and slowly meanders across the plate toward the positive point. As it goes it continually builds for itself, with the silver of the emulsion, a highly conducting path.

Rocket Lightning.—Many persons have observed what at least seemed to be a progressive growth in the length of a streak of lightning. In some cases[101] this growth or progression has appeared so slow as actually to suggest the flight of a rocket, hence the name.

At first one might well feel disposed to regard the phenomenon in question as illusory, but it has been too definitely described and too frequently observed to justify such summary dismissal. Naturally, in the course of thousands of lightning discharges. many degrees of ionization, availability of electric charge, and slopes of potential gradient are encountered. Ordinarily the growth of the discharge, doubtless, is in a geometric ratio and the progress of its end exceedingly swift, but it seems possible for the conditions to be such that the discharge can barely more than sustain itself, in which case the movement of the flash terminal may, possibly, be relatively slow, and the appearance of a rocket therefore roughly imitated.

Ball Lightning.—Curious luminous balls or masses, of which C. de Jans[102] probably has given the fullest account, have time and again been reported among the phenomena observed during a thunderstorm. Most of them appear to have lasted only a second or two and to have been seen at close range, some even passing through a house, but they have also seemed to fall, as would a stone,[103] like a meteor, from the storm cloud, and along the approximate path of both previous and subsequent lightning flashes. Others appeared to start from a cloud and then quickly return, and so on through an endless variety of places and conditions.

Doubtless many reported cases of ball lightning, probably the great majority, are entirely spurious, being either fixed or wandering brush discharges or else nothing other than optical illusions, due, presumably, to persistence of vision. But here. too, as in the case of rocket lightning, the amount and excellence of observational evidence forbid the assumption that all such

[101] Everett, *Nature*, London, 1903, 68, 599.
[102] *Ciel et terre*, Bruxelles, 1910, 31, 499.
[103] Violle, *Comptes rendus*, Paris, 1901, 132, 1537.

phenomena are merely subjective. Possibly in some instances, especially those in which it is seen to fall from the clouds, ball lightning may be only extreme cases of rocket lightning, cases in which the discharge for a time just sustains itself. A closely similar idea has been developed in detail by Toepler.[104] It might either disappear wholly and noiselessly, as often reported, or it could, perhaps, suddenly gain in strength and instantly disappear as sometimes observed, with a sharp, abrupt clap of thunder.

To say that all genuine cases of ball lightning, those that are neither brush discharges nor mere optical illusions, are stalled thunderbolts, certainly may sound very strange. But that, indeed, is just what they are, according to the above speculation, a speculation that recognizes no difference in kind between streak, rocket, and ball lightning; only differences in the amounts of ionization, quantities of available electricity and steepness of potential gradients.

Sheet Lightning.—When a distant thundercloud is observed at night one is quite certain to see in it beautiful illuminations, appearing like great sheets of flame, that usually wander, flicker and glow in exactly the same manner as does streak lightning, often for well-nigh a whole second, and occasionally even longer. In the daytime and in full sunlight the phenomenon when seen at all appears like a sudden sheen that travels and spreads here and there over the surface of the cloud. Certainly in most cases, so far as definitely known in all cases, this is only reflection from the body of the cloud of streak lightning in other and invisible portions. Often a blurred, yellowish streak is seen through the thinner portions of the intervening cloud. Occasionally, too, the cloud is wholly cleared in places where, of course, the discharge is white and dazzling. Conceivably a brush or coronal discharge may take place from the upper surface of a thunderstorm cloud, but one would expect this to be either a faint continuous glow or else a momentary flash coincident with a discharge from the lower portion of the cloud to earth or to some other cloud. But, as already stated, only reflection is definitely known to be the cause of sheet lightning. Coronal effects seem occasionally possible, but that they ever are the cause of the phenomenon in question has never clearly been established and

[104] *Annalen d. Physik*, Leipzig, 1900, 2, 623.

appears very doubtful. It has often been asserted, too, that there is a radical difference between the spectra of streak and sheet lightning, but even this appears never to have been photographically, or otherwise definitely established.

Beaded Lightning.—Many photographs showing streaks of light broken into more or less evenly spaced dashes have been obtained and reported as records of beaded lightning. Without exception, however, these seem certainly to be nothing other than photographs of alternating-current electric lights, taken with the camera in motion. On the other hand, it occasionally happens that a reliable observer reports that he has actually seen a discontinuous or beaded streak of lightning. Thus Professor O. J. Ferguson, of the University of Nebraska (Department of Electric Engineering), says:[105]

"In the spring of 1914 a violent thunderstorm swept over Lincoln at about nine o'clock at night. There were numerous vivid lightning displays. One of these discharges occurring in the storm front originated at an elevation of about 45 degrees from my viewpoint and struck almost vertically downward. I was watching the storm from the window of a dark room, and the flash occurred directly in front of me. It was a direct stroke of chain or streak lightning.

"However, in dying away, it took probably a full second to disappear; it broke up seemingly into detached portions, short and numerous. In fact, it gave a bead-like effect, and it would be very easy for one to have retained the latter impression and to have called the stroke bead lightning.

"In explanation of this phenomenon I would suggest that each bead probably represents the 'end-on' view of the irregular portions of the lightning path, and that they remained luminous during the subsequent lesser discharges, while the intermediate sections became non-luminous, because viewed from the side."

The explanation offered by Professor Ferguson and illustrated by Fig. 118 doubtless is entirely correct. Hence beaded or pearl lightning must be accepted as a real though unusual phenomenon, which probably would be more often seen if definitely watched for. Indeed, by close observation, the author has several times had that pleasure.

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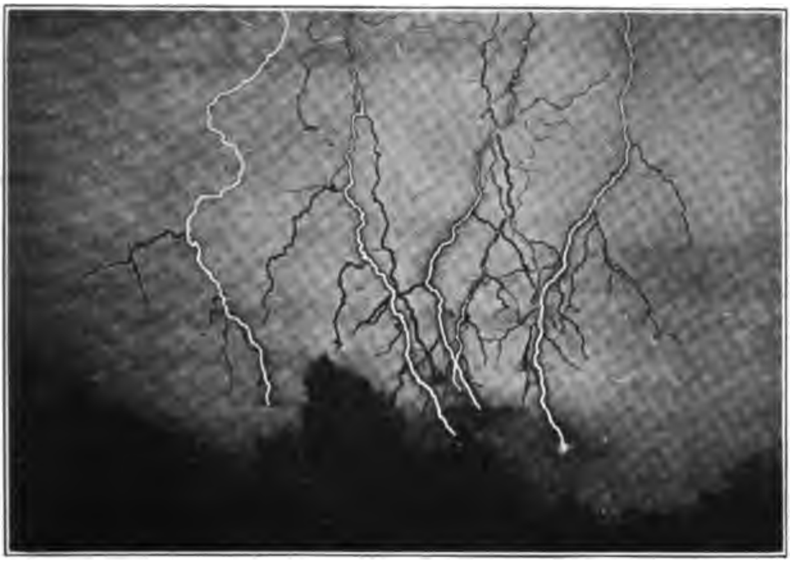
[105] JOURNAL OF THE FRANKLIN INSTITUTE, V. 179, p. 253, 1915.

Return Lightning.—This is commonly referred to as the return shock, and is only that relatively small electrical discharge that takes place here and there from objects on the surface of the earth coincidently with lightning flashes, and as a result of the suddenly changed electrical strain. This discharge is always small in comparison with the main lightning flash, but at times is sufficient to induce explosions, to start fires and even to take life.

FIG. 120.



Dark lightning. (F. Ellerman, photo.)

Dark Lightning.—When a photographic plate is exposed to a succession of lightning flashes it occasionally happens that one or more of the earlier streak images, on development, exhibits the "Clayden effect"—that is, appears completely reversed—while the others show no such tendency. Obviously, then, on prints from such a negative the reversed streaks must appear as dark lines (Fig. 120), and for that reason the lightning flashes that produced them have been called "dark lightning." There is, of course, no such thing as dark lightning, since the only invisible radiation to which the ordinary photographic plate is sensitive is the ultra-violet, which cannot be excited by electric discharges

in the atmosphere without at the same time producing visible radiation. Nevertheless, the photographic phenomenon that gives rise to the name "dark lightning," is real, interesting, and reproducible at will in the laboratory.[106]

Duration.—The duration of the lightning discharge is exceedingly variable, ranging from measured intervals of 0.0002 second,[107] and probably less, for a single flash to even a full second or more for a multiple flash consisting of a series of sequent discharges. On rare occasions a discharge of long duration appears *to the eye* to be steady like a glowing solid. Probably the best measurements of the shorter intervals were made by De Blois [108] with the aid of a high-frequency oscillograph. He found the durations of 38 single peaks, averaging 0.00065 second, to range from 0.0002 second to 0.0016 second. Flashes that last as long as a few tenths or even a few hundredths of a second are almost certainly multiple, consisting of a succession of apparently individual discharges occurring at unequal intervals. Occasionally a practically continuous discharge of varying intensity, but all the time strong enough to produce luminosity, will last a few hundredths of a second.

It must be remembered that the duration of even a single discharge and the length of time to complete the circuit, or ionize a path, from cloud to earth, say, are entirely different things. The latter seems usually (rocket and ball lightning may furnish exceptions) to be of exceedingly short duration, while the former depends upon the supply of electricity and the ohmic resistance directly, and upon the potential difference inversely.

Length of Streak.—The total length of a streak of lightning varies greatly. Indeed the brush discharge so gradually merges into the spark and the spark into an unmistakable thunderbolt that it is not possible sharply to distinguish between them, nor, therefore, to set a minimum limit to the length of a lightning path. When the discharge is from cloud to earth the length of the path is seldom more than 2 to 3 kilometres; in the case of low-lying clouds even much less, especially when they envelop a mountain peak.

On the other hand, when the discharge is from cloud to cloud

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[106] Wood, *Science*, New York (N.S.), 1899, 10, 717.

[107] De Blois.

[108] *Loc. cit.*

the path generally is far more tortuous and its total length much greater, amounting at times to 10, 15, and even 20 kilometres.

Discharge, Where to Where?—As already explained, lightning discharges may be between cloud and earth, between one part and another of the same cloud, or between cloud and cloud. But since the great amount of electrical separation, without which the lightning could not occur, takes place within the rain cloud, it follows that this is also likely to be the seat of the steepest potential gradients. Hence it would appear that lightning must occur most frequently between the lower and the upper portions of the same cloud, and this is fully supported by observations. The next in frequency, especially in mountainous regions, is the discharge between cloud (lower portion) and earth and the least frequent of all, ordinarily, those that take place between one and another entirely independent or disconnected clouds.

Since the electricity of the thunderstorm obviously is generated within the cumulus cloud and there mechanically separated into upper and lower layers it may not at first be clear how discharges can take place to earth at all. Of course, there will be some lines of force between the earth and each cloud charge, but these must be relatively few so long as the charges are equal and approximately superimposed and the resulting dielectric strain correspondingly feeble. However, as the upper charge is carried higher, and especially as it is drifted away from the lower by the winds into which it projects, the lines of force between cloud and earth become more and more numerous, and the strain progressively greater until suddenly relieved by the lightning's disruptive flash.

It would seem, therefore, that a marked difference between the wind velocities at the upper and lower storm levels would be especially favorable to frequency of cloud-to-earth discharges. Hence one would infer that heat thunderstorms, since they occur only when the general winds are light, are less dangerous—less likely to be accompanied by cloud-to-earth lightning—than those (presumably every other type) in which the wind velocity increases more rapidly with elevation. And from this one would further infer that tropical thunderstorms, since they commonly belong to the heat variety, are less dangerous than storms of equal electrical intensity of middle and higher latitudes, where the other or cross-current varieties prevail.

Unfortunately data are not at hand by which these deductions may be tested statistically. They are, however, in accord with the general impression [109] that thunderstorms are more dangerous in England than in India.

Discharges Direct, not Alternating.—Years ago some one for some reason or other, or for no reason, made the statement that the lightning flash is alternating and of high frequency, like the discharge of a Leyden jar, and forthwith, despite the fact that all evidence is to the contrary, it became a favorite dogma of the textbook, passed on unquestioned from author to author and handed down inviolate from edition to edition. True, there often are a number of successive discharges in a fraction of a second, as shown by photographs taken with a revolving camera, but these not only are along the same path but also in the same direction. This is obvious from the fact that side branches, whose trend with reference to the main trunk gives the direction of discharge, persisting as in Fig. 119, through two or more partial or sequent discharges, always follow the same paths. It is also proved by the direct evidence of the oscillograph.[110] In the case of each separate discharge also the direction seems constant; it may vary in strength, or pulsate, but, apparently, it does not alternate.

There are several reasons for concluding that lightning discharges, both single and multiple, are direct and not alternating, of which the following cover a wide range and probably are the best:

(*a*) Lightning operates telegraph instruments. If the discharge were alternating it would not do so, unless very heavily damped.

(*b*) At times it reverses the polarity of dynamos. This requires either a direct current or an alternating one so damped as to be quasidirect.

(*c*) The oscillograph[111] shows each surge or pulsation, as well as the whole flash, to be unidirectional.

(*d*) The relative values of the ohmic resistance, the self-induction, and the capacity, in the case of a lightning discharge,

[109] Symons, *Metrl. Magazine*, **49**, p. 114 and p. 164, 1914.

[110] De Blois, *loc. cit.*

[111] De Blois, *loc. cit.*

appear usually, if not always, to be such as to forbid the possibility of oscillations.

From the equation of a condenser discharge,

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=0$$

it may be shown[112] that whenever the product of the capacity by the square of the resistance is greater than four times the self-induction, or, in symbols, that whenever

$$CR^2>4L$$

oscillations are impossible. Undoubtedly all these terms vary greatly in the case of lightning discharges, but R, presumably, is always sufficiently large to maintain the above inequality and therefore absolutely to prevent oscillations.

To illustrate with perhaps a typical case, assume a cloud whose under surface is circular with a radius of 3 kilometres, and whose height above the ground is 1 kilometre, and let there be a discharge from the centre of the cloud base straight to the earth: Find a probable value for the self-induction and capacity, and from these the limiting value of the resistance to prevent oscillations, or the value of R in the equation

$$CR^2 = 4L.$$

To find L we have the fact that the coefficient of self-induction is numerically equal to twice the energy in the magnetic field per unit current in the circuit, and the further fact that per unit volume this energy is numerically equal to $\mu H^2/8\pi$, in which H is the magnetic force and μ the magnetic permeability of the medium. Let a be the radius of the lightning path and assume the current density in it to be uniform. Let b be the equivalent radius of the cylinder, concentric with the lightning path, along which the return or displacement current flows. In this case, μ being unity, the energy W of the magnetic field per unit current and per centimetre length of the discharge is given by the equation

$$W = \log\frac{b}{a} + \tfrac{1}{4}.$$

[112] J. J. Thomson. "Elements of Electricity and Magnetism," § Discharge of a Leyden Jar.

Let $b = 2$ kilometres and $a = 5$ centimetres. Then $W = \log_e 4 \times 10^4 + \frac{1}{4} = 11$, approximately. Hence the energy of the magnetic field per unit current for the whole length, 1 kilometre of the flash is represented by the equation

$$W 10^5 = 11 \times 10^5,$$

and the self-induction $= 22 \times 10^5 = 10^{-4}$ henry.

To find C, assume a uniform field between the cloud and the earth. As a matter of fact, this field is not uniform, and the calculated value of C, based upon the above assumption, is somewhat less than its actual value, but not greatly less. Assuming, then, a uniform field we have

$$C = \frac{A}{4\pi d} = \frac{\pi\, 9 \times 10^{10}}{4\pi \times 10^5} = 225 \times 10^3 = 25 \times 10^{-8} \text{ farad, } + \cancel{\text{about}}.$$

Hence, by substitution in the equation

$$CR^2 = 4L,$$

it appears that

$$R = 190 \text{ ohms per kilometre, } \cancel{\text{approximately}}.$$

Neither a, the radius of the lightning path, nor b, the equivalent radius of the return current, is accurately known, but from the obviously large amount of suddenly expanded air necessary to produce the atmospheric disturbances incident to thunder it would seem that 1 centimetre would be the minimum value for a. Also, from the size of thunder clouds, it appears that 10 kilometres would be the maximum value for b.

The substitution of these extreme values in the above equation gives

$$R = 200 \text{ ohms per kilometre, } \cancel{\text{roughly}}.$$

From the fact that C varies inversely and L directly as the altitude of the cloud it follows that, other things remaining equal, the height of the cloud has no effect on the value of R per unit length.

If the altitude is kept constant and the size of the cloud varied C will increase directly as the area, and L will increase directly as the natural logarithm of the equivalent radius of the cylinder of return current. Assuming the area of the cloud base

to be 1 square kilometre, which certainly is far less than the ordinary size, and computing as above it is found that

$$R = 850 \text{ ohms per kilometre, } + \ \cancel{\text{roughly}}.$$

Again, assuming the base area to be 1000 square kilometres, an area far in excess of that of the base of an ordinary thunderstorm cloud, the result is

$$R = 35 \text{ ohms per kilometre, roughly.}$$

It would seem, therefore, that a resistance along the lightning path of the order of 200 ohms per kilometre, or 0.002 ohm per centimetre, would suffice, in most cases, absolutely to prevent electrical oscillations between cloud and earth. In reality the total resistance includes, in addition to that upon which the above calculations are based, the resistance in parallel of the numerous feeders or branches within the cloud itself. In other words, the assumption that the resistance of the condenser plates is negligible may not be strictly true in the case of a cloud. Nor is this the only uncertainty, for no one knows what the resistance along the path of even the main discharge actually is; though, judging from the resistance of an oscillatory electric spark,[113] it, presumably, is many times greater than the calculated limiting value; and if so, then lightning flashes, as we have seen, must be unidirectional and not alternating.

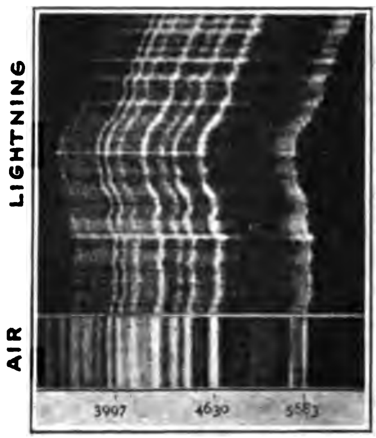
Temperature.—What the temperature along the path of a lightning discharge is no one knows, but it obviously is high, since it frequently sets fire to buildings, trees, and many other objects struck. In an ordinary electrical conductor the amount of heat generated in a given time by an electric current is proportional to the product C^2RT, in which C is the strength of the current, R the ohmic resistance, and T the time in question during which C and R are supposed to remain constant. In a spark discharge of the nature of lightning some of the energy produces effects, such as decomposition and ionization, other than mere local heating, but as experiment shows, a great deal of heat is generated, according, so far as we know, to the same laws that obtain for ordinary conductors. Hence extra heavy discharges, like extra large currents, produce excessive heating, and therefore

[113] Fleming, "The Principles of Electric Wave Telegraphy and Telephony," 2d ed. 1910, 8o, p. 228–237.

are far more liable than are light ones to set on fire any objects that they may hit.

Visibility.—Just how a lightning discharge renders the atmosphere through which it passes luminous is not definitely known. It must and does make the air path very hot, but no one has yet succeeded, by any amount of ordinary heating, in rendering either oxygen or nitrogen luminous. Hence it seems well-nigh certain that the light of lightning flashes owes its origin to something other than high temperature, probably to

FIG. 121.



Spectrum of lightning. (Fox.)

internal atomic disturbances induced by the swiftly moving electrons of the discharge, and to ionic recombination.

Spectrum.—Lightning flashes are of two colors, white, and pink or rose. The rose-colored flashes, when examined in the spectroscope, show several lines due to hydrogen which, of course, is furnished by the decomposition of some of the water along the lightning path. The white flashes, on the other hand, show no hydrogen lines or at most but faint ones. As one might suspect, the spectrum of a lightning flash and that of an ordinary electric spark in air are practically identical. This is well shown by Fig. 121, copied from an article on the spectrum of lightning by Fox,[114] in which the upper or wavy portion is due to the light-

[114] *Astrophysical Jr.*, 1913, 18, p. 293.

ning and the lower or straight portion to a laboratory spark in air. Fig. 122 is from an exceptionally fine photograph by Mr. Steadworthy of the Dominion Observatory, Ottawa, Canada. The heavy streak across the spectrum is not the parent, but an accidental stray that got in beside the prism.

FIG. 122.

Spectrum of lightning; and stray streak. (Steadworthy.)

It is often asserted that the spectrum of streak lightning consists wholly of bright lines, and that sheet lightning gives only nitrogen bands; and from this it is argued that the latter is not a mere reflection of the first. This assertion is not supported by Figs. 121–122, the brightest portions of which, the portions that would the longest be seen as reflection grew steadily feebler, coincide with strong nitrogen bands.

Thunder.—For a long while no one had even a remotely satisfactory idea in regard to the cause of thunder, and it is not a

rare thing even yet to hear such a childish explanation as that it is the noise caused by the bumping or rubbing of one cloud against another.

Nor are all the learned explanations wholly free from error. Thus it has been suggested that thunder is due to the mutual repulsion of electrons along the path of discharge, though there are several objections to this pleasing hypothesis. If such repulsion really occurred to the extent indicated, one might therefore expect a thread or rod of mercury carrying a current to spread out. Instead, however, it actually draws together, and, with a strong enough current, even pinches itself in two. Again, if mutual repulsion actually drove the electrons violently asunder one would expect the discharge instantly to dissipate, producing some kind of a brush effect, instead of concentrating along the familiar streak. Electronic repulsion, therefore, though it must exist to some extent, does not seem adequate, nor, as we shall see presently, is it necessary, to the production of heavy peals of thunder.

Another plausible but erroneous hypothesis in regard to the origin of thunder insists that it is caused by the collapse of the partial vacuum produced by the heat generated by the lightning. Obviously cooling in this case must be rapid, especially at the instant the discharge ceases, but probably not nearly rapid enough to create sound, nor, therefore, ever to produce any of the crashes and rumbling that always follow heavy lightning.

On the other hand, the heating of the atmosphere, the molecular agitation due to ionization, along the discharge path is so great and the resulting expansion so sudden as to simulate a violent explosion and therefore to send out a steep compression wave. Indeed, compression waves generated by electric sparks are so sharply defined that not only they themselves but even their reflections may be clearly photographed.[115] A compression wave, therefore, generated in the manner just explained, apparently is an adequate cause of thunder, and hence, presumably, its only cause.

Rumbling.—Probably the most distinctive characteristic of thunder is its long-continued rumbling and great variation in intensity. Several factors contribute to this peculiarity, among them:

[115] Wood, *Philosophical Magazine*, **48**, 218, 1899.

(*a*) Inequalities in the distances from the observer to the various portions of the lightning's path. Hence the sound, which ordinarily travels about 330 metres per second in the air, will not all reach him simultaneously, but continuously over an appreciable interval of time.

(*b*) Crookedness of path. Because of this condition it often happens that sections of the path here and there are, each through its length, at nearly the same distance from the observer or follow roughly the circumferences of circles of which he is the centre, while other portions are directed more or less radially from him. This would account for, and doubtless in a measure is the correct explanation of, some of the loud booming effects or crashes that accompany thunder.

(*c*) Succession of discharges. When, as often happens, several discharges follow each other in rapid succession there is every opportunity for all sorts of irregular mutual interference and reinforcement of the compression waves or sound impulses they send out. Occasionally they may even give rise to a musical note of short duration.

(*d*) Reflection. Under favorable conditions, especially when the lightning is at a considerable distance, the echo from clouds, hills, and other reflecting objects certainly is effective in accentuating and prolonging the noise and rumble of thunder. But the importance of this factor generally is overestimated, for ordinarily the rumble is substantially the same whether over the ocean, on a prairie, or among the mountains.

Distance Heard.—The distance to which thunder can be heard seldom exceeds 25 kilometres, while ordinarily, perhaps, it is not heard more than half so far. To most persons, familiar with the great distances to which the firing of large cannon is still perceptible, the relatively small distances to which thunder is audible is quite a surprise. It should be remembered, however, that both the origin of the sound and often the air itself as a sound conductor are radically different in the two cases. The firing of cannon or any other surface disturbance is heard farthest when the air is still and when, through temperature inversion or otherwise, it is so stratified as in a measure to conserve the sound energy between horizontal planes. Conversely, sound is heard to the least distance when the atmosphere is irregular in respect to either its temperature or moisture distribution, or both, for

these conditions favor the production of internal sound reflections and the dissipation of energy. Now the former or favorable conditions occasionally obtain during the production of ordinary noises, including the firing of cannon, but never during a thunderstorm. In fact, the thunderstorm is especially likely to establish the second set of the above conditions, or those least favorable to the far carrying of sound.

Then, too, when a cannon, say, is fired the noise all starts from the same place, the energy is concentrated, while in the case of thunder it is stretched out over the entire length of the lightning path. In the first case the energy is confined to a single shell; in the second it is diffused through an extensive volume. It is these differences in the concentration and the conservation of the energy that cause the cannon to be heard much farther than the heaviest thunder, even though the latter almost certainly produces much the greater total atmospheric disturbance.

The Ceraunograph.—Various instruments, based upon the principles of "wireless" receivers and known as ceraunographs, have been devised for recording the occurrence of lightning discharges, whether close by or so far away as to be invisible and their thunder unheard. Of course, the sensitiveness of the instrument, the distance, and the magnitude of the discharge, all are factors that affect the record, but by keeping the sensitiveness constant, or nearly so, it is possible with an instrument of this kind to estimate the approximate distance, progress, and to some extent even the direction and intensity of the storm. Nevertheless, there does not appear to be much demand for this information, and therefore at present the ceraunograph is but sparingly used.

Chemical Effects.—As is well known, oxides of nitrogen and even what might be termed the oxide of oxygen, or ozone, are produced along the path of an electric spark in the laboratory. Therefore, one might expect an abundant formation during a thunderstorm of these same compounds. And this is exactly what does occur, as observation conclusively shows. It seems probable, too, that some ammonia must also be formed in this way, the hydrogen being supplied by the decomposition of raindrops and water vapor.

In the presence of water or water vapor these several compounds undergo important changes or combinations. The nitro-

gen peroxide (most stable of the oxides of nitrogen) combines with water to produce both nitric and nitrous acids; the ozone with water gives hydrogen peroxide and sets free oxygen; and the ammonia in the main merely dissolves, but probably also to some extent forms caustic ammonia.

Symbolically the reactions seem to be as follows:

$$2NO_2 + H_2O = HNO_3 + HNO_2.$$
$$O_3 + H_2O = H_2O_2 + O_2.$$
$$NH_3 + H_2O = NH_4OH.$$

The ammonia and also both the acids through the production of soluble salts are valuable fertilizers. Hence, wherever the thunderstorm is frequent and severe, especially, therefore, within the tropics, the chemical actions of the lightning may materially add, as has recently been shown,[116] to the fertility of the soil and the growth of crops.

Explosive Effects.—As already explained, the excessive and abrupt heating caused by the lightning current explosively and greatly expands the column of air through which it passes, thereby shattering chimneys, ripping off shingles, and producing many other similar and surprising results. It also explosively vaporizes such volatile objects as it may hit that have not sufficient conductivity to carry it off. Hence trees are stripped by it of their bark or utterly slivered and demolished through the sudden volatilization of sap and other substances; wire fused and vaporized; holes melted through steeple bells and other large pieces of metal, and a thousand other seeming freaks and vagaries wrought.

Many of the effects of lightning appear at first difficult to explain, but, except the physiological, which, indeed, are but little understood, and probably some of the chemical, nearly all depend upon the sudden and intense heating along its path.

Crushing Effects.—One of the more surprising phenomena of the lightning discharge is the crushing of hollow conductors, an effect that gives some idea of the strength of current and quantity of electricity involved, and therefore deserves a full discussion.

Pollock and Barraclough[117] have described and explained this phenomenon in connection with a hollow copper cylinder: outside

[116] Capus, Guillaume, *Annales de géographie*, 1914, 23, p. 109.

[117] *Jr. and Proc. Roy. Soc. N. S. Wales*, 39, p. 131, 1905.

diameter 18 mm., inside 16 mm., lap join 4 mm. wide, 2 mm. thick. In what follows, however, reference will be had to a remarkable and even more instructive product of the same phenomenon kindly lent by Mr. West Dodd, of Des Moines, Iowa. Fig. 123 shows two originally duplicate (so reported), hollow, copper lightning rods, one uninjured (never in use), the other crushed by a discharge. The uninjured rod consists of two parts, shown assembled in Fig. 123, and separate in Fig. 124. The conical cap, nickel plated to avoid corrosion, telescopes snugly over the top of the cylindrical section, and when in place, where it is left loose or unsoldered, becomes the ordinary discharge point.

The dimensions are:

Section	Outside Diameter	Inside Diameter
Cylinder	16.0 mm.	14.65 mm.
Cone shank	17.4 mm.	16.0 mm.

Length of conical cap, cylindrical portion, 7 cm., total 19 cm.

Both the cylindrical and the conical portions of the rod are securely brazed along square joints.

The general effects of the discharge, most of which are obvious from the illustrations, were:

1. One or two centimetres of the point were melted off.

2. The conical portion of the top piece and all the cylindrical rod except the upper 2 centimetres, roughly, within the cap, were opened along the brazed joint.

3. The brazing solder appears to have been fused and nearly all volatilized—only patches of it remain here and there along the edges.

4. The upper end of the cylindrical rod was fused to the cap just below its conical portion.

5. The rod was fused off where it passed through a staple. Whether a bend in the conductor occurred at the place of fusion is not stated.

6. The collapse of the cylindrical rod extended up about 5 centimetres into the cap.

7. The cylindrical portion of the cap, about 7 centimetres in length, was uninjured, even the brazing was left in place.

What force or forces caused this collapse? Possibly it might occur to many that it was produced by the reaction pressure from

an explosion-like wave in the atmosphere due to sudden and intense heating. But however plausible this assumption may seem at first there, nevertheless, are serious objections to it, some of which are:

(*a*) While explosions with their consequent pressures may be obtained by passing a powerful current along a conductor they

FIG. 123.

Originally duplicate hollow copper lightning rods; one never used, the other crushed by a lightning discharge.

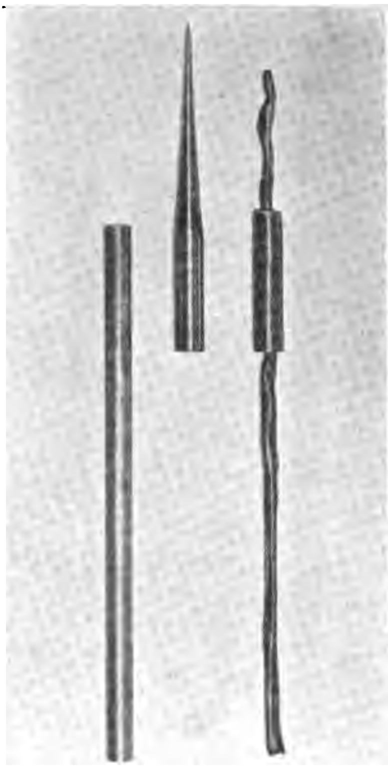
seem to occur only with the sudden volatilization of the conductor itself, which in this case did not take place.

(*b*) The heating of the enclosed air should have produced a pressure from within more or less nearly equal to the pressure simultaneously caused from without, and thereby have either prevented or at least greatly reduced the collapse.

(*c*) The assumption that the crushing of the conductor was due to mass inertia of the suddenly heated air offers no solution whatever of the collapse of the rod up into the shank of the cap.

For these reasons it seems that the idea that the collapse of the conductor may have been caused by the reaction pressure of

FIG. 124.



Same as Fig. 114, except unused rod is not assembled.

an explosion wave in the atmosphere due to sudden heating, is untenable.

Probably the correct explanation of the collapse, as already offered by Pollock and Barraclough,[118] an explanation that at least must involve an important factor, is as follows:

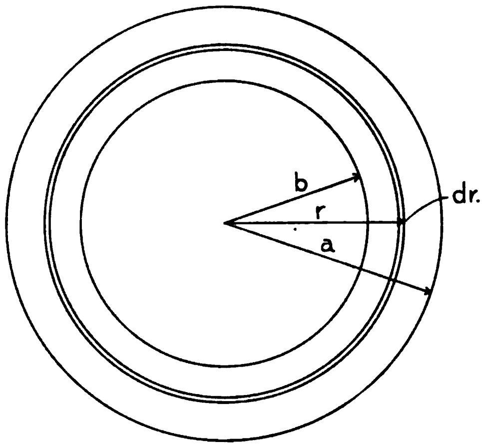
Each longitudinal fiber, as it were, of the conductor attracted

[118] *Loc. cit.*

every other such fibre through the interaction of the magnetic fields due to their respective currents, and the resulting magnetic squeeze on the hollow rod, whose walls were weakened by the heating of the current, caused it to collapse in the manner shown.

As is well known the force, f, in dynes per centimetre length, with which a straight wire carrying a current of I amperes is

FIG. 125.



Section of a hollow tubular conductor, inner radius, a, outer radius, b.

urged at right angles to the direction of the lines of force of a uniform magnetic field of intensity H is given by the equation

$$f = \frac{IH}{10}$$

Also, the value of H, r centimetres from a relatively very long straight conductor carrying I ampères, is given by the relation

$$H = \frac{2I}{10r}$$

Now, as developed by Northrup[119] in the theory of his heavy-current ammeters, let a, Fig. 125, be the outer, and b the inner radius of a tubular conductor, and let r be the radius of any intermediate tube of infinitesimal thickness, dr. Also let the conductor as a whole carry a uniformly distributed current of I am-

[119] *Trans. Amer. Electrochem. Soc.* 15, p. 303, 1909.

pères. Then the value of the magnetic force, at the end of the radius r, is given by the equation

$$H_r = \frac{2\,I\,(r^2-b^2)}{10\,r\,(a^2-b^2)}$$

which depends upon the fact that only those portions of the current less than r distant from the axis are effective—the forces due to the outer portions neutralizing each other. Also the strength of the current, dI, carried by the cylinder of radius r and infinitesimal thickness, dr, is given by the relation

$$d\,I = \frac{2\,I\,r\,dr}{(a^2-b^2)}$$

Hence, under the assumed conditions, the normal pressure, dP, per unit area on the cylinder of radius r and thickness, dr, may be determined by the equation

$$dP = \frac{2\,I\,r\,dr}{2\,\pi\,r\,10\,(a^2-b^2)} \times \frac{2\,I\,(r^2-b^2)}{10\,r\,(a^2-b^2)}$$

Hence the total normal pressure, P, per square centimetre of the *inner* surface is given by integrating the above expression between the limits b and a. That is,

$$P = \frac{2\,I^2}{100\,\pi\,(a^2-b^2)^2}\left(\int_b^a r\,dr - b^2\int_b^a \frac{dr}{r}\right)$$

$$= \frac{2\,I^2}{100\,\pi\,(a^2-b^2)^2}\left[\tfrac{1}{2}\,(a^2-b^2) + b^2\log_e\frac{b}{a}\right]$$

$$= \frac{I^2}{100\,\pi\,(a^2-b^2)}\left(1 + \frac{2\,b^2}{a^2-b^2}\log_e\frac{b}{a}\right)$$

Substituting for a and b their numerical values, 0.8 cm. and 0.7325 cm. respectively, it is found that

$$P = \frac{I^2}{379.1}$$

If we assume P, the pressure in dynes per square centimetre of the inner surface, to be 10^6, approximately one atmosphere, then

$I = 19{,}470$ ampères, approximately.

If the lightning discharge were alternating the current density would be greatest in the outer portions of the conductor, and therefore the total current would have to be still heavier than the above computed value to produce the assumed pressure. How-

ever, from reasons already given, it seems extremely probable that the discharge is unidirectional and not alternating, and therefore that the computed strength of current, though of minimum value, is substantially correct.

Quantity of Electricity in Discharge.—To determine the amount of electricity involved in a lightning discharge it is necessary to know both its duration and the average strength of current. Both factors and, therefore, the total charge are known to vary greatly, though actual measurements have been comparatively few and even these as a rule only crudely approximate.

It has often been stated that the duration of a single discharge, or single component of a multiple discharge, is not more than one one-millionth of a second. Some have computed a duration of roughly one one-hundred thousandth of a second, while others have estimated that it can not be greater than one forty thousandth or, at most, one thirty-five thousandth of a second. Possibly many discharges are as brief as some of these estimates would indicate, but there is ample reason to believe that others are much longer. Thus one occasionally sees a streak of lightning that lasts fully half a second without apparent flicker, while more or less continuous or ribbon discharges are often photographed by moving cameras. But in addition to these evidences we have also a number of time-measurements made by Rood[120] with a rotating disk, ranging from less than 1/1600 second up to 1/20 second; and others, 38 in all, by De Blois[121] with an oscillograph, ranging from 0.0002 second to 0.0016 second. In one case De Blois found the durations of 5 sequent discharges to be 0.0005, 0.0015, 0.0016, 0.0014 and 0.0012 second respectively, or 0.0062 second as the summation time of these principal components of the total discharge. Hence it seems probable that the actual time of a complete discharge, that is, the sum of the times of the several components, may occasionally amount to at least 0.01 second.

The second factor mentioned above, the strength of discharge, is even more difficult to determine, and but few estimates of it have been made.

Pockels,[122] adopting the ingenious method of measuring the

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[120] *Amer. Jr. Sci.*, vol. 5, p. 163, 1873.

[121] *Proceedings Am. Inst. Elec. Eng.*, vol. 33, p. 568, 1914.

[122] *Annalen d. Phys.*, 63, p. 195, 1897; 65, p. 458, 1898; *Met. Zeit.*, 15, p. 41, 1898; *Phys. Zeit.*, 2, p. 306, 1901.

residual magnetism in basalt near a place struck by lightning and comparing these quantities with those similarly obtained in the laboratory, concluded that the maximum strength of current in such discharges amounted occasionally to at least 10,000 ampères. However, the loss of magnetism before the measurements were made, and other unavoidable sources of error, indicate that the actual current strength probably was much greater than the estimated value—that the maximum strength of a heavy lightning discharge certainly amounts to many thousands of ampères, occasionally perhaps to even one hundred thousand.

Since the above estimates are very rough it would be well to check them, even though the check itself be equally crude. Hence it may be worth while further to consider the crushed lightning rod with this particular object in view.

From the dimensions already given of this rod, outside diameter 1.6 centimetres, inside diameter 1.465 centimetres, it follows that its cross-sectional area is about .325 square centimetre, and its weight, therefore, approximately 2.9 grams per centimetre length. Further, from the fact that the brazed joint was opened and most of the solder removed, apparently volatilized, and the further fact that the rod itself, in several places, indicates incipient fusion, it would seem that the final temperature may have been roughly 1050° C. If so the rod must have been heated about 1025° C., since its temperature just before being struck probably was approximately 25° C. But the average specific heat of copper over this temperature range is roughly 0.11 and therefore the calories generated per centimetre length about 327.

Now one ampère against one ohm generates 0.24 calories per second. Hence, since the resistance of the uninjured or check rod, as kindly measured by the Bureau of Standards, is practically that of pure copper, the average resistance of the crushed conductor over the assumed temperature range probably was about 17 microhms per centimetre length,[123] we have the equation

$$\frac{24}{10^2} I^2 \frac{17}{10^6} t = 327.$$

in which I is the average strength of current, and t the actual time of discharge. Assuming that $t = .01$ sec. we get, roughly,

$$I = 90{,}000 \text{ ampères.}$$

[123] Northrup, JOURNAL FRANKLIN INSTITUTE, 1914, 177, p. 15.

A current of this average value would indicate a maximum value of perhaps 100,000 ampères.

It was computed above that a current of 19.470 ampères in the given hollow conductor would produce on it a radial pressure of 10^6 dynes per square centimetre, or about one atmosphere. Hence 100,000 ampères would give a pressure of 2638×10^4 dynes per square centimetre, or approximately 400 pounds per square inch; enough, presumably, to produce the crushing that actually occurred.

A current of 90,000 ampères for .01 second would mean 900 coulombs or 27×10^{11} electrostatic units of electricity; certainly an enormous charge in comparison with laboratory quantities, but after all a surprisingly small amount of electricity, since it would electrolyze only .084 of a gram of water. It must be distinctly remembered, however, that these estimates are exceedingly rough, and further that this particular discharge presumably was exceptionally heavy since it produced an exceptional effect.

An interesting method of measuring the resultant electric exchange between earth and cloud incident to a lightning discharge recently has been used by C. T. R. Wilson.[124] Values up to about 50 coulombs were found, but it is not stated whether the discharges were single or mutliple, nor are their durations given.

From the above various observations and experiments, therefore, it appears that in some cases the strength of current in a lightning discharge probably amounts to many thousands of ampères, and that the total duration of the individual or partial discharges may be several thousandths of a second.

Danger.—It is impossible to say much of value about danger from lightning. Generally, it is safer to be indoors than out during a thunderstorm, and greatly so if the house has a well-grounded metallic roof or properly installed system of lightning rods. If outdoors it is far better to be in a valley than on the ridge of a hill, and it is always dangerous to take shelter under an isolated tree—the taller the tree, other things being equal, the greater the danger. An exceptionally tall tree is dangerous even in a forest. Some varieties of trees appear to be more frequently struck, in proportion to their numbers and exposure, than others, but no tree is immune. In general, however, the trees most

[124] *Proc. Roy. Soc. A.*, **92**, p. 555, 1916.

likely to be struck are those that have either an extensive root system, like the locust, or deep tap roots, like the pine, and this for the very obvious reason that they are the best grounded and therefore offer, on the whole, the least electrical resistance.

If one has to be outdoors and exposed to a violent thunderstorm, it is advisable, so far as danger from the lightning is concerned, to get soaking wet, because wet clothes are much better conductors, and dry ones poorer, than the human body. In extreme cases it might even be advisable to lie flat on the wet ground. In case of severe shock, resuscitation should be attempted through persistent (hour or more, if necessary) artificial respiration and prevention from chill.

As just implied, the contour of the land is an important factor in determining the relative danger from lightning because. obviously, the chance of a discharge between cloud and earth, the only kind that is dangerous, varies somewhat inversely as the distance between them. Hence thunderstorms are more dangerous in mountainous regions, at least in the higher portions, than over a level country. For this same reason, also (inverse relation between distance from cloud to earth and frequency of discharge between them), there exists on high peaks a level or belt of maximum danger, the level, approximately, of the base of the average cumulus cloud. The tops of the highest peaks are seldom struck, simply because the storm generally forms and runs its course at a lower level.

Clearly, too, for any given region the lower the cloud the greater the danger. Hence a high degree of humidity is favorable to a dangerous storm, partly because the clouds will form at a low level and partly because the precipitation, and probably therefore the electricity generated will be abundant. Hence, too, a winter thunderstorm, because of its generally lower clouds, is likely to be more dangerous than an equally heavy summer one. Finally, as already explained, cyclonic or other cross-current thunderstorms presumably are more dangerous, than those due to local heating, and therefore the thunderstorm of middle latitudes generally more dangerous than one of equal severity in the tropics.

It may also be interesting to note that the front edge of a thunderstorm probably is more dangerous than any other portion; more dangerous because it is immediately beneath the re-

gion of most active electrical generation, and because objects here often still are dry and therefore if struck more likely to be penetrated and fired than later when wet and thus partially shielded by conducting surfaces.

LIGHTNING PROTECTION.

If, as seems quite certain, the lightning discharge follows, or tends closely to follow, the instantaneous lines of electric force, then it is obvious that whatever changes the direction of this force must correspondingly alter the path the flash shall take. To the extent then that the direction of electric force near the surface of the earth can be changed, but in general to only this extent, lightning protection is possible. If also the strength of the field could materially be reduced, clearly, the discharges might be rendered less violent and even less frequent, but, as will be explained presently, there is no evidence that the strength of the field can greatly be altered by any practicable means. Hence it appears that protection from lightning must be sought through directional control, which is both possible and practical,[125] rather than through prevention.

Assume, in accordance with observation, that over an extended horizontal surface, a prairie for instance, the lines of electric force are vertical; determine how the field of force will be modified by the presence of a given structure. Obviously if the structure itself consists of such non-conducting materials as wood and stone there will be but little directional change of the electric force. If, however, it is made of a conducting substance the direction of the force will be changed, but to an extent and over an area that depend upon the size and shape of the structure in question. In general this effect is not calculable, but fortunately it may be definitely computed in the special case of a conducting semi-ellipsoid with vertical axis and standing, as would a right cone, on the conducting surface—the actual surface, if wet, somewhat below, if dry. The ground and all parts of the conductor, unless actively discharging, will have the same potential. Hence by varying the values of the three diameters of the semi-ellipsoid a fair approximation may be made to many ordi-

[125] O. S. Peters, "Protection of Life and Property Against Lightning." Technologic Paper, 56, Bureau of Standards, Washington, D. C., 1915. Also R. N. Covert, "Modern Methods of Protection Against Lightning," *Farmers' Bulletin 842*, Department of Agriculture, Washington, D. C., 1917.

nary structures and their effects on the electric field estimated, in some cases roughly, in others with even a high degree of accuracy. Thus, by making each of the horizontal diameters small and the vertical one relatively very large, the modification of the field by a single upright metallic rod may be computed very closely, and its efficiency as a protection against lightning approximately determined. This has recently been done by Sir J. Larmor and Mr. J. S. B. Larmor,[126] who say that

"In fact, if the undisturbed vertical atmospheric field is F, the modified potential

$$V = -F_z + A_z \int_\epsilon^\infty \frac{d\lambda}{(a^2+\lambda)^{1/2}(b^2+\lambda)^{1/2}(c^2+\lambda)^{3/2}}$$

will be null over the ground, and also null over the ellipsoid (a, b, c), provided

$$\frac{F}{A} = \int_0^\infty \frac{d\lambda}{(a^2+\lambda)^{1/2}(b^2+\lambda)^{1/2}(c^2+\lambda)^{3/2}}$$

"For our special case of a thin symmetrical semi-ellipsoid of height c, this gives

$$V = -F_z + A_z \int_\epsilon^\infty \frac{d\lambda}{\lambda\,(c^2+\lambda)^{3/2}}$$

"The value of this integral, however, increases indefinitely towards its lower limit as ϵ falls to zero, when a and b are null. Thus as the semi-ellipsoid becomes thinner the value of A diminishes without limit; that is, the modification of the field of force by a very thin rod is negligible along its sides unless close to it. A thin isolated rod thus draws the discharge hardly at all unless in the region around its summit."

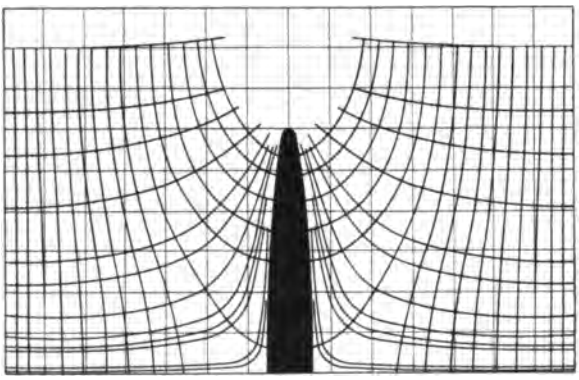
This is not to be taken as a condemnation of lightning rods in general. It only shows that a single vertical rod affords but little protection to things in its neighborhood, and thus explains why kite wires, for instance, are so seldom struck. When, however, the horizontal diameters of the semi-ellipsoid are of appreciable length the directions of the otherwise vertical lines of force are greatly changed for some distance on all sides, as illustrated by Fig. 126, adapted from the paper quoted above. Hence one method, and so far as known the only method, of at least partially

[126] *Proceedings Roy. Soc. London*, A 90, p. 314, 1914.

protecting an object from lightning consists in surrounding it by a hollow conductor, or by a well-grounded conducting cage. Perfect protection ordinarily is not practical, if even possible.

By this method the lightning that otherwise would hit at random is guided to the conducting system and through it, if all goes well, harmlessly to the ground. It must be clearly remembered, however, that this discharge, though in all probability unidirectional, is extremely abrupt and of great amperage and therefore possesses the dangerous voltage and inductive properties of alternating currents of high frequency and large volume. It should also be remembered that although the successive

FIG. 126.



Vertical field of electric force disturbed by a conducting, semi-ellipsoidal column.

partial discharges that make up the usual lightning flash follow the same ionized path, this path itself, shifted by the winds, probably often guides one or more of the secondary or sequent discharges to an entirely different object from that hit by the first.

From these fundamental principles it is easy to formulate general rules (details may be varied indefinitely) for the construction of an efficient system of lightning protection.

CONDUCTORS.

Since lightning discharges occasionally involve very heavy currents, it is necessary that the conductors of the protective system be sufficiently large to prevent fusion. Probably copper is the best material to use, mainly because non-corrosive, or prac-

tically so, in the atmosphere and therefore very durable, though aluminum and galvanized iron are also good. If copper, a weight of 370 grams per metre (4 ounces per foot) might suffice, but a greater weight possibly would be better. The shape of the cross section appears to be of comparatively small importance.

TERMINALS.

Because of the distortion of the electric field due to the object to be protected, house for instance, and to the system of conductors, each ridge, peak, chimney and other highest point should be capped or surmounted by a conductor that is well grounded.

It would be better if the conductor extended 2 metres or so above each of these salients, though the protection is still fair to good with much shorter projections, or even none at all. Whether or not each projection in turn is provided with the customary sharp points probably is of small importance—rather a matter of taste or sentiment than a necessity. To be sure, it often is asserted that sharp points discharge so freely that they thereby largely *prevent* lightning. But this assumption has little support from observation or experiment. Lodge,[127] for instance, says: "*I find that points do not discharge much till they begin to fizz and audibly spit; and when the tension is high enough for this, blunt and rough terminals are nearly as efficient as the finest needle points.* The latter, indeed, begin to act at comparatively low potentials, but the amount of electricity they can get rid of at such potentials is surprisingly trivial, and of no moment whatever when dealing with a thundercloud."

SYSTEM.

Because a single rod modifies the electric field only in its near neighborhood, and because the wind shifts the ionized or conducting path during the interval between successive partial discharges, it is obvious that the smaller the spaces left bare by the conductive covering the more effective the protection. A steel frame building with the framing well grounded from its lower portions and connected at all upper corners, and other places of near approach, to a metallic roof from which in turn conductors extended above the chimney tops and other protrusions, would, therefore, appear to be especially well protected from lightning damage.

[127] "Lightning Conductors and Lightning Guards," London, 1892, p. 370.

A stone or wooden building should have electrically continuous rods up each corner to the eave, thence to and along the ridge, with such side branches and elevated projections as the size and shape of the building, and other considerations, may require. In general, no place on the roof should be more than 3 metres (10 feet) from some portion of the protective system. Further, the principal and secondary conductors must be so placed that from any point the ground may be reached by a continuous *downward* course.

Protection would also be increased by surmounting each corner with a conducting rod 3 to 4 metres tall, properly connected to the rest of the system. Architectural considerations, however, might often forbid this additional precaution.

JOINTS.

To facilitate the discharge as far as possible the conductors should be as nearly as practicable continuous. Hence all necessary joints should be electrically good and mechanically secure. Well-made screw joints turned up tight appear to be the best.

BENDS.

Since electric surges tend to arc across sharp angles lightning rods must have no short bends. Changes in direction must be avoided as far as possible and wherever necessary be made gradually along a curve of 30 centimetres (one foot) radius, or more,—self induction must be kept at a minimum.

ATTACHMENT.

The rods should be attached to the building with holders of the same material as the rod itself. This prevents corrosion, and also secures electrical connection to the roof and sides which usually are wet and conducting during a thunderstorm.

GROUND CONNECTIONS.

Because of the considerable resistance of even very damp earth, ground connections should be as good and as many as practicable. Every descending rod, and there would better be one at each corner, and on large buildings even more, should be sunk straight down to perpetually damp earth, if convenient connected also to underground water pipes, and of course protected

from injury a couple of metres above ground. Generally copper is best for this purpose. If iron is used it should not be packed in coke or charcoal, since either would cause the iron more rapidly to corrode.

CONNECTION TO NEIGHBORING CONDUCTORS.

The high potential and strong induction of the lightning discharge require that not only gutters, waterspouts, and the like, on the outside, but also all internal conductors of large size or considerable length be connected with the outer system at their upper ends and wherever they come within even two or three metres of it, cross connected with each other at points of close approach, and, finally, well grounded, from their lower ends, either directly or by proper attachment to the main conductors.

It is often stated that leaky gas pipes should be excepted from such connections. Possibly so, but in the first place gas pipes should not be allowed to leak.

SPECIAL DANGERS.

Overland wires, telephone, telegraph, light and power, necessarily are sources of danger unless provided with proper lightning arresters. However, appropriate devices of this nature commonly are installed, and therefore danger from electric wires usually is negligible. Nevertheless, it must be remembered that this distinctly is a case where the price of protection is proper forethought and adequate precaution.

A much greater source of danger, because seldom if ever provided with an efficient lightning arrester, is the harmless-looking wire clothes line running from some part of the house to a convenient tree. The obvious remedy in this important case is either to use a cotton or other fibre rope, or else to avoid connection with the house altogether.

Still another common source of danger, especially to stock, is the ordinary wire fence. But here, too, approximate safety is easy of attainment. It is only necessary that good ground connections be made at intervals of every 100 metres (20 rods), or less—the shorter the better, so far as safety is concerned.

Finally the question of shade trees is of some importance. None is safe, but in general the danger they imply increases both with their own height and with the elevation of the ground above adjacent regions.

PART II.

ATMOSPHERIC ELECTRICITY AND AURORAS

CHAPTER I

ATMOSPHERIC ELECTRICITY.

THREE manifestations of atmospheric electricity, lightning (discussed in connection with the thunderstorm), the aurora polaris, and St. Elmo's fire—a "brush" discharge from elevated objects—have long been known; the first two, of course, from the beginning of human existence, and the last, as an object of the sailor's superstition, certainly since the days of ancient Greece and Rome.[128] Their identification, however, as electrical phenomena is very modern.

The following list of contributions to the science of atmospheric electricity, though fragmentary, will, perhaps, give some idea of its slow but accelerated course of development:

(*a*) The suspicion of the electrical nature of lightning by Hawksbee, who says,[129] "Sometimes I have observed the light to break from the agitated [electrified] glass in as strange a form as lightning." And also,[130] "I likewise observed that . . . it was but approaching my hand near the surface of the outer glass [a rotated open receiver containing an exhausted vessel] to produce flashes of light like lightning in the inner one"; by Wall,[131] . . . "by holding a finger a little distance from the [electrified] amber, a crackling is produced, with a great flash of light succeeding it . . . and it seems, in some degree, to represent thunder and lightning"; by Gray[132] . . . "this electric fire, which, by several of these experiments, seems to be of the same nature with that of thunder and lightning"; and by many others.

(*b*) The devising by Franklin,[133] in 1749, of a simple means "to determine the question, whether the clouds that contain lightning are electrified or not."

(*c*) The proof, May 10, 1752, by Dalibard[134] (following

[128] Brand's "Antiquities," Castor and Pollux.

[129] *Phil. Trans.*, 1705.

[130] *Phil. Trans.*, 1707.

[131] *Phil. Trans.*, 1708.

[132] *Phil. Trans.*, 1735.

[133] "Experiments on Electricity," edition 1769, p. 66.

[134] Franklin, "Experiments on Electricity," edition 1769, p. 107.

Franklin's suggestion), that clouds in which lightning appears are electrified.

(*d*) The proof, July, 1752, by Le Monnier[135] that a tall, insulated metallic conductor becomes electrified even when the sky is absolutely clear.

(*e*) The inauguration in 1757 by Beccaria[136] of systematic and long continued (15 years) observations of atmospheric electricity.

(*f*) The invention by Thomson[137] (Lord Kelvin) of the quadrant electrometer in 1855, and the "water-dropper," about the same time, that greatly increased the delicacy and accuracy of the measurements of atmospheric electricity.

(*g*) The discovery by Linss[138] in 1887 that even the most perfectly insulated conductors lose their charges, when exposed to the air, in a manner that shows the atmsophere itself to be a conductor of electricity.

(*h*) The discovery in 1900 by C. T. R. Wilson[139] and also by H. Geitel[140] of spontaneous ionization in the atmosphere.

(*i*) The discovery in 1902 independently by Rutherford and Cooke,[141] and McLennon and Burton,[142] of a penetrating radiation in the lower atmosphere, presumably from radioactive substances near the surface of the earth.

(*j*) The discovery in 1905 by Langevin[143] of slow moving or large ions in the atmosphere.

(*k*) The discovery by Simpson[144] in 1908 and 1909 that the electric charge on thunderstorm rain, and precipitation generally, is prevailingly positive.

(*l*) The discovery by Kolhörster[145] that an extremely hard or penetrating radiation exists in the atmosphere that comes from the outside—chiefly, apparently, from the sun.

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[135] *Acad. Roy. des. Sciences*, 1752, p. 233.
[136] "Dell' Elettricità Terrestre Atmospherica a Cielo Sereno," Torino, 1775.
[137] *B. A. Rept.*, 1855 (2), p. 22.
[138] *Met. Zeit.*, **4**, p. 345, 1887.
[139] *Proc. Cambr. Phil. Soc.*, **2**, p. 52, 1900.
[140] *Phys. Zeitsch.*, **2**, p. 116, 1900.
[141] *Phys. Rev.*, **16**, p. 183, 1903.
[142] *Phys. Rev.*, **16**, p. 184, 1903.
[143] *C. R.*, **140**, p. 232, 1905.
[144] *Memoirs Indian Meteorl. Dept.*, Simla, 1910, **20**, pt. 8.
[145] *Deutsche Phys. Gesel.*, July 30, 1914.

ELECTRICAL FIELD OF THE EARTH.

The experiments of Franklin and others with kites and insulated vertical rods revealed a persistent difference of electric potential between the earth and the atmosphere, that soon became, and still is, the object of innumerable measurements.

Instruments.—The instruments essential for accurate measurements of the difference of potential between the earth and any point in the atmosphere are a "collector" and an electroscope. The "collector" is merely an insulated conductor provided with an adequate means of electric discharge—sharp point, flame, ionizing salt, or "dropper"—that brings it and all other conductors with which it is electrically connected to the potential in the air at the point of discharge.

The electrometer, one element of which is connected to the "collector" and thus brought to its potential while the other is grounded, or connected to a "collector" at a different level, may be any one of several types. Those generally used at present are the Thomson quadrant, Bendorf registering (adaptation of the Thomson quadrant). Wulf bifilar, and Einthoven single-fibre. The quadrant type must be kept stationary, but the others are not so restricted and give good results even on shipboard and in balloons.

Potential Gradient Near the Surface.—The vertical potential gradient near the surface of the earth varies greatly, with location, season, hour, and weather conditions—occasionally even reversing sign during storms—but the general average over level areas and during fine weather appears to be of the order of 100 volts per metre, in response to a negative surface charge.

Location Effect.—Since the earth is a conductor it is obvious that the distribution on its surface and the resulting vertical potential gradient will be so modified by topography as to be smaller in narrow valleys than on the neighboring ridges. Over level regions of the same elevation the gradient appears to be largest in the interior of continents of the temperate zones and least within the tropics, and also, perhaps, in very high latitudes.

Annual Variation.—The annual variation of the vertical potential gradient near the surface of the earth differs greatly from place to place. In general it is comparatively small in tropical regions, and also anywhere on mountain tops, but large, as much in some cases as twice the annual average value, in the temperate

zones where the gradient changes are roughly as follows: An increase during the fall and early winter to a maximum of perhaps 250 volts per metre followed by a rapid decrease during spring to a moderately constant summer minimum of roughly 100 volts per metre.

Diurnal Variation.—The diurnal variation of the potential gradient, in some places fully equal to the average gradient, changes with place, season, and altitude. Its amplitude is greater along middle latitudes in the interior of continents than along low latitudes, or anywhere over the ocean; greater during winter, when it is single-crested, than summer, when double-crested. At moderate elevations, half a kilometre or less, the gradient has only a single daily maximum and minimum, whatever its surface periods.

In all cases a minimum gradient occurs about 4 o'clock in the morning. If the variation is double diurnal the second, but less pronounced minimum, occurs about mid-afternoon; the first maximum at 9 o'clock, roughly, in the forenoon, and the second at about 8 to 9 o'clock in the evening. If the variation is only diurnal, as in the winter, the maximum is attained during the afternoon. Typical examples of such curves are given in Fig. 127 after Bauer and Swann.[146]

From the above facts it appears that the single daily variation of the potential gradient is fundamental, and that the summer afternon minimum that develops a double diurnal variation is only a shallow disturbance due, presumably, in part at least, to dust, since any material caught up from the earth obviously must carry along some of the negative surface charge and thereby decrease the gradient in the lower air.

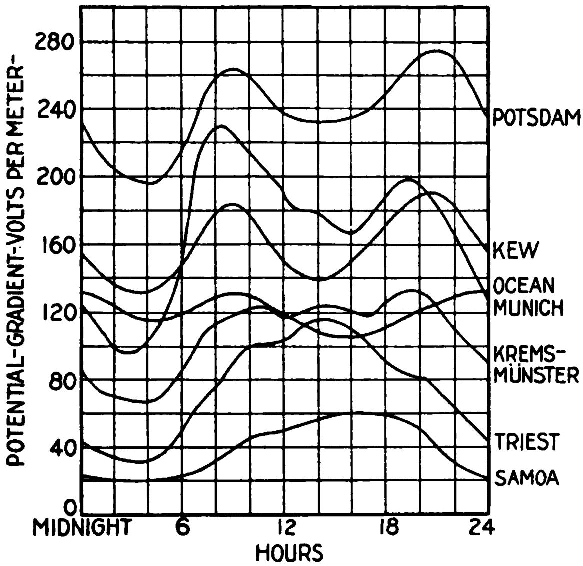
Potential Gradient and Meteorological Elements.—Many efforts have been made to find what relations obtain between the potential gradient and the various meteorological elements, but the results in most cases are inconclusive, especially in respect to temperature, humidity, and pressure changes. Strength and direction of wind both are important through their effect on the amount of smoke, dust, factory fumes, et cetera, in the air at the place of measurement. Fog, rain, and other forms of precipitation are nearly always electrically charged and therefore

[146] Publication No. 175 (Vol. III) of the Carnegie Institution of Washington.

often greatly modify and occasionally even reverse the potential gradient as do also heavily charged or thunderstorm clouds. Cirrus and other types of high, fair-weather clouds produce little or no effect.

Potential Gradient and Elcvation.—Measurements of the potential gradient from free balloons have shown that it varies greatly and irregularly through the low dust-laden stratum, and that above this layer it decreases less and less rapidly to a com-

FIG. 127.



Diurnal variation of potential gradient

paratively small value at an altitude of only a few kilometres. If the surface gradient is 100 volts per metre, it may be 25 volts per metre at an elevation of 1.5 kilometres, 10 at an elevation of 4 kilometres, 8 at 6 kilometres elevation, with similar decreases for greater heights.

Surface and Volume Charges, et cetera.—From the simple equation,

$$\frac{dV}{dn} = \frac{100 \text{ volts}}{\text{metre}} = f = 4\pi\sigma,$$

giving the electric force, f, or rate of change of potential normal to the surface, in terms of the surface charge σ per unit area, it

follows that when the potential gradient at the surface of the earth is 100 volts per metre the charge is 2.65×10^{-4} negative electrostatic units per square centimetre, or 4.5×10^5 coulombs, roughly, for the total surface charge of the earth.

Similarly, from the equation,

$$\frac{d^2V}{dn^2} = \frac{df}{dn} = 4\pi\rho$$

between the volume charge ρ and the ratio of change of the electric force to change of elevation, it appears that near the surface of the earth the net charge of the air is roughly 0.1 electrostatic unit of positive electricity per cubic metre.

ELECTRICAL CONDUCTIVITY OF THE ATMOSPHERE

It is well known that an electrified conductor exposed to the air gradually loses its charge, however carefully it may be insulated. This phenomenon was first investigated by Coulomb,[147] who found the important law that the rate of loss of charge is proportional to the existing charge, or rate of drop of potential proportional to the existing potential. In symbols,

$$\frac{dQ}{dt} = -aQ \text{ and } \frac{dV}{dt} = -aV$$

or

$$Q_t = Q_0 e^{-at}$$

and

$$V_t = V_0 e^{-at}$$

where Q_0 and V_0 are the charge and potential, respectively, at any given instant, Q_t and V_t the corresponding values t seconds, or other units of time, later, e the base of the natural logarithms, and a a constant.

The loss of charge was explained by Coulomb, and his explanation was accepted for more than a century, as due to the charging by contact of neutral molecules of air and their subsequent repulsion.

From the work begun by Linss[148] and extended by others it is now known, however, that the discharge coefficient a varies more or less from hour to hour and from season to season, and, further, that generally it is not the same for charges of opposite

[147] *Mém. de l'Acad. de Paris*, 1785, p. 616.

[148] *Met. Zeit.*, 4, 345 ,1887.

sign. Hence the loss of charge in addition to that which may be accounted for by imperfect insulation, is due to neutralization by numerous minute charges of the opposite sign normally present in the atmosphere—charges that render it conductive. It is also known that the values of these charges are either that of the electron or multiples thereof. Swann[149] has shown that whatever the shape of the charged body the rate of its loss of charge is given by the equation,

$$\frac{dQ}{dt} = -4\pi Q n e v$$

where Q is the charge on the object, n the number of ions per cubic centimetre of sign opposite to that of Q, v the specific velocity of these ions and e the ionic charge. In other words, the rate of supply of electricity by the ions to the charged body is $4\pi\lambda\ C\ V$, in which C is the capacity of the charged object and λ the conductivity of the air for electricity of sign opposite to that of the charge.

The conductivity, therefore, of the atmosphere may be conveniently measured by noting the rate of potential drop of a charged cylinder concentrically surrounded by a relatively large tube through which a good circulation of fresh air is maintained. Fig. 128 indicates the equipment used for this purpose on the *Carnegie* during the cruises of 1915–1916.[150] As explained in the publication referred to, if C_1 is the capacity of the whole apparatus, including the electroscope, and C_2 the measured capacitv of the concentric cylinders, including that portion of the supporting rod A that is exposed to the air current, then

$$-C_1\frac{dV}{dt} = 4\pi\lambda C_2 V$$

and

$$4\pi\lambda C_2 = \frac{C_1}{T}\log_e \frac{V_1}{V_2}$$

in which T is the time required for the potential to fall from V_1 to V_2. Hence both conductivities, λ_+ and λ_-, corresponding respectively to the positive and negative ions, are easily determinable.

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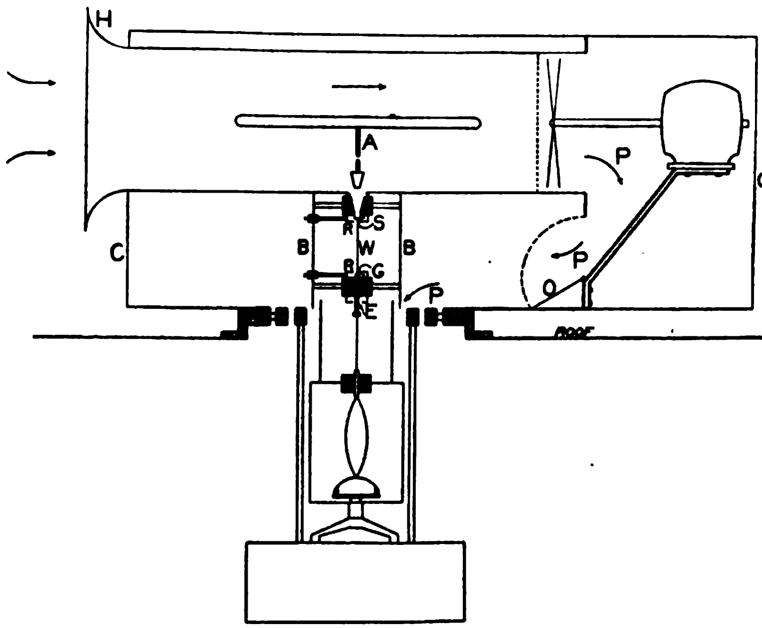
[149] *Terr. Mag. and Atmos. Elec.*, **19**, p. 81, 1914.

[150] Bauer and Swann, Publication 175 (Vol. III) of the Carnegie Institution of Washington, p. 385.

The average value of the conductivities found during the above-mentioned cruise of the *Carnegie* were $\lambda_+ = 1.44 \times 10^{-4}$ and $\lambda_- = 1.19 \times 10^{-4}$. These are also approximately the values found over land during clear weather.

Annual Variation.—In general the conductivity is greater during the summer than during winter—the reverse of the potential gradient.

FIG. 128.



Conductivity apparatus.

Diurnal Variation.—The diurnal variation of the conductivity is quite irregular, but is more or less the reverse of the potential gradient, that is, high in the early morning and low in the evening.

Relation to Weather.—The conductivity of the atmosphere is very small when the air is either dusty or foggy; nearly all the ions being then attached to masses so large that the velocity factor, v, in the current equation, and consequently the current itself, is quite small. On the other hand, when the air is clean and dry, the conductivity is relatively large.

Conductivity and Elevation.—Through the first kilometre the

conductivity of the atmosphere varies irregularly, owing, presumably, at least in part, to corresponding variations in the dust content. Beyond about that level it generally increases rather rapidly, so that at the elevation of 6 kilometres it may have roughly 20 times the surface value.

IONIC CONTENT OF THE AIR

Ionic Density.—The number of ions of either sign per unit volume of the atmosphere may be found by passing a known volume of air through a cylindrical condenser, sufficiently charged to catch all the ions of opposite sign, and noting the drop in potential.

Let $n+$ and $n-$ be the number of positive and negative ions respectively per cubic centimetre of the air examined, e the ionic charge, V the initial potential, and δV the drop in potential on passage of A cubic centimetres of air through the condenser, then, neglecting, or allowing for the leakage,

$$n_{\pm} = \frac{C\delta V}{eA}$$

The value of n varies greatly, being very small during foggy and dusty weather, and relatively large when the air is clear. In general it is larger during summer than winter, larger during the day time than at night, and larger when the temperature is high than when it is low. It also increases with elevation through at least the first few kilometres, but to what maximum value, and where, is not known.

Through the lower atmosphere the fair-weather values of n_+ and n_- generally are of the order of 800 and 680, respectively, per cubic centimetre.

Ionic Velocity.—The velocities v_1 and v_2, of the positive and negative ions respectively, may be computed from the corresponding values of the current, n_+ev, and ionic density n_+, since the value of e is a known constant. The average value of v_+ in the lower air is of the order of $1 \frac{\text{cm/sec}}{\text{volt/cm}}$, and of v_-, $1.2 \frac{\text{cm/sec}}{\text{volt/cm}}$. Both values increase with decrease of pressure—at half the pressure the velocity is double, approximately—and therefore with increase of elevation.

Large, or Langevin Ions.—After the atmosphere is deprived of all its ions of molecular size it still is slightly conductive, be-

cause, as discovered by Langevin,[151] of the presence of relatively slow moving and therefore comparatively massive ions. The number of such ions per cubic centimetre varies greatly. In the open country this number appears to be comparatively small, but it is very great over large cities, perhaps many times that of the ordinary or molecular ions.

ELECTRIC CURRENTS IN THE ATMOSPHERE

At least four different electric currents exist in the atmosphere —two always and everywhere, or nearly so, and two sporadically in time and place. These are:

(*a*) The lightning discharge, of very brief duration, but often rising to a strength of many thousand ampères.

(*b*) Precipitation currents, or currents due to the falling of charged rain, snow, hail, et cetera. The average strength of such current may be found from the rate of precipitation and charge, usually positive, per cubic centimetre, say, of the rain, or its equivalent in the case of snow or hail. During non-thunderstorm rains this current often averages about 10^{-16} ampère per square centimetre of surface. During violent thunderstorms, however, it is far greater, even as much as 10^{-12} ampère per square centimetre for brief intervals has been reported.

(*c*) Convection currents, due to the mechanical transfer of the ions in the atmosphere from one place to another by winds, including vertical convection. The strength of such current per unit area at right angles to the direction of the wind is obtained by multiplying the wind velocity by the net density of the charge.

This density may be found either by multiplying the ionic charge by the difference between the numbers of ions of opposite sign per cubic centimetre, or from the equation,

$$\rho = -\frac{1}{4\pi}\frac{d^2 V}{d h^2}$$

in which ρ is the density required, and dV/dh the vertical potential gradient.

The value of ρ varies greatly, but through much of the atmosphere the convection current is of the order 10^{-16} ampère per square centimetre cross section of the wind, per metre/second velocity.

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[151] *C. R.*, 140, p. 232. 1905.

(*d*) Conduction current, due to the downward flow of one set of ions, usually the positive, and the simultaneous upward flow of the other in response to the vertical potential gradient. The density of this current, or strength per square centimetre cross section, may be computed from the potential gradient and the conductivity, or, with suitable apparatus, may be measured directly. The average value of this conduction current is of the order of 3×10^{-16} ampère per square centimetre of, apparently, the entire surface of the earth. It generally is less during the day than at night, and less in summer than winter; but always of such value that the sum total of the current for the entire earth is roughly 1500 ampères. How this constant current, always, on the whole, in the same direction, is maintained is one of the greatest problems of atmospheric electricity.

RADIOACTIVE CONTENT OF THE ATMOSPHERE

The first evidence that the atmosphere normally contains one or more radioactive substances was obtained in 1900 when Geitel [152] and C. T. R. Wilson [153] independently found that an insulated electrified conductor gradually loses its charge even when inside a closed vessel. Later Elster and Geitel[154] showed that a bare wire exposed to the air and charged negatively to a high voltage gradually becomes coated with radioactive material. In 1904 Bumstead[155] showed that the radioactive substance of the atmosphere consists essentially of radium and thorium emanations, which, it is now known, occur in widely varying proportions. On the average, however, they appear to produce about the same amount of ionization, that is, near the surface and over land, roughly 2 ions each, of each sign, per cubic centimetre per second.

The emanations, which are heavy, radioactive gases, are several fold more abundant in mines and cellars than in the open and obviously get into the atmosphere by diffusion from the earth where they are generated by the spontaneous decomposition of radium and thorium. They may be absorbed from a known volume of air by cocoanut charcoal, liquified by low tem-

[152] *Phys. Zeit.*, **2**, p. 116, 1900.
[153] *Proc. Camb. Phil. Soc.*, **2**, p. 52, 1900.
[154] *Phys. Zeit.*, **2**, p. 590, 1901.
[155] *Amer. Jr. Sci.*, **18**, p. 1, 1904.

peratures ($-150°$ C. or lower), or caught up by a conductor charged to a high negative potential. In any case the nature of the deposit can be determined from the decay curve, from which, together with the saturation current and the volume of air used, the amount of active material per unit volume may be determined. In this way it has been found[156] that the radioactive emanations in the atmosphere over the Pacific Ocean, Sub-Antarctic Ocean, and land (average) amount to 3.3×10^{-12}, 0.4×10^{-12}, and 88×10^{-12} curie per cubic metre, respectively. Or, since the volume of one curie of emanation at standard temperature and pressure is 0.59 cubic millimetre,[157] the emanation gases constitute, in these several regions, 1.95×10^{-19}, 0.24×10^{-19}, and 51.9×10^{-19} of the atmosphere, respectively.

The amount of these emanations appears to be sufficient to account for the measured ionization (ions of molecular size) on the land, but quite insufficient over the oceans to maintain the ionization of these regions. Perhaps, as the slow ions are so very numerous over land areas, it may account for only a small part of the ionization in either case.

PENETRATING RADIATION

It has been found that the air within a closed metallic vessel remains fully conductive, even when deprived of all emanations and when the inner walls of the vessel have been cleaned, as far as possible, of radioactive materials. By surrounding this vessel with thick screens, or sinking it in water, the conductivity of the enclosed air is more or less reduced.[158] It is, therefore, inferred that the conductivity in question is produced by penetrating radiation of the γ type from the outside. One obvious source of such radiation is the radium and thorium, and their decomposition products, that seem to be more or less prevalent everywhere near the surface of the earth, especially over land.

That a portion, at least, of the ionization giving this conductivity is produced by the γ rays of ordinary radioactive substances

[156] Bauer and Swann, Publication 175 (Vol. III) Carnegie Institution of Washington, p. 422.

[157] Rutherford, "Radioactive Substances and Their Radiations," Cambridge University Press, 1913, p. 480.

[158] Rutherford and Cook, *Phys. Rev.*, **16**, p. 183, 1903; McLennan and Burton, *Phys. Rev.* **16**, p. 184, 1903.

in the earth and lower atmosphere is evident from the fact that it decreases with elevation up to about 1.5 kilometres above the surface. From this level, however, up to the greatest elevation at which it has been reported, 9 kilometres, the ionization increases very rapidly, and to several fold its surface value.[159] Hence there appears also to be a γ radiation of extremely high penetrating power that enters the lower atmosphere from somewhere above it.

ORIGIN AND MAINTENANCE OF THE EARTH'S CHARGE

Numerous hypotheses have been made to account for the negative charge of the earth and to explain how that charge is maintained in spite of the conductivity of the atmosphere, but no satisfactory explanation of either has yet been found. As Simpson[160] has explained, since the vertical current is constant up to at least 1800 metres, the greatest altitude at which it has been determined, it follows that the negative charge of the earth can not be supplied from the air below that level. Neither can it be supplied by electrical separation within the earth, as that would quickly lead to a positive instead of the prevailing negative surface charge.

Simpson[161] suggests that the negative charge of the earth may be maintained by a bombardment from the upper atmosphere, or even cosmical space, of negative ions of much greater penetrating power than any now known. But, it is stated, this is only a suggestion and not a solution of the greatest, perhaps, of the problems of atmospheric electricity.

The latest and most satisfactory explanation of the origin of the earth's charge is the following, by Swann[162]:

" Measurements of the variation of the penetrating radiation, with altitude, point to the upper atmosphere as the origin of a part of this radiation. The whole of the penetrating radiation is probably of the γ-ray type, but the part which reaches the earth's surface from the outer atmosphere is naturally the most penetrating part. Indeed, it is so penetrating that it passes through a thickness of air which would be equivalent, in absorp-

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[159] Kolhörster, *Deutsch. Phys. Gesell.*, **16**, p. 719, 1914.

[160] *M. W. R.*, **44**, p. 115, 1916.

[161] *Loc. cit.*

[162] *Phys. Rev.*, **9**, p. 555, 1917.

tive action, to a column of mercury 76 cm. high, if absorption coefficients were simply proportional to density and were independent of material. The γ-ray radiation from the outer layers of the atmosphere will consequently be very 'hard,' and, in accordance with the known results of laboratory experiments, we must conclude that the negative corpuscles which it emits from the air molecules are emitted almost entirely in the direction of the radiation, and further, that they can have a range in air at least equal to that of the swiftest β-rays from radium products, a range, for example, of 8 metres. The emission of corpuscles by these γ-rays will consequently result, at each point of the atmosphere, in a downward current of negative electricity, which we shall call the corpuscular current. This corpuscular current will charge the earth until the return conduction-current balances the corpuscular current at each point of the atmosphere.

"Taking, for the purpose of this abstract, a simplified case where the penetrating radiation considered is all directed vertically downwards, if q is the number of corpuscles liberated per c.c. per second by the penetrating radiation and h the average distance which a corpuscle travels from its point of origin, the corpuscular current density will be

$$i = qeh,$$

where e is the electronic charge.

"If q be taken as 2, which is probably about equal to the number of pairs of ions produced per c.c. per second in a closed vessel as a result of the part of the penetrating radiation in question, and if h be taken as 8 metres, we have

$$i = 2 \times 4.8 \times 10^{-10} \times 800 = \text{about } 8 \times 10^{-7} \text{ E. S. U./cm.}^2,$$

which is just of the order of magnitude of the air-earth current density, so that on this view, the penetrating radiation from the outer layers of the atmosphere provides a sufficient basis for the explanation of the maintenance of the earth's charge.

"The corpuscular current-density, and consequently the conduction current-density, will not necessarily be independent of the altitude, for the factors upon which i depends, viz., the intensity and quality of the penetrating radiation, the number of molecules per c.c. available for possible ionization by the radiation, and the range of the corpuscles set free all alter with the altitude.

"A few minor difficulties present themselves if the above view be adopted. Thus, for example, near the surface of the earth, a considerable portion of the whole penetrating radiation comes from the soil, and is directed upwards, but this difficulty disappears when it is remembered that the average 'hardness' of the radiation from the soil is very much less than that of the radiation which reaches the earth from the outer layers of the atmosphere. Again, it might appear that the corpuscles set free by the penetrating radiation should, on account of their great energy, produce in the atmosphere many more ions per second than are actually found to be produced. This difficulty, and others of allied nature become greatly reduced in magnitude, however, when considered in the light of our present knowledge of the action of very swift β-rays when passing through a gas."

Chapter II

AURORA POLARIS

The aurora polaris is a well-known but imperfectly understood luminous phenomenon of the upper atmosphere, of which Figs. 129 and 130, from Störmer's numerous photographs, are good examples.

Types.—While no two auroras are exactly alike, several types have been recognized, such as arcs, bands, rays, curtains or draperies, coronas, luminous patches, and diffuse glows. The arcs normal to the magnetic meridian, often, but not always, reach the horizon. Their under edge is rather sharply defined, so that by contrast the adjacent portion of the sky appears exceptionally dark. The rays, sometimes extending upward from an arch, at other times isolated, are parallel to the lines of magnetic force. Many auroras are quiescent, others exceedingly changeable, flitting from side to side like wandering searchlights, and in some cases even waving like giant tongues of flame.

Latitude Variation.—The aurora of the northern hemisphere occurs most frequently, about 100 per year, at the latitudes 60° (over the North Atlantic and North America) to 70° (off the coast of Siberia). Its frequency appears to be less within this boundary, while with decrease of latitude it falls off so rapidly that even in southern Europe it is a rare phenomenon. At the same latitude it is distinctly more frequent in North America than in either Europe or Asia.

The distribution of auroras in the southern hemisphere is not so well known, but it appears to be similar, in general, to that of the northern.

Periodicity.—It is well established that on the average auroras are more numerous during years of sun spot maxima than during years of spot minima. They also appear to be more numerous before midnight than after. Relations of frequency to phase of the moon, season, et cetera, have also been discussed, but with no conclusive results.

Color.—Many auroras are practically white. Red, yellow and green are also common auroral colors. Some streaks and bands are reddish through their lower (northern) portion, then yellowish, and finally greenish through the higher portions.

FIG. 129.

Aurora, February 28, 1910. (Störmer.)

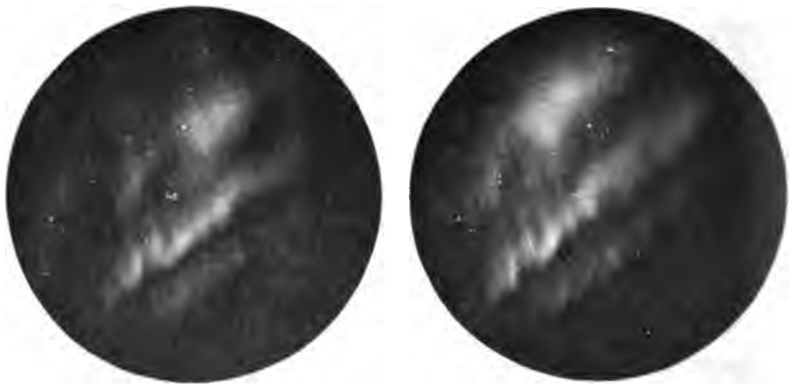
FIG 130.

Aurora, March 3 1910. (Störmer.)

Much of the light is due to nitrogen bands, but the source of the most prominent line of the auroral spectrum, λ .5578 μ (green), is not known. It has often been attributed to krypton, but other conspicuous krypton lines are absent; besides krypton is too heavy to exist at auroral heights in sufficient abundance to produce a spectrum of such brilliance.

There is good evidence that this green light, the light that produces the "auroral line," is always present in the sky, though whether wholly of auroral origin, or due in part to bombardment by meteoric dust, or to some other cause, is not known.

FIG. 131.



Parallactic auroral photographs for determining altitude. (Störmer.)

Height.—The problem of the height of auroras has often been investigated, but only recently solved. By simultaneously photographing the same aurora from two stations against a common background of stars, Fig. 131, and measuring the parallax obtained, Störmer,[163] and Vegard and Krogness[164] have secured many excellent height measurements. The upper limits of the auroral light vary from about 100 kilometres to over 300 kilometres; and the lower limits from perhaps 85 kilometres to 170 kilometres, with two well-defined maxima, one at 100 kilometres, the other at 106 kilometres.

Cause.—The fact that brilliant shifting auroras are accompanied by magnetic storms renders it practically certain that they,

[163] *Terr. Magnet. and Atmos. Elec.*, 21, p. 157, 1916.
[164] *Terr. Magnet. and Atmos. Elec.*, 21, p. 169, 1916.

and presumably therefore all auroras, are due to electric discharges; and the further fact that they vary in frequency with the sunspot period indicates that this current either comes from or is induced by the sun. For some time it was thought probable that auroras are caused by negative particles shot off from the sun, and entrapped by the magnetic field of the earth. On the other hand, Vegard [165] has given strong arguments in favor of the α particle which is positively charged, and Störmer[166] has found at least one case that required the positive charge to account for the observed magnetic disturbance.

The evidence, then, while not conclusive, indicates that auroras are due to streams of α particles in the upper atmosphere shot off by radioactive substances in the sun.

The seeming convergence of the auroral rays on a point far short of the magnetic pole, towards which they actually do converge, is due to perspective. Similarly, their apparent divergence from the magnetic zenith, thus forming a corona, is also a phenomenon of perspective, for here one is looking out along a bundle, or tube, of rays that, following the lines of magnetic force, surround him in every direction. The rapid, upward pulses of light along these rays, however, are quite real, and due, presumably, to progressive electric discharges.

[165] *Phil. Mag.*, **23**. p. 211. 1912; *Ann. der Phys.*, **50**, p. 853, 1916.

[166] *Terr. Magnet. and Atmos. Elec.*, **20**, p. 1, 1915.

PART III.

ATMOSPHERIC OPTICS

INTRODUCTION—CLASSIFICATION.

MANY curious and beautiful phenomena, of which the mirage, the rainbow, the halo, the azured sky, and the twilight glow, are some of the more conspicuous, are due to the optical properties of the air and the foreign substances suspended in or falling through it. All, or nearly all, of them have been the objects of innumerable observations and many careful studies, the results of which, fortunately, have been summarized and discussed by various authors. The most extensive discussion, however, of this subject is by Pernter and Exner, whose work, "Meteorologische Optik," therefore, will be largely, but by no means exclusively, drawn upon for the material of this section.

When one's chief or only purpose in discussing the optics of the air is to describe the phenomena seen, it is convenient to divide them into such general classes as mirages, rainbows, halos, coronas, etc. For explanatory purposes it is more convenient, perhaps, to group them according to (*a*) their objective or material causes, namely: atmosphere, raindrops, water droplets, ice crystals, etc.; or (*b*) their physical causes, such as reflection, refraction, diffraction, etc.

Each of the above classifications has its advantages and disadvantages. On the whole, however, the division according to physical causes seems best suited to the needs of an explanatory discussion and therefore is here adopted.

CHAPTER I.

PERSPECTIVE PHENOMENA.

Apparent Stair-step Ascent of Clouds.—The stair-step appearance of the echelon cloud (Fig. 132) is, perhaps, the simplest sky phenomenon due to perspective. The exact manner by which the stair-step or terrace illusion is brought about is shown by Fig. 132, in which *O* is the position of the observer, *H* his horizon, 1, 2, 3, etc., evenly spaced flat-bottomed cumuli of the same base elevation—flat-bottomed and of constant level because of the approximately uniform horizontal distribution of moisture.

Since the clouds are at a higher level than the observer, each successive cumulus, as the distance increases, is seen at a lower angle than its predecessor; and the dark bases of any two adjacent clouds appear to be connected with each other by the lighter side of the farther one. Besides, their general resemblance to stair-steps often leads one into the error of "seeing" the connection between any two adjacent bases to be at right angles to both. That is, starting with base *a*, the light side of cloud 3 appears as a vertical surface at *b*, and its base as a dark horizontal surface at *c*; the side and base of cloud 4 appear as the next vertical and horizontal surfaces, *d* and *e*, respectively, and so on for the other clouds; the whole effect merging into the appearance of a great stairway, consisting of the horizontal treads, *a, c, e*, etc., connected by the seemingly vertical risers, *b, d, f*, etc.

Apparent Arching of Cloud Bands.—Occasionally a narrow cloud band is seen to stretch almost, if not entirely, from horizon to horizon, but although its course is practically horizontal and its direction often nearly straight, it usually appears arched. If even the nearest portion of the cloud still is far away, the apparent arching is slight. On the other hand, when the cloud is near the arching is great. The apparent curve is neither circular nor elliptical, but resembles rather a conchoid whose origin is at the observer and whose asymptote is his horizon.

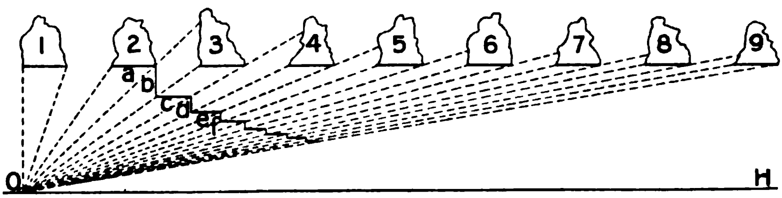
The angle of elevation at which different segments of the cloud are seen clearly varies from a minimum for the more distant portions to a maximum for the nearest. Hence, the phenomenon in question, the apparent arching of the band along its nearest portions, is only an optical illusion, due entirely to the

projection of the cloud (above the observer's level) onto the sky.

When several such bands or streaks occur in parallel, they appear to start from a common point at or beyond the horizon, to terminate, if long enough, in a similar opposite point, and progressively to arch and spread apart as they approach the observer's zenith. They thus form the perspective effect often called " Noah's Ark " or polar bands.

Apparent Divergence and Convergence of Crepuscular Rays (Sunbeams).—Everyone is familiar with the beautiful phenomenon of the " sun drawing water "—sunbeams that, finding their way through rifts in the clouds, are rendered luminous by the dust in their courses. Equally familiar and equally beautiful are also those streaks and bands of pearly lights (where the lower atmosphere is illuminated) and azure shadows (where only the

FIG. 132.



Cloud echelon effect.

upper atmosphere is illuminated) that often at twilight and occasionally at dawn radiate far out from the region of the sun, and at times even converge towards the opposite point of the horizon. These, too, are only beams of sunlight and shadow caused by broken clouds or irregular horizon.

All such crepuscular rays, whether their common origin, the sun, be below or above the horizon, seem first to diverge, while the few that cross the sky appear also to arch on the way and finally to converge towards the antisolar point.

Here, again, the facts are not as they seem, for the rays, all coming, as they do, from the sun, some 93,000,000 miles away, necessarily are practically parallel. Their apparent divergence, convergence, and arching are all illusions due to perspective, just as are the apparent divergence, convergence, and arching of the rails on a long straight track.

Apparent Divergence of Auroral Streamers.—Anyone at all familiar with the appearance of auroral streamers will recall that

at most localities they seem to radiate from some place far below the horizon. In reality they do diverge (or converge, if one prefers) slightly since they follow, approximately, the terrestrial lines of magnetic force. Indeed, their rate of convergence is about the same, on the average, as that of the geographic meridians at the same latitudes, and therefore far less than one would infer from their apparent courses. That is, their seeming rapid convergence is only another illusion due to perspective, just as is the apparent divergence of the crepuscular rays, as above explained.

Apparent Shape (Flat Vault) of the Sky.—To everyone the sky looks like a great blue dome, low and flattish, whose circular rim rests on the horizon and whose apex is directly overhead. So flat, indeed, does this dome appear to be that points on it estimated to lie half-way between the rim and apex generally have an elevation of but little more than 20°, instead of 45°, as they would if it seemed spherical.

That the rim of the sky dome should appear circular is obvious enough. It is simply because the horizon, where land and sky come together, itself is circular, except when conspicuously broken by hills or mountains.

To understand the other and more important feature, that is, why the dome looks so flat, consider (1) a sky filled from horizon to horizon with high cirrus clouds. These seem nearest overhead for the simple reason that that is just where they are nearest. As the horizon is approached, the clouds merge, through perspective, into a uniform gray cover that appears to rest on the land at the limit of vision, whether this limit be fixed by the curvature of the earth or by haze, and the whole cloud canopy may seem arched just as and for the same reason that cloud streaks and crepuscular rays seem arched, as above explained. But (2) even a thin cirro-stratus veil whose parts are well-nigh indistinguishable produces a similar effect, the nearest portions appearing nearest, largely because they are the most clearly seen. Similarly, when there are no clouds the sky overhead also appears nearest because it is clearest; and that unconscious inference, based on endless experience, is correct—it is clearest because nearest. As the eye approaches the horizon, the increasing haze produces the impression of greater distance; and this impression is entirely correct, for the blue sky seen in any such direction is

farther away than the sky overhead. In short, the spring of a cloudless sky dome is "seen" to rest on the distant horizon and its ceiling to come closer and closer, in proportion to increasing clearness, as the zenith is approached. The shape, then, of this dome should not always appear the same, and it does not—not the same on a clear night, for instance, as on a clear day.

Impressions, therefore, of the "shape" of the sky are, perhaps, not so erroneous as sometimes they are said to be. Indeed, they usually conform surprisingly well to the actual facts.

Change, with Elevation, of Apparent Size of Sun and Moon.—One of the most familiar, as also one of the most puzzling, of optical illusions is the change between the apparent sizes of the full moon, say, or of the sun, at rising or setting, and at or near culmination. It is, however, only a phenomenon of perspective.

Since the solid angle subtended at any place on the earth by the moon, as also that subtended by the sun, is sensibly constant throughout its course from rising to setting, it follows that its projection, and, therefore, its apparent size, must be relatively large, or small, as the place of projection (sky dome) is comparatively far away or nearby. But, as already explained, the sky dome, against which all celestial objects are projected and along which they therefore appear to move, seems to be farther away, and is farther away, near the horizon than at places of considerable elevation. Hence the moon and the sun must look much larger when near the horizon than when far up in the heavens, and the fact that they do so look, is, as stated, merely a phenomenon of perspective.

The familiar fact that the moon appears of one size to one person and a different size to another clearly is also due to perspective. The one who judges it large imagines his comparison object to be at a greater distance than does the one who judges it small. But such estimates usually are very erroneous; the moon may seem a foot in diameter, for instance, and three miles away, whereas at that distance a 144-foot circle would just cover it.

Change, with Elevation, of Apparent Distance Between Neighboring Stars.—The generally recognized fact that the distance between neighboring stars appears much greater when they are near the horizon than when well up is also a phenomenon of perspective. Its explanation is identical with that of the change, under similar circumstances, of the apparent diameter of the moon, and therefore need not be given in further detail.

CHAPTER II.

REFRACTION PHENOMENA: ATMOSPHERIC REFRACTION

Astronomical Refraction.—It is well known that because of astronomical refraction the zenith distance of a star, or other celestial object, is greater than it seems, except when zero, to an extent that increases with that distance. To understand this important phenomenon, it is necessary to recall two experimental facts: (*a*) that in any homogeneous medium light travels in sensibly straight lines, and (*b*) that its velocity (velocity pertaining to any given wave frequency) differs from medium to medium.

Let, then, the parallel lines *AB* and *DE* (Fig. 133) be the intersections of the boundaries between three homogeneous media, 1, 2, 3, by a plane normal thereto and to the wave front, *BC*. Let the velocities in these media of a given monochromatic light be v_1, v_2, and v_3, respectively. Hence, when the light disturbance at *C* has travelled the distance *CA* in the first medium, that at *B* will have gone the distance *BE* in the second, where $CA/BE = v_1/v_2$, and *AE* will be the new wave front. Similarly, *DF* will be the wave front in the third medium, and so on for any additional media that may be traversed.

If i is the angle between the normal to the interface, *AB*, and the direction of the light, both in medium 1, and r the corresponding angle in medium 2, then, as is obvious from the figure,

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}, \text{ or } \sin i = \frac{v_1}{v_2} \sin r.$$

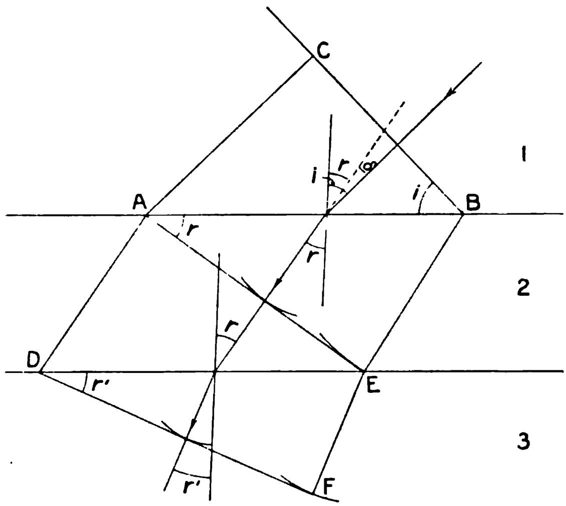
Similarly, $\sin r = \dfrac{v_2}{v_3} \sin r'$. Hence, $\sin i = \dfrac{v_1}{v_3} \sin r'$. That is, the total change in direction of the light depends solely on its velocities in the first and final media, respectively, and the initial angle of incidence. The optical densities of the intermediate layers may abruptly change by large amounts, as indicated, and thus cause the light to follow a perceptibly broken course, as from air to water, for instance; or they may change so gradually that the path is a smooth curve, even to the closest observation.

Since the ratio of the velocity of light in space to its velocity

in any given gas, or definite mixture of gases (the refractive index of that medium), increases directly with density, it follows that all rays of light that cross the atmospheric shell, except those that enter it normally, must follow continuously curved paths, somewhat as shown to an exaggerated extent in Fig. 134.

To determine the shape of such a curve through the atmosphere, let ϕ (Fig. 134) be the angle between the radii from the

FIG. 133.



Refraction of light on change of media.

centre of the earth at the place of observation and any other point along the course of a refracted ray.

As before,

$$\mu_2 \sin r_1 = \mu_1 \sin i_1$$

in which μ_1 and μ_2 are the refractive indices (with reference to space) of media, or layers, 1 and 2, respectively, i_1 the angle of incidence and r_1 the angle of refraction at the interface between these media or layers. But, corresponding to the radii R_1 and R_2,

$$\frac{\sin i_2}{\sin r_1} = \frac{R_1}{R_2}$$

Hence,

$$R_1 \mu_1 \sin i_1 = R_2 \mu_2 \sin i_2,$$

or, in general,

$$R \mu \sin i = C, \text{ a constant.}$$

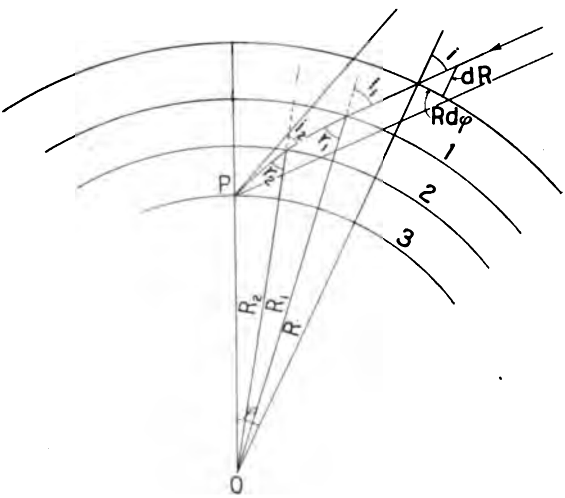
Further,
$$\frac{dR}{Rd\phi} = \cot i.$$

But cot $i =$
$$\frac{\cos i}{\sin i} = \sqrt{\frac{1 - \sin^2 i}{\sin^2 i}} = \sqrt{\frac{\mu^2 R^2}{C^2} - 1}$$

Hence,
$$\frac{dR}{d\phi} = R\sqrt{\frac{\mu^2 R^2}{C^2} - 1},$$

and
$$\phi = \int_{R_0}^{R} \frac{dR}{R\sqrt{\dfrac{\mu^2 R^2}{C^2} - 1}}$$

FIG. 134.



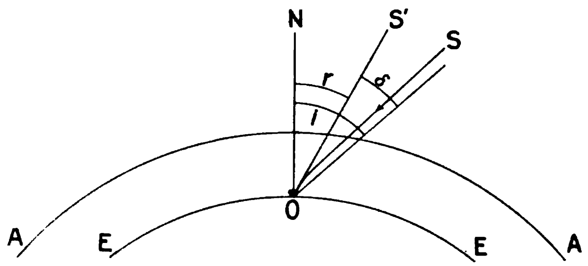
Path of light through the atmosphere.

Clearly, then, the value of ϕ corresponding to a definite value of R, or the value of R appropriate to a definite value of ϕ, depends upon the relation of μ to R, or, very nearly, the relation of the density of the atmosphere at any point to the altitude of that point. Hence, refraction curves may be drawn for different angles of incidence, or, if preferred, for different apparent alti-

tudes, according to any assumed distribution of atmospheric density—a distribution fairly well known.

The approximate value of astronomical refraction, that is, its value generally to within one second of arc, through all zenith distances up to at least 60°, may easily be obtained as follows: Assume the atmosphere to be flat, as it nearly is, over the restricted area through which stars may be seen whose zenith distances are within 60°, or thereabouts. Let *O* (Fig. 135) be the position of the observer, *S* the true position of a star and *S′* its apparent position.

FIG. 135.



Approximate astronomical refraction.

As explained above,

$$\sin i = \mu \sin r$$

in which μ is the refractive index of the air at the point of observation, i the actual and r the apparent zenith distance. But

$$i = r + \delta$$

in which δ is the angle of deviation.

Hence,

$$\sin (r + \delta) = \sin r \cos \delta + \cos r \sin \delta = \mu \sin r$$

When the angle of incidence is 60°, or less, δ is always very small, and

$$\sin \delta = (\mu - 1) \tan r, \text{ nearly.}$$

Expressed in seconds of arc this gives

$$\delta'' = 206265'' (\mu - 1) \tan r$$

in which the numerical coefficient is the approximate number of seconds in a radian.

For dry air at 0° C. and 760 mm. pressure, the average value of μ is about 1.000293.

Hence, also

$$\delta'' = 60''.4 \tan r.$$

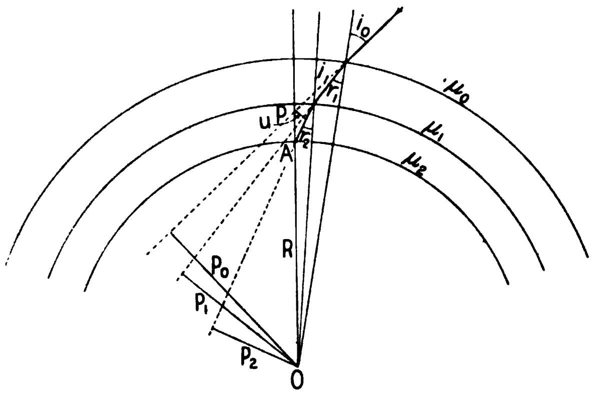
But for gases $\mu - 1 = K\rho$, very closely, in which K is a constant and ρ the density.

Hence, finally,

$$\delta'' = \frac{21''.7\ B}{T} \tan r,$$

in which B is the height of the barometer in millimetres, and T the absolute temperature in degrees C.

FIG. 136.



Astronomical refraction (Lord Rayleigh).

As a matter of fact, the atmospheric shell is not plane, even over small areas, but slightly curved, and therefore the complete formula for astronomical refraction, such as is needed for the construction of tables to be used in the most accurate measurements of star positions, is rather complicated. Probably the briefest and simplest derivation of a formula adequate for all zenith distances to at least 75° is due to Lord Rayleigh,[167] and is essentially as follows:

Let p_0, p_1, p_2, etc., be the normals from the centre of the earth onto the tangents of a ray path through the atmosphere at the

[167] *Phil. Mag.*, 36, p. 141, 1893.

points where the refractive indices are μ_0, μ_1, μ_2, etc., respectively. Let i_0, i_1, i_2, etc., be the angles of incidence, and r_1, r_2, etc., the corresponding angles of refraction. Then (see Fig. 136),

$$\frac{\mu_0}{\mu_1} = \frac{\sin r_1}{\sin i_0} = \frac{p_1}{p_0},$$

$$\frac{\mu_1}{\mu_2} = \frac{\sin r_2}{\sin i_1} = \frac{p_2}{p_1}, \text{ etc.}$$

Hence,

$$\mu \, p = \text{constant.}$$

Let the tangent to the ray path, where it enters the atmosphere, meet the vertical at the distance, C, above the point of observation, A; let μ_s be the refractive index at A; θ the apparent zenith distance; $\delta\theta$ the total refraction; and R the radius of the earth. Then, since the refractive index of space is 1,

$$\mu_s \, p_s = p_0, \text{ or } \mu_s \, R \sin \theta = (R + C) \sin (\theta + \delta\theta) \; . \; . \; . \; . \; . \; (A)$$

Obviously, the refraction, $\delta\theta$, could be determined from this equation directly if the value of C were known. But

$$C = \frac{u}{\sin \theta} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (B)$$

in which u, the total linear deviation of the ray, may be substituted by known terms. Hence, C and, therefore, $\delta\theta$ are determinable.

To determine u, let α be the angle which the ray makes with the direction of most rapid increase of index of refraction (at the surface α equals θ); z the vertical coördinate; and v the velocity of light. Now consider a wave front moving through the atmosphere in any direction except vertical. The portion in the higher or thinner air will move faster than that in the denser air and the path will be curved. If ρ is the radius of curvature and $d\rho$ is regarded as positive when measured towards the centre, then, as is obvious from Fig. 137,

$$\frac{d\rho}{\rho} = -\frac{dv}{v}$$

Also, since the refractive index, μ, is inversely proportional to the velocity, v,

$$\frac{d\rho}{\rho} = \frac{d\mu}{\mu}$$

Hence, calling the path s, and since

$$\frac{d\mu}{d\rho} = \frac{d\mu}{dz}\sin\alpha,$$

$$\frac{1}{\rho} = \frac{d\log\mu}{d\rho} = \frac{d\log\mu}{dz}\sin\alpha = \frac{d\log\mu}{ds}\tan\alpha = \frac{d^2u}{ds^2}$$

To a close approximation,

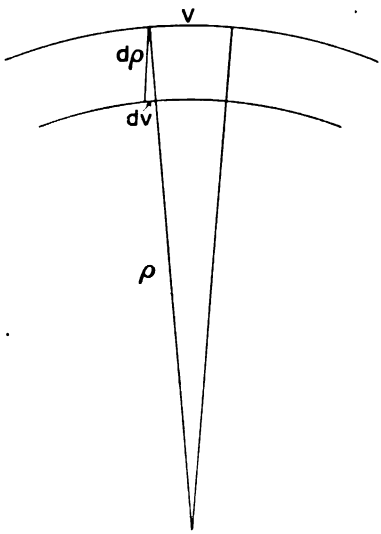
$$\frac{du}{ds} = \int \tan\alpha\, d\log\mu = \tan\alpha\,(\mu - 1) + a,$$

in which a is the constant of integration and $\mu - 1$ is substituted for $\log_e\mu$, μ being but little greater than 1, and

$$u = \tan\alpha\int(\mu - 1)\,ds + as + b,$$

$$= \frac{\sin\alpha}{\cos^2\alpha}\int(\mu - 1)\,dz + as + b \ldots\ldots \text{(C)}$$

FIG. 137.



Curvature of a light path in the air.

But $\mu - 1$ is directly proportional to the density. Hence, if the height, h, in "the homogeneous atmosphere" be such that air below it is the same as below z in the actual atmosphere, and if the origin be taken at the surface where $\alpha = \theta$, and $\mu = \mu_s$

$$u = \frac{\sin\theta}{\cos^2\theta}\int_0^h(\mu - 1)\,dz = \frac{\sin\theta}{\cos^2\theta}(\mu_s - 1)\,h.$$

For the limit of the atmosphere, and any point beyond, $h = H$, the height of the homogeneous atmosphere, or about 7.990×10^5 cm.

For stars, therefore, viewed from the surface of the earth,

$$u = (\mu_s - 1) H \frac{\sin \theta}{\cos^2 \theta} \quad \ldots\ldots\ldots\ldots\ldots \text{(D)}$$

Substituting this value of u in (B) we get

$$C = \frac{(\mu_s - 1) H}{\cos^2 \theta}.$$

Hence, substituting in (A),

$$\mu_s R \sin \theta = \left\{ R + \frac{(\mu_s - 1) H}{\cos^2 \theta} \right\} \left\{ \theta + \delta \theta \right\}.$$

With this equation it is only a matter of arithmetic to compute a table of corrections, that is, values of $\delta\theta$ for every value of θ and μ_s; θ being the apparent or observed zenith distance, and μ_s the current refraction at the surface, given by the equation

$$\mu_s = \frac{273 \mu_0 B}{760 T}$$

in which B is the height of the barometer in millimetres, T the absolute temperature in degrees C., and μ_0 atmospheric refraction at 0° C. and 760 mm. pressure.

Since $\delta\theta$ is small, we have from (A), to a close approximation,

$$\begin{aligned}
\delta\theta = \sin \delta\theta &= \frac{\mu_s \tan\theta}{1 + C/R} - \tan \theta \cos \delta\theta \\
&= \tan \theta \left(\frac{\mu_s}{1 + C/R} - \sqrt{1 - \sin^2 \delta\theta} \right) \\
&= \tan \theta \left\{ \frac{\mu_s}{1 + C/R} - 1 + \tfrac{1}{2} (\delta\theta)^2 \right\} \\
&= \tan \theta \left\{ \mu_s - \frac{\mu_s C}{R} - 1 + \tfrac{1}{2} (\delta\theta)^2 \right\}, \text{ nearly.}
\end{aligned}$$

But, from the laws of refraction,

$$\mu_s \sin \theta = \sin (\theta + \delta\theta)$$

from which

$$\delta\theta = (\mu_s - 1) \tan \theta, \text{ nearly.}$$

Hence,

$$\delta\theta = (\mu_s - 1) \tan \theta \left\{ 1 - \frac{\mu_s C}{(\mu_s - 1) R} + \tfrac{1}{2} (\mu_s - 1) \tan^2 \theta \right\}, \text{ approximately.}$$

Substituting for C its value and noting that

$$\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$$

this equation reduces to

$$\delta\theta = (\mu_s - 1)\left(1 - \frac{\mu_s H}{R}\right)\tan\theta - (\mu_s - 1)\left(\frac{\mu_s H}{R} - \frac{\mu_s - 1}{2}\right)\tan^3\theta$$

$$= (\mu_s - 1)\left(1 - \frac{H}{R}\right)\tan\theta - (\mu_s - 1)\left(\frac{H}{R} - \frac{\mu_s - 1}{2}\right)\tan^3\theta, \text{ nearly.}$$

If $H = 7.990 \times 10^5$ cm.; $R = 6.3709 \times 10^8$ cm.; and $\mu_s - 1 = .0002927$, all closely approximate values, then,

$$\delta\theta = 60''.29 \tan\theta - 0''.06688 \tan^3\theta.$$

This is Lord Rayleigh's final equation, and it appears to be exceedingly accurate for all values of θ up to at least 75°, or as far, perhaps, as irregular surface densities generally allow any refraction formula to be used with confidence.

Since the index of refraction varies from color to color, it follows that star images are drawn out into vertical spectra. The amount of this effect, however, is small. Thus the difference between the refractions of red and blue-green is only about one one-hundredth of the total refraction of yellow (D) light. Hence, the approximate angular distance between the red and blue-green images of a star at the zenith distances, 30°, 45°, 60° and 75° are 0''.35, 0''.60, 1''.04, and 2''.24, respectively. Possibly this may account, in part at least, for the fact that stellar declinations, as determined from the northern and southern hemispheres, respectively, are not quite the same.

Scintillation or Twinkling and Unsteadiness of Stars.—The scintillation or twinkling of stars, that is, their rapid changes in brightness and occasionally also in color, especially when near the eastern or western horizon, is a well-known, and now well-understood, phenomenon that for many centuries, certainly since the days of Aristotle (384–322 B.C.), who noted the fact that the fixed stars twinkle while the planets shine with comparatively steady lights, has been observed, investigated, and discussed. The most systematic and complete observations, however, of scintillation are those made by Respighi [168] with a spectroscope

[168] Assoc. Française pour l'Avancement des Sciences, I, 1872, p. 148.

during the years 1868–1869, and which he summed up substantially as follows:

(1) In the spectra of stars near the horizon more or less broad and distinct dark and bright bands sweep with greater or less velocity from the red to the violet and from the violet to the red or oscillate from the one to the other; and this whatever the direction of the spectra from the horizontal to the vertical.

(2) When the conditions of the atmosphere are normal the dark and bright bands of western stars travel regularly from the red to the violet, and of eastern from the violet to the red; while in the neighborhood of the meridian they usually oscillate from the one color to the other, or even are limited to a portion of the spectrum.

(3) On examining the horizontal spectrum of a star sensibly parallel dark and bright bands more or less inclined to the axis (transverse) of the spectrum are seen passing from the red to the violet or reversely, according as the star is in the west or east.

(4) The inclination of the bands, or angle between them and the axis (transverse) of the spectrum, depends upon the altitude of the star; increasing rapidly from 0° at the horizon to 90° at an elevation of 30° to 40°, where they, therefore, are longitudinal.

(5) The inclination of the bands, reckoned from above, is towards the violet end of the spectrum.

(6) The bands, most distinct at the horizon, become less conspicuous with increase of elevation. Above 40° the longitudinal bands reduce to mere shaded streaks and often can be seen in the spectrum only as slight general changes of brightness.

(7) With increase of elevation the movement of the bands becomes more rapid and less regular.

(8) On turning the spectrum from the horizontal the inclination of the bands to the transversal continuously diminishes until it becomes zero, when the spectrum is nearly vertical. They also become less distinct, but continue always to move in the same direction.

(9) The bright bands are less frequent and more irregular than the dark, and are well-defined only in the spectra of stars near the horizon.

(10) In the midst of this general and violent movement of light and shade over the spectra of stars, the Fraunhofer lines peculiar to the light of each star remain quiescent or are subject to only very slight oscillations.

(11) When the atmospheric conditions are abnormal, the bands are fainter and more irregular in form and movement.

(12) When the wind is strong, the bands usually are quite faint and ill-defined; the spectra even of stars near the horizon showing mere changes of brightness.

(13) Good definition and regular movement of the bands appear to indicate the continuance of fair weather, while varying definition and irregular motion seem to imply a probable change.

These observations show that the dark bands are due to temporary deflection of light from the object glass by irregularities in the density of the atmosphere. For stars near the horizon, the linear separation of the rays of different color is so great as they pass through the atmosphere to the observer that successive portions of the spectrum may be deflected from or concentrated onto (light deficiencies in a transparent medium must be balanced by light concentrations, and *vice versa*) the telescope or eye. Hence, the progression of bright and dark bands along the spectra of low altitude stars, and their rapid change of color to the unaided eye.

Further, since the path of the more refrangible light necessarily lies above that of the less refrangible (Montigny's principle) it follows that an atmospheric irregularity travelling or

rotating with the earth would affect the different-colored rays from stars in the west in the order of red, green, blue, violet, and rays from stars in the east in the reverse order. If, then, the separation of the extreme rays is large in comparison to the effective dimension of the air irregularity, the resulting band will, at any given instant, cover only a portion of the spectrum. But the approximate amount of this separation is readily obtained from equation (D), page 438. Thus, for the limit of the atmosphere,

$$du = d\mu_s H \frac{\sin\theta}{\cos^2\theta}$$

Hence, at the zenith distance, 80°, the red and violet rays simultaneously received by the observer from the same star will be separated at the limit of the atmosphere (assuming the dispersion between these rays to be one-fiftieth the refraction of yellow light) by about 156 centimetres, and proportionately for levels below which definite fractions of the total mass of the atmosphere lie. For the zenith distance, 40°, however, the corresponding separation at the limit of the atmosphere is only about 5 centimetres, and for 20° about 2 centimetres. Inequalities in the atmosphere may, therefore, interfere with only a portion at a time of the spectrum of a star near the horizon and thus produce the phenomenon of a travelling band, while in the case of a star whose zenith distance is 40°, or less, the interference will include nearly or quite the entire spectrum, and thus produce mere changes of brightness.

It is generally stated that the direction of travel of the bands during fine weather, red to violet for stars in the west, violet to red for stars in the east, and irregularly or simultaneously over the entire spectrum for stars near the meridian, is directly dependent upon the west to east rotation of the earth. It is correctly stated (on assumption of a stationary atmosphere) that this rotation would cause an atmospheric irregularity to affect the red rays first and the violet last, violet first and red last, and all rays more or less simultaneously, of stars in the west, east, and near the meridian, respectively. But the order would be the same if the earth were at rest and the air travelling from west to east. As a matter of fact, over most of the earth outside the tropics the west to east angular velocity of the general winds, as seen by the observer, is several times that of the earth. Hence,

the rate at which the disturbance drifts across the line of sight presumably depends much more on the direction of the prevailing winds than upon the rotation of the earth. Indeed, in tropical regions, where the prevailing winds are from easterly points, the usual direction of travel of the bands probably (if the above reasoning is correct) is reversed.

The disappearance of distinct bands with high winds is due, of course, to the more complete mixing of the atmosphere at such times.

In the same general way, atmospheric inequalities produce "unsteadiness," or rapid changes in the apparent positions of stars as seen in a telescope. In reality, this is a telescopic form of scintillation which, because never amounting to more than a very few seconds of arc, the unaided eye cannot detect. On the other hand, the great changes in brightness and color so conspicuous to the naked eye are scarcely if at all noticeable in a large telescope. This is because the object glass is so large that, in general, light deflected from one portion of it is caught in another.

Scintillation of the Planets, Sun, and Moon.—It is commonly stated that the planets do not scintillate—that the light from the several portions of their disks follow such different paths through the atmosphere that not all nor even any large portion of it can be affected at any one time. It is true that because of their sensible disks the scintillation of planets is much less than that of fixed stars, but under favorable circumstances their scintillation is quite perceptible. Even the rims of the sun and the moon boil or "scintillate" while, of course, any fine marking on either or on a planet is quite as unsteady as the image of a fixed star.

Nature of Irregularities.—It is well known that the atmosphere, generally, is so stratified that with increase of elevation many more or less abrupt changes occur in temperature, composition, density, and, therefore, refrangibility. As such layers glide over each other, billows are formed, and the adjacent layers thereby corrugated. The several layers frequently also heat unequally, largely because of disproportionate vapor contents, and thereby develop, both day and night, and at various levels, innumerable vertical convections; each moving mass differing, of course, in density from the surrounding air, and by the chang-

ing velocity being drawn out into dissolving filaments. Optically, therefore, the atmosphere is so heterogeneous that a sufficiently bright star shining through it would produce on the earth a somewhat streaky pattern of light and shade.

Shadow Bands.—A striking proof of the optical streakiness of the atmosphere is seen in the well-known shadow bands that at the time of a total solar eclipse appear immediately before the second, and after the third, contact.

Terrestrial Scintillation.—A bright terrestrial light of small size, such as an open electric arc, scintillates when seen at a great distance, quite as distinctly as do the stars and for substantially the same reason, that is, optical inequalities due to constant and innumerable vertical convections and conflicting winds.

Shimmering.—The tremulous appearance of objects, the common phenomenon of shimmering, seen through the atmosphere immediately over any heated surface, is another manifestation of atmospheric refraction, and is due to the innumerable fibrous convections that always occur over such an area.

Optical Haze.—The frequent indistinctness of distant objects on warm days when the atmosphere is comparatively free from dust, and ascribed to optical haze, is due to the same thing, namely, optical heterogeneity of the atmosphere, that causes that unsteadiness or dancing of star images that so often interferes with the positional and other exact work of the astronomer. Both are but provoking manifestations of atmospheric refraction.

Times of Rising and Setting of Sun, Moon, and Stars.—An interesting and important result of astronomical refraction is the fact that the sun, moon, and stars rise earlier and set later than they otherwise would. For places at sea level the amount of elevation of celestial objects on the horizon averages about 35′, and therefore the entire solar and lunar disks may be seen before (on rising) and after (on setting) even their upper levels would have appeared, in the first case, or disappeared, in the second, if there had been no refraction. This difference in time of rising, or setting, depends on the angle or inclination, α, of the path to the horizon. In general, it is given by the equation,

$$t = 140^s \csc\alpha, \text{ about.}$$

The minimum time, therefore, occurs when the path is normal to the horizon and is about $2^m\ 20^s$, while the maximum, which

obviously occurs at the poles, is infinite in the case of stars that just clear the horizon, and, for the sun, about a day and a half, the time required near equinox for the solar declination to change by 35′.

Green Flash.—As the upper limb of the sun disappears in a clear sky below a distant horizon its last star-like point often is seen to change rapidly from pale yellow or orange to green and finally blue, or, at least, a bluish-green. The vividness of the green, when the sky is exceptionally clear, together with its almost instant appearance, has given rise to the name "green flash" for this phenomenon. The same gamut of colors, only in reverse order, occasionally is seen at sunrise.

The entire phenomenon has been described by some as merely a complementary after-image effect, that is, the sensation of its complementary color that frequently follows the sudden removal of a bright light. This explanation, however, cannot account for the reverse order of the colors as seen at sunrise. Neither does it account for the twinkling of the "flash" close observation now and then reveals, nor for the fact that when the sun is especially red the "flash" is never seen.

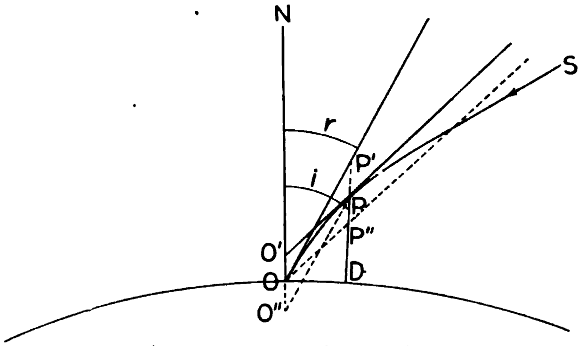
It is not, indeed, a physiological effect, but only the inevitable result of atmospheric refraction, by virtue of which as a celestial object sinks below the horizon its light must disappear in the order of refrangibility; the red first, being least refrangible, then the green, and finally the blue or most refrangible. Violet need not be considered, since only a comparatively small portion of it can penetrate so far through the atmosphere.

It may properly be asked, then, why these color changes do not apply to the whole solar disk. The answer is, because the angular dispersion, due to the refraction of the atmosphere, between the several colors is very small—between red and green, for instance, only about 20″, even when the object is on the horizon, so that any color from a given point on the sun is reinforced by its complementary color, thus giving white, from a closely neighboring point. Hence, color phenomena can appear only when there are no such neighboring points, or when only a minute portion of the disk is above the horizon. It must further be noted that color effects, due to the general refraction of the atmosphere, occur only when the source (brilliant point) is on the horizon. Stars above the horizon are not permanently drawn out

into rainbow bands, and that for the simple reason that the red light by one route is supplemented by the green, blue, etc., by others and the whole blended into white, yellow, or whatever the real color of the star may be. This multiplicity of routes and consequent blending of color, is not possible for rays of light from objects just sinking below or rising above the horizon, and, therefore, under such circumstances, they pass through a series of color changes.

Terrestrial Refraction.—The curving of rays of light is not confined to those that come from some celestial object, but applies also to those that pass between any points within the atmosphere, whether at the same or different levels. This latter

FIG. 138.



Approximate terrestrial refraction.

phenomenon, known as terrestrial refraction, causes all objects on the earth or in the atmosphere to appear to be at greater altitudes than they actually are, except when the surface air is so strongly heated as to cause an *increase* of density with elevation and thus produce the inferior mirage, described below.

Terrestrial refraction is also a matter of great importance, especially to the geodesist, and its complete analysis, from which practical tables may be constructed, essentially the same as that of astronomical refraction.[168a] It will be instructive, however, to consider a few graphical corrections of apparent elevations.

Let *ON* (Fig. 138) be normal to the surface, *OD*, of the earth, and let *P* be observed from *O*. Obviously *P*, whose horizontal distance, *OD*, may be supposed known, will seem to be at *P'* and thus its apparent altitude greater than the actual by the distance

[168a] McLeod, *Phil. Mag.* 38, p. 546, 1919.

PP'. From the angle r, the density of the air at the observer's position, O, and the approximate density at P (known from the approximate height of P) it is easy to draw OP'' parallel to the tangent at P to the refraction curve, SPO. This gives P'', necessarily below P. Hence P lies somewhere between P' and P''.

A more exact determination could be had by drawing the curve of refraction, OPS, corresponding to the angle r and noting its intersection with the normal at D.

If the observer happens to be at P, the point O will appear elevated to O'. Clearly, however, from a knowledge of the angle $PO'N$, and the approximate air densities at P and O, one may draw PO'' parallel to $P'O$, and thus locate O somewhere between the two determined points O' and O''. Here, too, it would be more accurate to use the refraction curve, SPO.

Even initially horizontal rays normally curve down towards the surface of the earth, so that objects at the observer's own level, as well as those above and below it, appear elevated. To understand this phenomenon, consider a wave front normal to the surface of the earth and, consequently, moving horizontally. If, now, the density of the air at the place in question decreases with increase of elevation, as it nearly always does, the upper portion of the wave front will travel faster than the lower, and the path will be bent down towards the earth along a curve whose radius depends upon the rate of this density decrease. For example, let the corrected height of the barometer be 760 mm., the temperature 17° C., and the rate of temperature decrease with elevation 5° C. per kilometre; conditions that not infrequently obtain at sea level. On substituting these values in the density-elevation equation, it appears that the density gradient would be such that if continuous the limit of the atmosphere would be reached at an elevation of about 10 kilometres. Hence, under these circumstances, the velocity of light at an elevation of 10 kilometres would be to its velocity at the surface in the ratio of 1,000,276 to 1,000,000, approximately, since the refractive index of the lower air would be 1.000,276, about. The radius of curvature, r, therefore, is closely given in kilometres by the equation,

$$\frac{r}{r + 10} = \frac{1,000,000}{1,000,276}.$$

Hence, $r = 36{,}232$ kilometres, or approximately 5.7 times the radius of the earth.

It is conceivable, therefore, that the size of a planet and the vertical density gradient of its atmosphere might be such that one's horizon on it would include the entire surface—that he could look all the way round and, as some one has said, see his own back.

The distance to the horizon, corresponding to a given altitude, obviously depends upon the rate of vertical density decrease in such manner that when the latter is known the approximate value of the former can easily be computed. Thus, let the density decrease be such that the radius of curvature of a ray tangent to the surface shall be 5.7 R, $R = 6366$ kilometres, being the radius of the earth; let α be the angle between the radii from the centre of the earth to the observer and a point on his unobstructed horizon respectively; let h be the observer's height in metres above the level of his horizon; and let r be the distance, in kilometres, measured over the surface from the horizon to a point on the same level below the observer; then, by trigonometry, to a close approximation;

$$h = 6366000\ (\sec\alpha - 1) - 36286200\ \left(\sec\frac{\alpha}{5.7} - 1\right)\sec\alpha$$

and

$$r = 6366\ \alpha,\ \alpha \text{ in radians}$$

A few values of the distance to the horizon from different elevations, computed by the above formula, are given in the following table:

Distance to Horizon.

Distance in Kilometres.	1	2	5	10	20	50	100
Elevation in metres.	.061	.263	1.613	6.856	25.901	161.918	647.604

Looming.—Since the extension of the actual beyond the geometrical horizon depends, as just explained, upon the density decrease of the atmosphere with increase of elevation, it follows that any change in the latter must produce a corresponding variation of the former. An increase, for instance, in the normal rate of decrease, such as often happens over water in middle to high latitudes, produces the phenomenon of looming, or the coming into sight of objects normally below the horizon, a classical in-

stance of which was described by Latham.[169] Similar changes in the rate of density decrease with increase of elevation also are common in valleys, but here looming, in the above sense, is rendered impossible by the surrounding hills or mountains.

Towering.—The condition of the atmosphere that produces looming, in the sense here used, or would produce it if the region were level, often gives rise to two other phenomena, namely, unwonted towering, also usually called looming, and the consequent apparent approach of surrounding objects.

The more rapid the downward curvature of the ray paths at the observer the more elevated, clearly, will objects appear to be, and such curvature may, indeed, be very considerable. Thus, a temperature inversion near the surface of the earth of 1° C. per metre change of elevation bends down a ray along an arc whose radius is about 0.16 that of the earth, while an inversion of 10° C. per metre—a possible condition through a shallow stratum—gives a radius of only about 0.016 that of the earth, or, say, 100 kilometres. If now, as occasionally happens, the inversion layer is so located that rays to the observer from the top of an object are more curved than those from the bottom, it will appear not only elevated but also vertically magnified—it will tower and seem to draw nearer.

Sinking.—Instead of increasing the curvature of rays the temperature distribution may be such as, on the contrary, to decrease it and thereby cause objects normally on the horizon to sink quite beyond it. Such phenomena, exactly the reverse of looming, are also most frequently observed at sea.

Stooping.—Occasionally rays from the base of an object may be curved down much more rapidly than those from the top, with the obvious result of apparent vertical contraction, and the production of effects quite as odd and grotesque as those due to towering. Indeed, since the refraction of the atmosphere increases, in general, with the zenith distance, it is obvious that the bases of objects nearly always apparently are elevated more than their tops, and themselves, therefore vertically shortened. The normal effect, however, is small and seldom noticed except, perhaps, in connection with the slightly flattened shape of the sun and moon when on the horizon.

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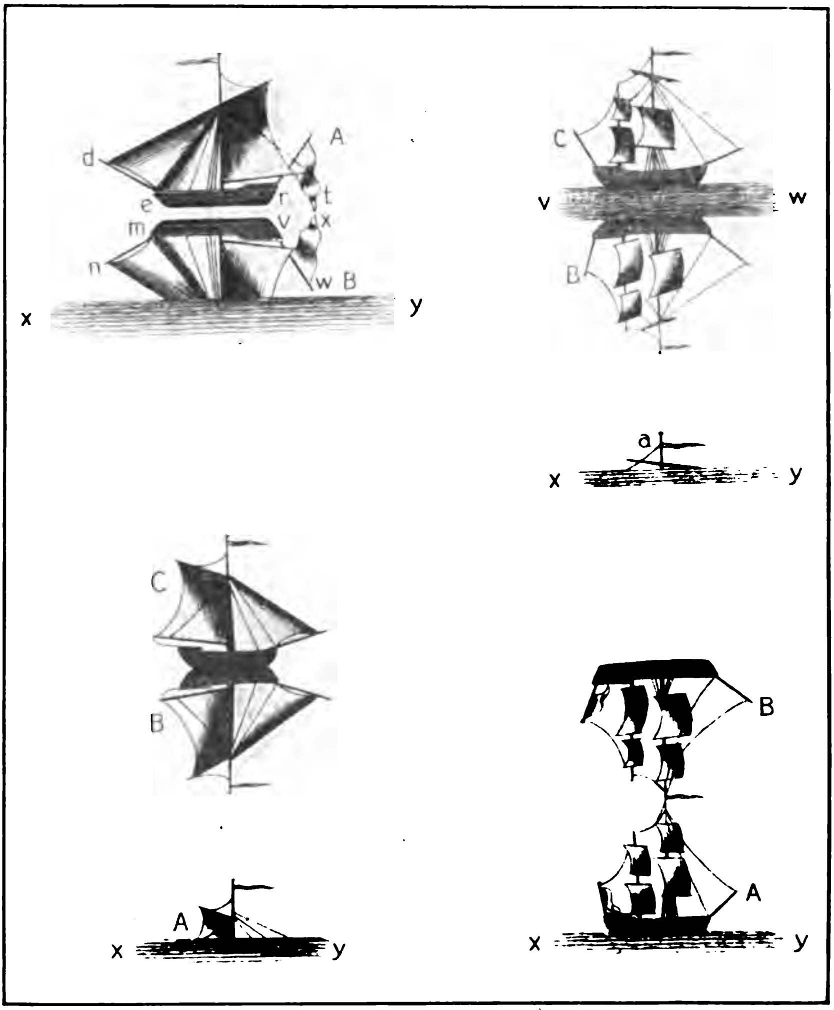
[169] *Phil. Trans.*, v. **88**, 1798; abridged, v. **18**, p. 337. See also Everett, "Nature," v. **11**, p. 49, 1874.

Superior Mirage.—It occasionally happens that one or more images of a distant object, a ship for instance, are seen directly

FIG. 139.



Examples of superior mirage (Vince).

above it, as shown in Fig. 139, copied from Vince's [170] well-known description of exceptionally fine displays of this phenomenon.

[170] *Phil. Trans.*, 89 (8, abridged, p. 436), 1799.

The image nearest the object always is inverted and therefore appears as though reflected from an overhead plane mirror—hence the name "superior mirage"—and, indeed, many seem to assume that this image really is due to a certain kind of reflection, that is, total reflection, such as occurs at the undersurface of water. It is obvious, however, that this assumption is entirely erroneous, since the atmosphere can never be sufficiently stratified in nature to produce the discontinuity in density (adjacent layers are always interdiffusible) this explanation of the origin of the proximate inverted image presupposes. Another apparently simple explanation of mirage phenomena is furnished by drawing imaginary rays from the object along arbitrary paths to the observer. But, in reality, this is no explanation at all, unless it is first demonstrated that the rays must follow the paths assumed. It is allowable, of course, to assume any *possible* distribution of atmospheric density and to trace the rays from an object accordingly. If the assumed distribution follows a simple law, the rays may be traced mathematically, as by Mascart,[171] though such discussions, when at all thorough, necessarily are long.

A simple explanation of mirage that admirably accounts for the phenomena observed has been given by Hastings,[172] in substance as follows:

Let the air be calm and let there be a strong temperature inversion some distance, 10 metres, say, above the surface—conditions that occasionally obtain, especially over quiet water. Obviously the ratio of decrease of density to increase of elevation is irregular in such an atmosphere, and therefore the velocity of light travelling horizontally through it must increase also irregularly with increase of elevation. Thus, beginning with the under surface of the inversion layer, the rate of velocity increase with elevation must first grow to a maximum and then diminish to something like its normal small value at and beyond the upper surface of this layer. Hence, that portion of an originally vertical, or approximately vertical, wave front that lies within the inversion layer must soon become doubly deflected, substantially as indicated in Fig. 140.

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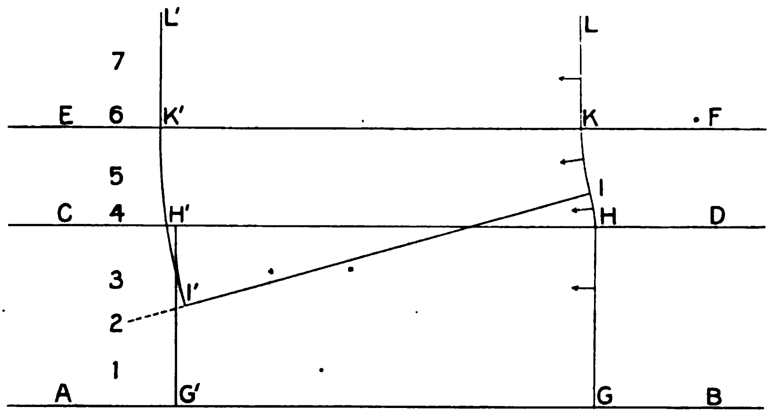
[171] "Traité d'Optique," v. 3, pp. 305–308.

[172] "Light," Chapter 7, New York, Chas. Scribner's Sons, 1902.

Let *AB* (Fig. 140) be the surface of water, say, *CD* the under and *EF* the upper surface of a strong inversion layer, and let *GHIKL* be the distorted wave front of light, travelling in the direction indicated, from a distant source. The future approximate positions of the wave front, of which *G'H'I'K'L'* is one, are readily located from the fact that its progress is always normal to itself, and the appearance of the distant object from which these wave fronts are proceeding easily determined. At 1, for instance, the object seems upright and at its proper level, no images are seen and the whole appearance is normal; at 2 confused elevated images appear, in addition to the object itself; at 3 the

FIG. 140.



Wave fronts giving a superior mirage (Hastings).

object and two distinct images are seen, the lower produced by the segment *HI* of the wave front inverted, the other erect; at 4, the under surface of the inversion layer, the inverted image blends with the object and disappears; at 5 only the erect image can be seen, and, indeed, may be seen even when the object itself normally would be below the horizon; at 6, the upper surface of the inversion layer, the vertical image merges with the object and disappears; while everywhere beyond the upper surface only the object itself is visible, as at 1, with no evidence whatever of abnormal refraction and mirage.

An additional inversion layer obviously might produce other images, while more or less confused layers might produce multiple and distorted images, such as shown in Fig. 141, copied from

Scoresby's account [178] of a certain telescopic view of the east coast of Greenland.

Inferior Mirage.—It is a very common thing in flat desert regions, especially during the warmer hours of the day, to see below distant objects and somewhat separated from them their apparently mirrored images—the inferior image. The phenomenon closely simulates, even to the quivering of the images, the reflection by a quiet body of water of objects on the distant shore, the "water" being the image of the distant low sky, and therefore frequently leads to the false assumption that a lake or bay

FIG. 141.

Telescopic appearance of the coast of Greenland, at the distance of 35 miles, when under the influence of an extraordinary refraction. July 18, 1820. Lat. 71° 20', Long. 17° 30' W.

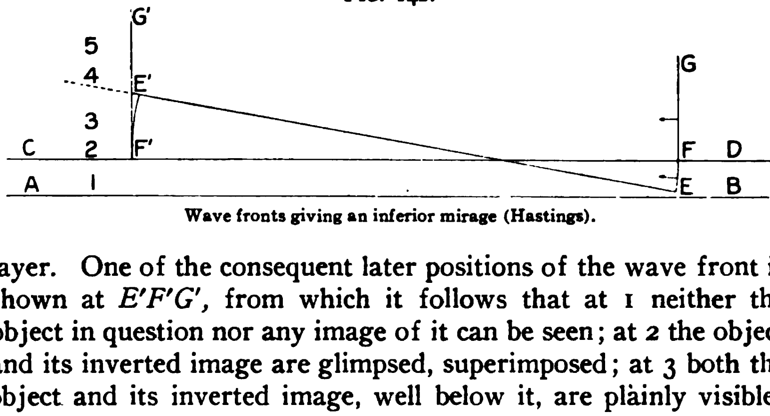
is close by. This type of mirage is very common on the west coast of Great Salt Lake. Indeed, on approaching this lake from the west one can often see the railway over which he has just passed apparently disappearing beneath a shimmering surface. It is also common over smooth-paved streets provided one's eyes are just above the street level. An under-grade crossing in a level town, for instance, offers an excellent opportunity almost any warm day of seeing well-defined small images that are apt to arouse one's surprise at the careless way his fellow-citizens wade through pools of water!

[178] *Trans. Roy. Soc. Edinburgh*, v. 9, p. 299, 1823.

Since the inferior mirage occurs only over approximately level places and there only when they are so strongly heated that for a short distance the density of the atmosphere increases with elevation it follows that its explanation is essentially the same as that of the superior mirage. Of course, the surface air is in unstable equilibrium and rising in innumerable filaments, but its rarefied state is maintained so long as there is an abundant supply of insolation. A wave front, therefore, from an object slightly above the general level soon becomes distorted through the greater speed of its lower portion, as schematically indicated in Fig. 142.

Let *AB* of this figure be the surface of the earth and *CD* the upper level of the superheated stratum. Let *EFG* be the position of a wave front travelling as indicated, the lower portion curved forward as a result of its greater speed in the rarefied

FIG. 142.



Wave fronts giving an inferior mirage (Hastings).

layer. One of the consequent later positions of the wave front is shown at *E'F'G'*, from which it follows that at 1 neither the object in question nor any image of it can be seen; at 2 the object and its inverted image are glimpsed, superimposed; at 3 both the object and its inverted image, well below it, are plainly visible; at 4 the image is just disappearing; while at 5 there is no evidence of a mirage, unless of objects more distant than the one under consideration.

Great uncertainty may exist, therefore, in regard to the exact positions of objects seen in (or perhaps hidden by) a mirage. Thus, in his official report of the battle of April 11, 1917, between the English and the Turks in Mesopotamia, General Maude, the British commander, says: "The fighting had to be temporarily suspended owing to a mirage."

Lateral Mirage.—Vertical sheets of abnormally dense or abnormally rare atmosphere obviously would produce lateral mirages in every way like those due to similar horizontal layers,

and indeed such mirages are occasionally seen along walls and cliffs whose temperatures differ widely from that of the air a few metres from them.

Fata Morgana.—Morgana (Breton equivalent of sea woman), according to Celtic legend and Arthurian romance, was a fairy, half-sister of King Arthur, who exhibited her powers by the mirage. Italian poets represent her as dwelling in a crystal palace beneath the waves. Hence, presumably, the name Fata Morgana (Italian for Morgan le Fay, or Morgan the fairy) was given, centuries ago, to those complicated mirages that occasionally appear over the strait of Messina moulding the bluffs and houses of the opposite shore into wondrous castles that alike tower into the sky and sink beneath the surface; nor is it strange that this poetical name should have become generic, as it has, for all such multiple mirages wherever they occur.

According to Forel,[174] this phenomenon, to which he has given much attention, results from the coexistence of the temperature disturbances peculiar to both the inferior and superior mirages, such as might be produced by a strong inversion over a relatively warm sea. This, of course, implies a marked increase of density with elevation to a maximum a short distance above the surface, followed by a rapid density decrease—an unstable condition and therefore liable to quick and multiform changes. Obviously, too, such a cold intermediate layer in addition to producing a double mirage acts also as a sort of cylindrical lens that vertically magnifies distant objects seen through it.

No wonder, then, that under such circumstances the most commonplace cliffs and cottages are converted, through their multiple, distorted, and magnified images, into magic castles, or the marvellous crystal palaces of Morgan le Fay!

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[174] *Archives des Sciences Phys. et Nat.*, v. 32, p. 471, 1911.

Chapter III.

REFRACTION PHENOMENA: REFRACTION BY WATER DROPS—RAINBOW.

Principal Bows.—It may seem entirely superfluous to describe so common a phenomenon as the rainbow, or to offer more than the simple explanation of it that may be found in innumerable text-books. But rainbows differ among themselves as one tree from another, and besides some of their most interesting features usually are not even mentioned—and naturally so, for the "explanations" generally given of the rainbow may well be said to explain beautifully that which does not occur and to leave unexplained that which does.

The ordinary rainbow, seen on a sheet of water drops—rain or spray—is a group of circular or nearly circular arcs of colors whose common centre is on the line connecting the observer's eye with the exciting light (sun, moon, electric arc, etc.) or rather, except rarely, on that line extended in the direction of the observer's shadow. A very great number of rainbows are theoretically possible, as will be explained later, and doubtless all that are possible actually occur, though only three (not counting supernumeraries) certainly have been seen on sheets of rain. The most brilliant bow, known as the *primary*, with red outer border of about 42° radius and blue to violet inner border, appears opposite the sun (or other adequate light); the next brightest, or the *secondary* bow, is on the same side of the observer, but the order of its colors is reversed and its radius, about 50° to the red, is larger; the third or *tertiary* bow, having about the same radius as that of the primary and colors in the same order, lies between the observer and the sun, but is so faint that it is rarely seen in nature. Obviously the common centre of the primary and secondary bows is angularly as far below the observer as the source (sun generally) is above, so that usually less than a semicircle of these spectral arcs is visible, and never more, except from an eminence.

The records of close observations of rainbows soon show that not even the colors are always the same; neither is the band of any color of constant angular width; nor the total breadth of the several colors at all uniform: similarly the purity and brightness of the different colors are subject to large variations. The great-

est contrast, perhaps, is between the sharply-defined brilliant rainbow of the retreating thunderstorm and that illy-defined faintly tinged bow that sometimes appears in a mist—the " white bow " or " fog bow."

All these differences depend, as will be explained later, essentially upon the size of the drops, and therefore inequalities often exist between even the several portions, especially top and bottoms, of the same bow, or develop as the rain progresses. Additional complications occasionally result from the reflection of bows and from bows produced by reflected images of the sun, but though unusual and thus likely to excite wonder and comment such phenomena are easily explained.

Supernumerary Bows.—Rather narrow bands of color, essentially red, or red and green, often appear parallel to both the primary and the secondary bows, along the inner side of the first and outer of the second. These also differ greatly in purity and color, number visible, width, etc., not only between individual bows but also between the several parts of the same bow. No such colored arcs, however, occur between the principal bows; indeed, on the contrary, the general illumination here is perceptibly at a minimum.

Deviation in Direction of Emerging from Entering Ray.—Since a raindrop is spherical, its action on an enveloping wave front may be obtained by determining first the effects in the plane of a great circle containing an entering ray, and then revolving this plane about that line in it that bisects the angle between the incident and emerged paths of any given ray in the same plane. Let, then, ABC (Fig. 143) be the plane of a great circle of an enlarged raindrop and let $S\ A\ B\ C\ E$ be the path of a ray in this plane, entering the drop at A and emerging at C. The changes in direction at A and C are each $i-r$, in which i is the angle of incidence and r the angle of refraction, and the change at B, as also at every other place of an internal reflection, when there are more than one, is $\pi - 2r$. Hence, the total deviation, D, is given by the equation,

$$D = 2\,(i - r) + n\,(\pi - 2r) \quad \ldots\ldots\ldots\ldots (1)$$

in which n is the number of internal reflections.

Minimum Deviation.—The above general expression for the deviation shows that it varies with the angle of incidence. There is also a minimum deviation, corresponding to a particular angle

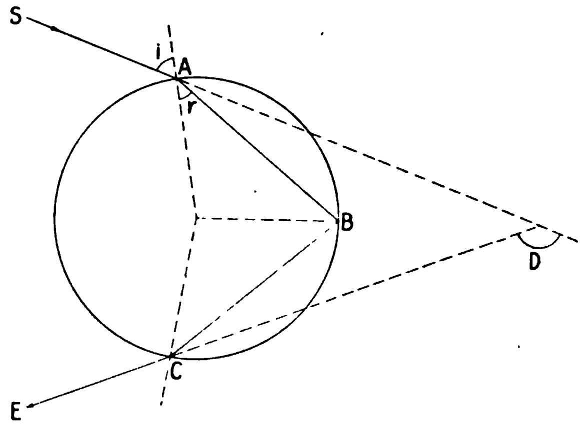
of incidence, as may be shown in the usual way. Thus by equation (1),

$$dD = 2di - 2(n + 1)\,dr$$

which, on putting $dD = 0$, the condition of stationary (maximum or minimum) deviation, gives,

$$di = (n + 1)\,dr \ldots\ldots\ldots\ldots\ldots \quad (2)$$

FIG. 143.



Change in light direction by raindrop.

But $\sin i = \mu \sin r$, in which μ is the refractive index of water with reference to air, and, therefore,

$$\cos i\, di = \mu \cos r\, dr.$$

Hence, by (2)

$$\mu \cos r = (n + 1) \cos i$$

and

$$\cos i = \sqrt{\frac{\mu^2 - 1}{n^2 + 2n}}.$$

This value of the angle of incidence corresponds, as stated, to a stationary deviation, but whether of maximum or minimum value may be determined by noting the sign of the second differential, which gives:

$$\frac{d^2D}{di^2} = -2(n + 1)\frac{d^2r}{di^2}$$

But

$$\frac{dr}{di} = \frac{\cos i}{\mu \cos r},$$

and

$$\frac{d^2r}{di^2} = \frac{(1 - \mu^2)\sin i}{\mu^3 \cos^3 r},$$

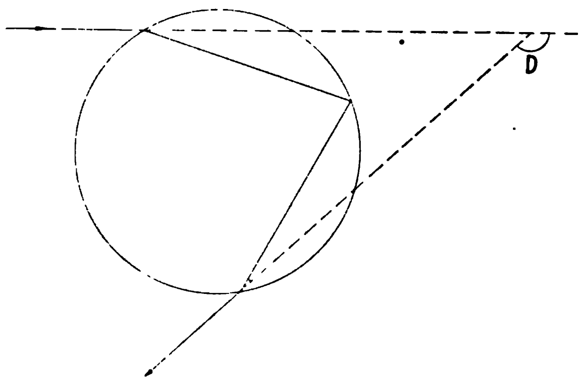
Hence, as μ is greater than unity, this latter value is negative and therefore the second differential of D with respect to i is positive, and the corresponding value of D is a minimum.

The following table gives these values for the primary, secondary, tertiary, quaternary, and quinary rainbows corresponding to 1, 2, 3, 4, and 5 internal reflections, respectively, as shown in Figs. 144, 145, 146, 147, and 148.

		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
Violet, H	i	$58° 48'$	$71° 30'$	$76° 36'$	$79° 27'$	$81° 17'$
$\lambda=3968.5$	r	$39° 33'$	$44° 54'$	$46° 23'$	$47° 2'$	$47° 22'$
$\mu=1.3435$	D	$\pi-40° 36'$	$2\pi-126° 24'$	$2\pi-37° 52'$	$3\pi-131° 26'$	$3\pi-45° 50'$
Yellow	i	$59° 23'$	$71° 50'$	$76° 50'$	$79° 38'$	$81° 26'$
$\lambda=$ 5800.0, about	r	$40° 12'$	$45° 27'$	$46° 55'$	$47° 32'$	$47° 52'$
$\mu=\frac{4}{3}$	D	$\pi-42° 2'$	$2\pi-129° 2'$	$2\pi-41° 40'$	$3\pi-136° 4'$	$3\pi-51° 32'$
Red, $H\alpha$	i	$59° 31'$	$71° 54'$	$76° 53'$	$79° 40'$	$81° 28'$
$\lambda=6562.9$	r	$40° 21'$	$45° 34'$	$47° 2'$	$47° 39'$	$47° 59'$
$\mu=1.3311$	D	$\pi-42° 22'$	$2\pi-129° 36'$	$2\pi-42° 30'$	$3\pi-137° 10'$	$3\pi-52° 52'$

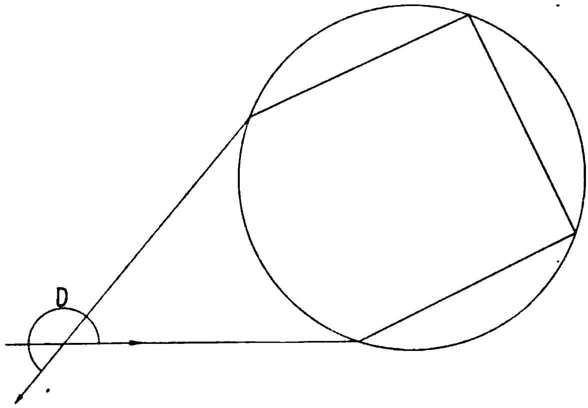
Entering and Emerging Rays.—Since a raindrop is spherical, it is obvious that its effect on incident radiation from the sun, or other spherical or point source, is symmetrical about an axis through the centre of the drop and the luminous object. Hence, in the study of the rainbow, it is sufficient to use only a single plane containing this axis, tracing the rays incident over one

FIG. 144.



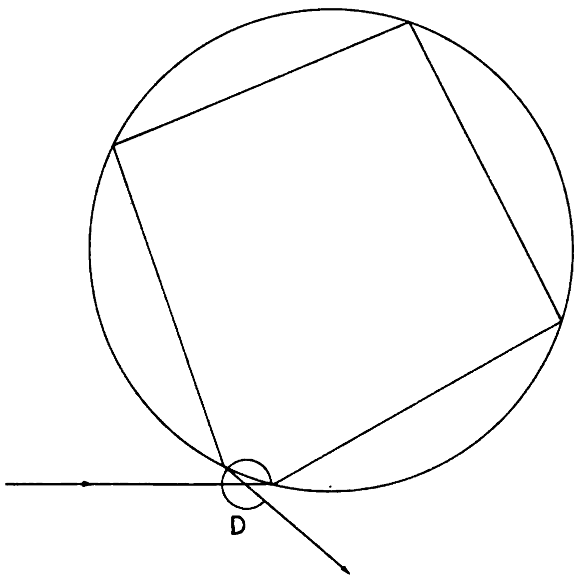
Change of light direction in primary rainbow.

FIG. 145.



Change of light direction in secondary rainbow.

FIG. 146.

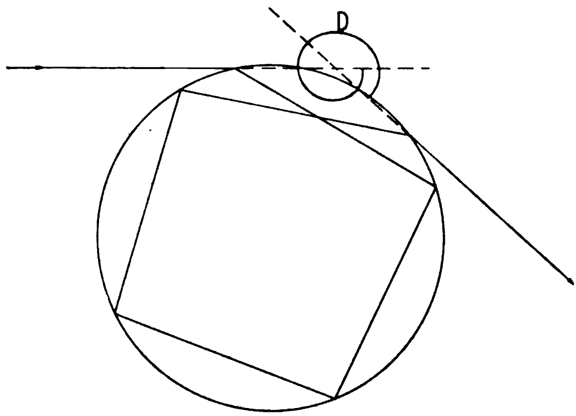


Change of light direction in tertiary rainbow.

quadrant of the intersection circle and noting the resulting phenomena. It is also obvious that, neglecting sky light, solar rays are parallel to within the angular diameter of the sun, 0.5° about, and that as a first approximation they may be regarded as

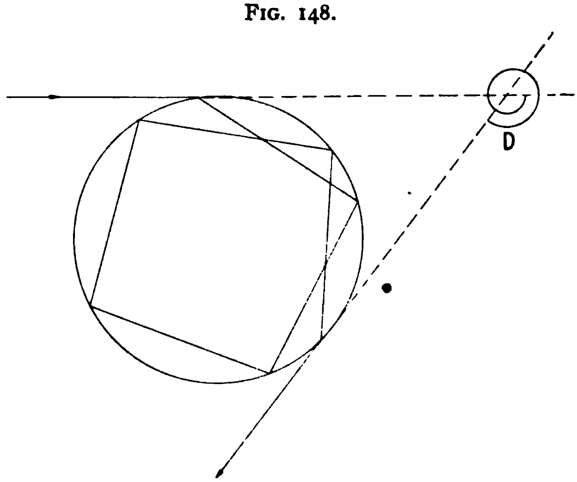
strictly parallel. Let, then, the plane of Fig. 149 pass through the centres of a raindrop and the sun, and let *AB* be the wave front of parallel rays incident, as shown, above the normal or axial ray (ray passing through centre of drop). An equal amount of light

FIG. 147.



Change of light direction in quaternary rainbow.

FIG. 148.

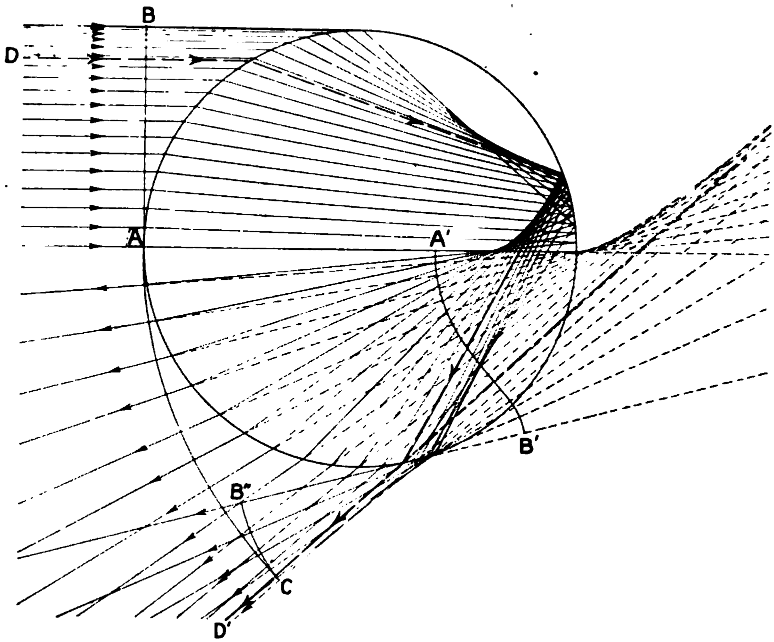


Change of light direction in quinary rainbow.

clearly enters below the normal ray, but for simplicity this is omitted. Similarly that portion reflected from the outer surface is ignored, as is also all that is internally reflected more than once. This reduces the problem of the rainbow to its simplest

terms, but loses none of its generality, since additional internal reflections merely change angular dimensions and brightness. The heavy line shows the course of the Descartes ray, or ray of minimum deviation for light of air-water refractive index, 4/3. The courses of other rays are, very approximately, as indicated. Since the deviations of the rays incident between the axial and the Descartes rays are greater than that of the Descartes, it follows

FIG. 149.



Course of light through a raindrop and the corresponding wave fronts.

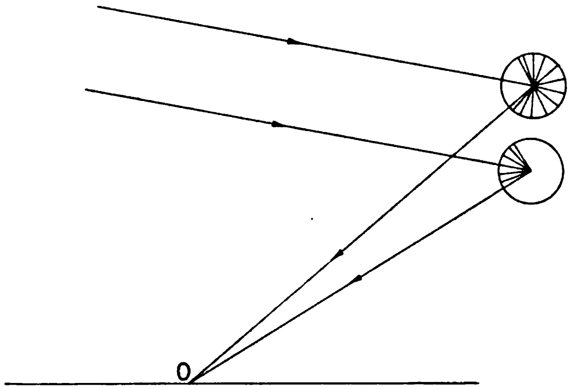
that their exits are, as shown, between those of the same two rays. Similarly, all rays that enter beyond the Descartes ray are likewise more deviated, and, therefore, while they leave the drop beyond this ray, they do so in such direction as sooner or later also to come between it and the axial ray, substantially as shown. Clearly, then, the once reflected light is diffuse and feeble except near the path of minimum deviation, and confined, as indicated, to the region between this path and the axial ray.

Formation of the Bow.—From the course, just given, of light through raindrops, it is clear that maximum brightness will be

produced by all illuminated drops along the elements of a right circular cone whose vertex is at the eye, whose axis passes through the sun, and whose angular opening, corresponding to a given number of internal reflections, is determined by the wave-length. Hence, the rainbow exhibits a number of concentric circular arcs of different colors whose centres are angularly as far below the observer as the sun is above him.

Minimum Brightness Between Primary and Secondary Bows.—Careful observers often note the fact that the region between the primary and secondary bows is slightly darker than any other in the same general direction. The explanation of this phenomenon is very simple. As the deviation of no ray can be less than

FIG. 150.



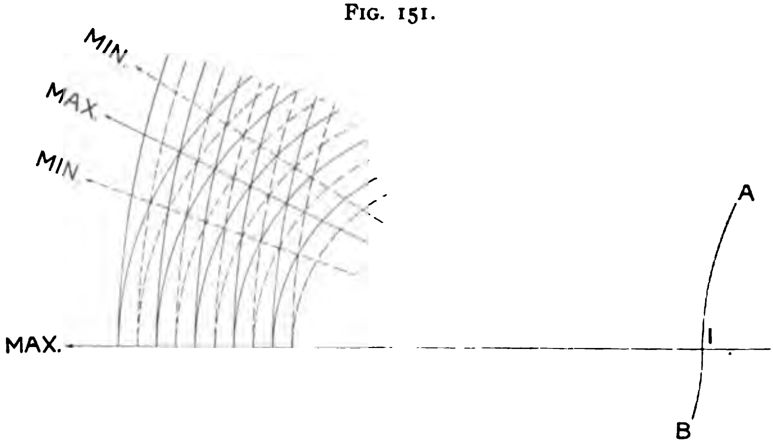
Minimum light between primary and secondary rainbows.

that of the Descartes, it is clear that all light pertaining to any given number of internal reflections must leave a drop within a cone formed by the rotation of a corresponding Descartes ray about the axial ray, keeping the angle between them constant as shown in Fig. 150 for light of 1, and 2 internal reflections, respectively. Hence, once reflected light reaches an observer from drops along and within his primary bow, but none from those without, while twice reflected light reaches him from along and without the secondary bow, but none from within it. The region between the two bows, therefore, and it alone, is devoid of both the once and the twice reflected rays and in consequence is comparatively dark.

Origin of Supernumerary Bows.—Since wave fronts are

normal to the corresponding rays, it is clear that the incident front, AB (Fig. 140), will, at the moment of complete emergence, appear as ACB''—exactly as though it had come from the virtual front, $A'B'$, the locus of the terminus of a line of constant length, AA', as it travels normally over the emerging wave front, ACB''. Further, since the rays here lie on both sides of the one of minimum deviation, it is obvious that this ray divides $A'B'$ into two portions curved in opposite directions. That portion of the front that is convex forward will, of course, remain convex, but with increasing radii of curvature, while the part that is concave forward will later become convex, and although neither portion is

FIG. 151.



Interference giving supernumerary rainbows.

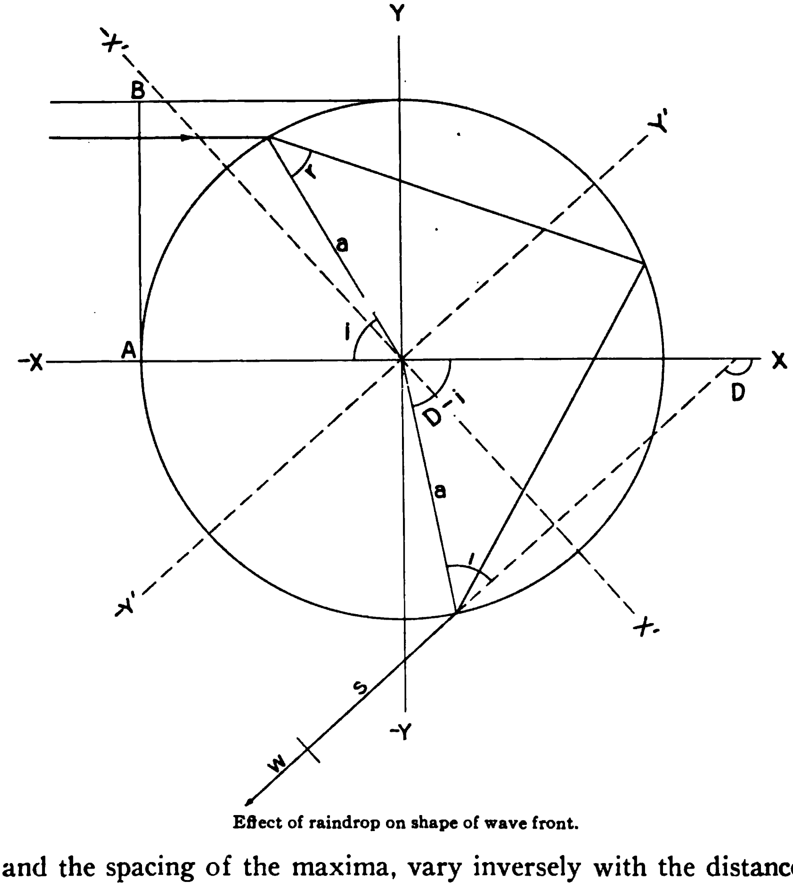
strictly the arc of a circle, the results they produce at a considerable distance, at the position of the observer, for instance, are qualitatively as though they were.

Let AIB, then (Fig. 151), be such a wave front, I being the point of inflection where the front is normal to the ray of minimum deviation. Let the full and dotted curves be opposite phase positions of the resulting cusped wave front. By inspection, it is obvious that soon after leaving the drop all the light must lie on one side of the ray of minimum deviation, thus making the observed angular radius of an arc of any given color slightly less than that of the Descartes ray. It is also obvious that with increasing angular distance from the Descartes ray the two branches of the cusped front are alternately, and with increasing

frequency, in opposite and like phases, thus producing alternate arcs of minimum and maximum brightness, within and without the circle of the primary and the secondary bow, respectively. These additional maxima, of which several frequently are visible, constitute the familiar supernumerary bows.

Clearly, from Fig. 151, the widths of all the color bands,

FIG. 152.



Effect of raindrop on shape of wave front.

and the spacing of the maxima, vary inversely with the distance between the centres of the interfering waves, or size of the drop.

Equation of Portion of Outgoing Wave Front Near Ray of Minimum Deviation.—In deriving this equation, originally due to Airy,[175] the more direct and elementary method of Wirt-

[175] *Cambridge Phil. Trans.*, vi, 380.

inger [176] will be followed with only such modifications as appear to make for clearness.

Let AB (Fig. 152) be an incident wave front tangent to a raindrop of radius a; let w be a small section of the emitted front near the minimum deviation ray—more distant portions of the front need not be considered as they are formed by rays too divergent to produce anything more than a slight general illumination; let v_1 be the velocity in air of the light under examination and $v_1\mu$ its velocity in water. Clearly, then, from the constancy of the time interval between corresponding points on the wave fronts,

$$\frac{a\,(1 - \cos i)}{v_1} + \frac{4\mu\, a \cos r}{v_1} + \frac{s}{v_1} = T, \text{ a constant.}$$

in which s is the distance between the drop and the corresponding point on w; i, and r, the angles of incidence and refraction, respectively.

Let the completely emitted front be also tangent to the drop, as shown in Fig. 149. Then

$$T = \frac{4\mu\, a}{v_1}$$

and

$$s = a. \left\{4\mu\,(1 - \cos r) - (1 - \cos i)\right\}$$

Let the centre of the drop be the intersection of coördinates as indicated. Then, if D is the angle of deflection, a point on w is given by the values,

$$x = a \cos (D - i) + s \cos D,$$

and

$$y = -a \sin (D - i) - s \sin D.$$

By turning the coördinates clockwise through the angle $D_1 - \pi/2$, in which D_1 is the change in direction of the ray of minimum deviation, the projection angles are correspondingly reduced and the new y axis brought parallel to the emerged Descartes ray. Hence, in terms of the new coördinates, writing d for $D - D_1$,

$$x' = -a \sin (d - i) - s \sin d$$
$$-y' = a \cos (d - i) + s \cos d.$$

[176] Berichte des naturw. medicin. Vereines an der Universität Innsbruck, xxi Jahrg., 1896–97. See also Pernter-Exner, Meteorologische Optik.

But as only rays very near that of minimum deviation need be considered, d is so small that to a sufficiently close approximation,

$$\cos d = 1, \text{ and } \sin d = d.$$

Hence,

$$x' = -ad\cos i + a\sin i - sd,$$

and

$$-y' = a\cos i + ad\sin i + s.$$

From Fig. 149 it is obvious that the small section, w (Fig. 152), of the emerged wave front is very nearly parallel to the x' axis, and that the x' coördinate, therefore, is extremely sensitive to changes in y', while the y' coördinate is relatively but little affected by the changes in x'. Hence, as d is very small, points on w are sufficiently closely given by the expressions,

$$x' = a\sin i$$

and

$$-y' = a\cos i + ad\sin i + a\left\{4\mu(1-\cos r) - (1-\cos i)\right\}.$$

Let I and R be the angles of incidence and refraction, respectively, corresponding to the minimum deviation, D_1, and let

$$i = I + \alpha, \text{ and } r = R + \beta$$

in which α and β are quite small, since only rays near that of minimum deviation are considered. Further, to make the problem entirely general, let n be the number of internal reflections. Then,

$$x' = a\sin(I+\alpha) = a\sin I + a\,\alpha\cos I$$

$$-y' = a\cos i + ad\sin i + a\left\{2\mu(n+1)(1-\cos r) - (1-\cos i)\right\}$$

$$= 2a\left\{\cos i + \frac{d}{2}\sin i - \mu(n+1)\cos r + \mu(n+1) - \tfrac{1}{2}\right\}.$$

Treating d and β as functions of α and developing we get, neglecting powers of α higher than the third,

$$\cos i = \cos(I+\alpha) = \cos I - \alpha\sin I - \frac{\alpha^2}{2}\cos I + \frac{\alpha^3}{6}\sin I + \cdots\cdots$$

$$\frac{d}{2} = \tfrac{1}{2}(D - D_1) = (I+\alpha) - (R+\beta) + \frac{n}{2}\left\{\pi - 2(R+\beta)\right\} - \left\{I - R + \frac{n}{2}(\pi - 2R)\right\}$$

$$= \alpha - (n+1)\beta = \alpha - (n+1)\left(\frac{d\beta}{d\alpha}\right)_0\alpha - (n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0\frac{\alpha^2}{2} - (n+1)\left(\frac{d^3\beta}{d\alpha^3}\right)_0\frac{\alpha^3}{6}$$

Since

$$\frac{d}{2} = \alpha - (n+1)\beta,\ d\frac{d}{2} = d\,\alpha - (n+1)\,d\beta, \text{ and } \left(\frac{d\beta}{d\alpha}\right)_0 = \frac{1}{n+1},$$

and, since

$$i = I + \alpha, \text{ or } \sin i = \sin I + \alpha \cos I,$$

therefore,

$$\frac{d}{2}\sin i = -(n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0 \frac{\alpha^2}{2}\sin I - (n+1)\left(\frac{d^3\beta}{d\alpha^3}\right)_0 \frac{\alpha^3}{6}\sin I - (n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0 \frac{\alpha^3}{2}\cos I \ldots.$$

Finally,

$$\cos r = \cos\left\{R + f(\alpha)\right\} = \cos R + \alpha\frac{d\cos r}{d\alpha} + \frac{\alpha^2}{2}\frac{d^2\cos r}{d\alpha^2} + \frac{\alpha^3}{6}\frac{d^3\cos r}{d\alpha^3} + \ldots\ldots$$

But

$$\frac{d\cos r}{d\alpha} = -\sin r\frac{d\beta}{d\alpha} = -\frac{\sin i}{\mu}\frac{d\beta}{d\alpha},$$

$$\frac{d^2\cos r}{d\alpha^2} = -\frac{\cos i}{\mu}\frac{d\beta}{d\alpha} - \frac{\sin i}{\mu}\frac{d^2\beta}{d\alpha^2}$$

$$\frac{d^3\cos r}{d\alpha^3} = \frac{\sin i}{\mu}\frac{d\beta}{d\alpha} - \frac{\cos i}{\mu}\frac{d^2\beta}{d\alpha^2} - \frac{\cos i}{\mu}\frac{d^2\beta}{d\alpha^2} - \frac{\sin i}{\mu}\frac{d^3\beta}{d\alpha^3}$$

At the limits

$$\left(\frac{d\beta}{d\alpha}\right)_0, \left(\frac{d^2\beta}{d\alpha^2}\right)_0, \left(\frac{d^3\beta}{d\alpha^3}\right)_0, \text{ where } i = I,$$

$$-\mu(n+1)\cos r = -\mu(n+1)\cos R - (n+1)\left(\frac{d\beta}{d\alpha}\right)_0\left[-\alpha\sin I - \frac{\alpha^2}{2}\cos I + \frac{\alpha^3}{6}\sin I\right] - (n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0\left[-\frac{\alpha^2}{2}\sin I - \frac{\alpha^3}{3}\cos I\right] + (n+1)\left(\frac{d^3\beta}{d\alpha^3}\right)_0\left[\frac{\alpha^3}{6}\sin I\right]$$

Hence, by addition of the above terms, remembering that,

$$\mu\cos R = (n+1)\cos I$$

and that

$$\left(\frac{d\beta}{d\alpha}\right)_0 = \frac{1}{n+1}$$

$$-y' = 2a\left\{1 - (n+1)^2\right\}\cos I - a(n+1)\left(\frac{d^2\beta}{d\alpha^2}\right)_0\frac{\alpha^3}{3}\cos I + a\left\{2\mu(n+1) - 1\right\}$$

Since

$$\sin i = \mu \sin r$$

$$\cos i = \mu \cos r \frac{d\beta}{d\alpha}$$

and

$$-\sin i = -\mu \sin r \left(\frac{d\beta}{d\alpha}\right)^2 + \mu \cos r \left(\frac{d^2\beta}{d\alpha^2}\right)$$

or

$$-\sin I = -\mu \sin R \left(\frac{d\beta}{d\alpha}\right)_0^2 + \mu \cos R \left(\frac{d^2\beta}{d\alpha^2}\right)_0$$

$$= -\frac{\mu \sin R}{(n+1)^2} + \mu \cos R \left(\frac{d^2\beta}{d\alpha^2}\right)_0$$

But

$$\mu \sin R = \sin I, \text{ and } \mu \cos R = (n+1) \cos I.$$

Therefore,

$$\left(\frac{d^2\beta}{d\alpha^2}\right)_0 = -\frac{\sin I}{\cos I} \frac{n^2 + 2n}{(n+1)^2}$$

and

$$-y' = 2a \left[-(n^2 + 2n) \cos I + \mu (n+1) - \tfrac{1}{2} \right] + a\alpha^3 \frac{n^2 + 2n}{3 (n+1)^2} \sin I,$$

$$x' = \left[a \sin I\right] + a \alpha \cos I.$$

The first terms in the equations for x' and y' are the coördinates of the point of inflection on the curve w. Taking this point as the origin and calling the coördinates x_1 and y_1 we have for points on w,

$$x_1 = a \alpha \cos I$$

$$y_1 = a \alpha^3 \frac{n^2 + 2n}{3 (n+1)^2} \sin I.$$

But

$$\alpha^3 = \frac{x_1^3}{a^3 \cos^3 I},$$

hence,

$$y_1 = \frac{1}{3a^2} \frac{n^2 + 2n}{(n+1)^2 \cos^3 I} x_1^3 \sin I.$$

As previously shown,

$$\cos I = \sqrt{\frac{\mu^2 - 1}{n^2 + 2n}},$$

hence,

$$y_1 = \frac{1}{3 a^2} \frac{(n^2 + 2n)^2}{(n+1)^2 (\mu^2 - 1)} \sqrt{\frac{(n+1)^2 - \mu^2}{\mu^2 - 1}} x_1^3.$$

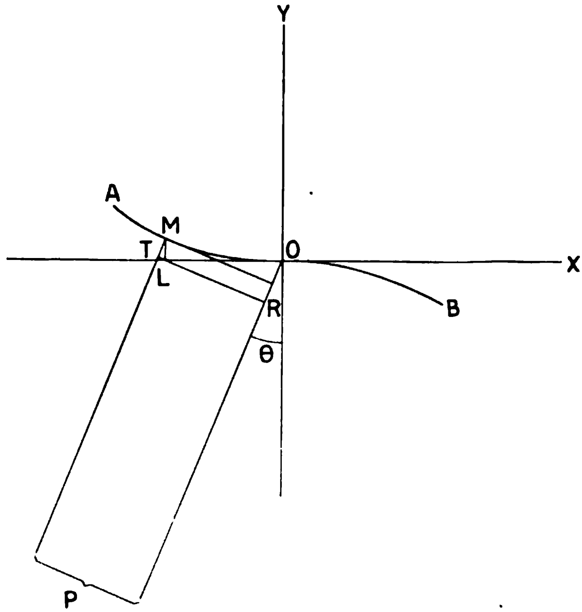
Putting

$$\frac{(n^2 + 2n)^2}{(n + 1)^2 (\mu^2 - 1)} \sqrt{\frac{(n + 1)^2 - \mu^2}{\mu^2 - 1}} = h,$$

$$y_1 = \frac{h}{3a^2} x_1^3.$$

This equation, then, represents a curve very nearly coincident with that portion of the wave front to which the rainbow phenomena are due, and since the effects computed from it substantially agree with those observed, as will be seen presently, it is clear that the approximation thus obtained is sufficient for most, if not all, practical uses; indeed, the assumption that raindrops are perfectly spherical involves, perhaps, a greater error.

FIG. 153.



Variation of intensity with angular distance from ray of minimum deviation.

Intensity and Its Variation with Angular Distance from the Ray of Minimum Deviation.—This, too, was first determined by Airy.[177] The following discussion, however, is essentially that of Mascart[178] and Pernter-Exner.[179]

[177] *l. c.*

[178] *Traité d'Optique*, i.

[179] *Meteorologische Optik.*

Let O (Fig. 153) be the point of inflection of an emitted wave front near a drop; let P be a distant point in the direction θ from the ray of minimum deviation. Then the difference in phase, ΔF, between the light vibrations at P, due respectively to an element, ds, of the front at O and an equal element at M is given by the equation,

$$\Delta F = 2\pi \frac{OR - MT}{\lambda} = 2\pi \frac{x \sin\theta - y \cos\theta}{\lambda}$$

in which λ is the wave-length.

Hence, substituting for y its value, $\frac{hx^3}{3a^2}$, and dx for ds, which is allowable over the effective portion of the wave front, the vibration at P is given by the equation,

$$V = k\int \sin 2\pi \left[\frac{t}{T} - \left(\frac{\frac{hx^3}{3a^2}\cos\theta - x\sin\theta}{\lambda}\right)\right] dx$$

in which T is the period and k the amplitude per unit length of front.

Or, putting

$$\frac{2\pi}{\lambda}\left(\frac{hx^3}{3a^2}\cos\theta - x\sin\theta\right) = \delta$$

$$V = k\int \sin\left(2\pi\frac{t}{T} - \delta\right) dx$$

$$= k\int \cos\delta\, dx \sin 2\pi\frac{t}{T} - k\int \sin\delta\, dx \cos 2\pi\frac{t}{T}.$$

Since $\cos\delta dx$ and $\sin\delta dx$ appear in this equation as amplitudes it follows, if

$$A = k\int \cos\delta\, dx$$

and

$$B = k\int \sin\delta\, dx$$

that the resultant amplitude $C = \sqrt{A^2 + B^2}$.

But to find A and B it is necessary to integrate the given expressions over the effective portion of the wave front, or through limits that produce essentially the same results. Such limits may be determined as follows:

Let x and x_1 be so situated that the difference between their distances from P is $\frac{\lambda}{2}$, and their combined result at that point, therefore, zero. That is, let

$$\frac{h}{3a^2}(x^3_1 - x^3)\cos\theta - (x_1 - x)\sin\theta = \frac{\lambda}{2}$$

or

$$x_1 - x = \frac{\lambda}{2\cos\theta} \cdot \frac{3a^2}{h(x_1^2 + x x_1 + x^2) - \tan\theta}$$

Considering the primary bow in which h has its least value, 4.89, nearly, for $\mu = \frac{4}{3}$; assuming the radius, a, of a drop to be 1 mm., and writing δx for $x_1 - x$, it follows, since θ is small, that

$$x^2\delta x = .0002 \text{ mm}^3\text{, roughly, for yellow light.}$$

Hence, δx decreases rapidly with increase of x (even when $x = .1$ mm., $\delta x = .02$ mm.), and the successive portions of the curve beyond a very short distance from the inversion point, O (Fig. 153), completely neutralize each other, and, therefore, no error will be introduced by integrating between infinity limits instead of between the *unknown* limits of the effective portion of the wave front.

Hence,

$$A = k\int_{-\infty}^{+\infty} \cos\delta\, dx$$

and

$$B = k\int_{-\infty}^{+\infty} \sin\delta\, dx$$

But as the sign of B reverses with that of x, while that of A remains unchanged,

$$B = 0$$

and

$$A = 2k\int_0^\infty \cos\frac{2\pi}{\lambda}\left(\frac{hx_3}{3a^2}\cos\theta - x\sin\theta\right)dx.$$

Putting

$$\frac{2h}{3a^2\lambda}x^3\cos\theta = \frac{u^2}{2},$$

from which

$$dx = \left(\frac{3\ a^2\ \lambda}{4h \cos\theta}\right)^{1/3} du,$$

and

$$\frac{2x}{\lambda} \sin\theta = \frac{zu}{2}$$

$$A = 2k \int_0^\infty \left(\frac{3a^2\lambda}{4h\ \cos\theta}\right)^{1/3} \cos\frac{\pi}{2}\ (u^3 - zu)\ du,$$

which is Airy's rainbow integral, in which

$$\frac{\pi}{2}\ (u^3 - zu)\ du = \delta,$$

the difference in phase.

Putting

$$\int_0^\infty \cos\frac{\pi}{2}\ (u^3 - zu)\ du = f(z)$$

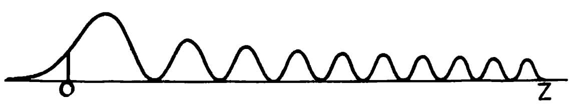
$$A = 2k \left(\frac{3a^2\lambda}{4h \cos\theta}\right)^{1/3} f(z) = M\ f(z), \text{ say},$$

and the intensity,

$$A^2 = M^2 f^2\ (z).$$

The evaluation of $f(z)$ is not simple, but it has been accomplished through mechanical quadrature by Airy [180] and through development in series by both Airy [181] and Stokes.[182]

FIG. 154.



Periodic changes of intensity of monochromatic light.

The following table and its graph (Fig. 154), both from Pernter-Exner's "Meteorologische Optik," give certain values of z and $f^2(z)$, corresponding to monochromatic light of a particular wave-length and drops of a definite size.

[180] *Cambridge Phil. Trans.*, vi, p. 379.
[181] *Cambridge Phil. Trans.*, viii, p. 595.
[182] *Cambridge Phil. Trans.*, ix, part i.

Values of $f^2(z)$ for Different Values of z.

z	$f^2(z)$	z	$f^2(z)$	z	$f^2(z)$
−2.0	0.006	3.4	0.609	8.8	0.189
−1.8	0.011	3.6	0.586	9.0	0.373
−1.6	0.018	3.8	0.436	9.2	0.320
−1.4	0.030	4.0	0.225	9.4	0.100
−1.2	0.048	4.2	0.051	9.6	0.001
−1.0	0.074	4.4	0.003	9.8	0.054
−0.8	0.113	4.6	0.104	10.0	0.240
−0.6	0.168	4.8	0.297	10.2	0.360
−0.4	0.239	5.0	0.465	10.4	0.220
−0.2	0.331	5.2	0.501	10.6	0.022
0.0	0.443	5.4	0.379	10.8	0.013
0.2	0.571	5.6	0.172	11.0	0.170
0.4	0.706	5.8	0.014	11.2	0.338
0.6	0.836	6.0	0.022	11.4	0.270
0.8	0.941	6.2	0.174	11.6	0.050
1.0	1.001	6.4	0.370	11.8	0.004
1.2	0.996	6.6	0.450	12.0	0.140
1.4	0.914	6.8	0.353	12.2	0.320
1.6	0.758	7.0	0.141	12.4	0.256
1.8	0.547	7.2	0.010	12.6	0.045
2.0	0.319	7.4	0.046	12.8	0.006
2.2	0.125	7.6	0.230	13.0	0.136
2.4	0.014	7.8	0.394	13.2	0.314
2.6	0.016	8.0	0.363	13.4	0.202
2.8	0.131	8.2	0.150	13.6	0.013
3.0	0.317	8.4	0.010	....	.....
3.2	0.502	8.6	0.038	....	.....

Maxima and Minima.

Maxima	z	$f^2(z)$	Minima	z
1.	1.0845	1.005	1.	2.4955
2.	3.4669	0.615	2.	4.3631
3.	5.1446	0.510	3.	5.8922
4.	6.5782	0.450	4.	7.2436
5.	7.8685	0.412	5.	8.4788
6.	9.0599	0.384	6.	9.6300
7.	10.1774	0.362	7.	10.7161
8.	11.2364	0.345	8.	11.7496
9.	12.2475	0.330	9.	12.7395
10.	13.2185	0.318	10.	13.6924

It will be noticed that the first maximum does not coincide with $z = 0$, nor, therefore, with $\theta = 0$, the direction of the ray of minimum deviation; that the intensity of the first maximum, corresponding to a principal bow, is much the greatest; and that the succeeding maxima, corresponding to the supernumerary

bows, and also the angular intervals between successive maxima, continuously decrease, at a decreasing rate, with the increase of θ.

Distribution of Colors in the Rainbow.—The above discussion of the distribution of light intensity applies, as stated, to monochromatic light. When the source of light simultaneously emits radiations of various wave-lengths, as does the sun, a corresponding number of bows, each consisting of a sequence of maxima and minima, are partially superimposed on each other. In this way different colors are mixed, and thus the familiar polychromatic rainbow produced.

The particular mixing of colors that obtains is the result of several coöperating causes. Thus the distribution of intensity, as illustrated by Fig. 154, depends on phase difference, as given by the expression,

$$2\pi \frac{\frac{h\,x^3}{3a^2}\cos\theta - x\sin\theta}{\lambda}.$$

The angular intervals between maxima, say, increase, therefore, with λ, and, consequently, coincident distribution of the intensities of any two colors is impossible. Again, since the direction of the ray of minimum deviation varies with the refractive index, as already explained, and that in turn with the wave-length or color, it follows that the direction of the zero point on the intensity curve, near which the first maximum lies, correspondingly varies. Obviously, then, these two causes together produce all sorts of color mixings that in turn arouse widely varied sensations.

To determine, however, just what color mixtures are induced by drops of any given size, it obviously is necessary to express the values of the abscissa, z, of the intensity curve (Fig. 154) in terms of angular deviation from the corresponding principal ray, since the direction of each such ray fixes the position of origin of its particular curve.

Let, then, as before,

$$\frac{z\,u}{2} = \frac{2\,x}{\lambda}\sin\theta$$

or

$$z^3 = \left(\frac{4\,\sin\theta}{\lambda}\,\frac{x}{u}\right)^3$$

Also, as before, let

$$\left(\frac{x}{u}\right)^3 = \frac{3a^2\lambda}{4\,h\,\cos\theta}$$

hence,

$$z^3 = \frac{48\, a^2}{h\,\lambda^2} \sin^2\theta \tan\theta.$$

But whatever the value of θ from 0° to 30°

$$\frac{\sin^2\theta \tan\theta}{\theta^3} = 1, \text{ to within } .0055.$$

Hence, approximately,

$$z^3 = \frac{48\, a^2}{h\,\lambda^2}\, \theta^3,$$

or

$$\theta = \frac{z}{2a^{\frac{2}{3}}} \left(\frac{h\,\lambda^2}{6}\right)^{\frac{1}{3}}$$

From this equation it appears that the angular distance between any two successive intensity maxima varies directly as the cube root of the square of the wave-length and inversely as the cube root of the square of the diameter, or other linear dimension of the parent drop. That is, this interval is greater for red light than for blue, and greater for small drops than for large ones.

The following table, copied with slight changes, from Pernter-Exner's "Meteorologische Optik," gives the values of θ in minutes of arc per 0.2 z, for lights of different wave-length and drops of different size.

Angle in Minutes per 0.2 z, Primary Bow.

a = in microns	5	10	15	20	25	30	40	50	100	150	250	500	1000
λ, in microns	Angle in minutes												
.687	85.8	54.0	41.2	34.0	29.3	26.0	21.4	18.5	11.7	8.9	6.32	3.98	2.51
.656	83.0	52.3	39.9	32.9	28.4	25.1	20.7	18.0	11.0	8.6	6.10	3.84	2.43
.589	77.0	48.5	37.0	30.5	26.4	23.3	19.3	16.6	10.5	7.9	5.67	3.57	2.26
.527	71.2	44.9	34.2	28.2	24.4	21.6	17.8	15.4	9.6	7.4	5.25	3.31	2.10
.494	68.1	42.8	32.6	27.0	23.1	20.6	17.0	14.7	9.3	7.1	5.02	3.15	2.03
.486	67.2	42.3	32.3	26.7	22.8	20.3	16.8	14.5	9.1	7.0	4.94	3.12	1.99
.449	63.4	40.0	30.5	25.2	21.7	19.2	15.9	13.7	8.6	6.6	4.67	2.93	1 88
.431	51.5	38.8	29.6	24.4	21.0	18.6	15.4	13.3	8.3	6.4	4.53	2.87	1.82

By the aid of this table; a table of intensity distribution, $M^2f^2(z)$, along the coördinate z; and the following relative intensities,

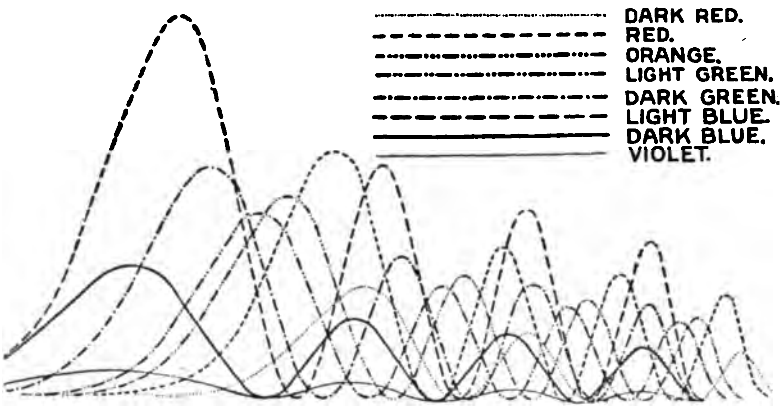
λ	.687	.656	.589	.527	.494	.486	.449	.431
I	20	86	250	152	121	134	163	74

Pernter has constructed Figs. 155 and 156 that show the intensity distributions of these several colors due to drops of .5 mm. and .05 mm. radius, respectively.

It still remains to determine the color at any particular point at which the relative intensities of the several colors are known. This can be done by the aid of Maxwell's [183] color triangle, as explained in detail by Pernter.[184]

Relation of size of Drop and Wave-length to Intensity.—Since the Airy expression for the amplitude of the vibration produced at a distant point by the effective portion of the emitted wave front involves the factor, $(\lambda a^2)^{1/3}$, it is evident that the cor-

Fig. 155.



Distributions of colors by drops of 0.5 mm. radius.

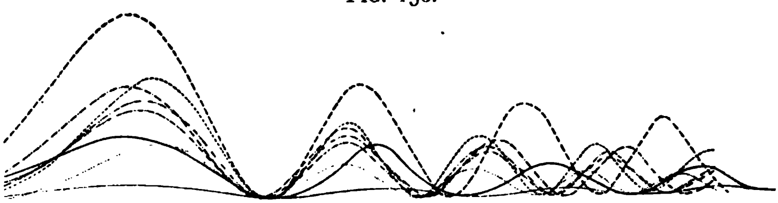
responding intensity, which is proportional to the square of the amplitude, will be proportional to $(\lambda a^2)^{2/3}$. This, however, is based on the assumption that the effective light from the drop comes strictly from the *line* of a great circle. As a matter of fact, it actually comes from a narrow belt whose effective angular width, as measured from the centre of the drop, is inversely proportional to the curvature, or directly to the radius *a*, and inversely proportional to the wave-length.[185] Hence, the actual intensity is proportional to $\lambda^{-1/3} a^{7/3}$.

[183] *Trans. Roy. Soc.*, p. 57, 1860.
[184] *Meteorologische Optik.*
[185] Mascart, C. R., V. 115, p. 453, 1892.

A larger fraction of the short wave-length light is effective, therefore, than of the long. Further, the rainbow bands produced by very small droplets are not only broad, as previously explained, but also feeble, and as their colors necessarily are faint they frequently are not distinguished—the bow appearing as a mere white band.

Popular Questions About the Rainbow.—A few popular questions about the rainbow need perhaps to be answered. "What is the rainbow's distance?" In the sense of its proximate origin, the drops that produce it, it is nearby or far away, according to their respective distances, and thus extends from the closest to the farthest illuminated drops along the elements of the rainbow cone. Indeed, the rainbow may be regarded as consisting of coaxial, hollow conical beams of light of different colors seen edgewise from the vertex, and thus having great depth or extent in the line of sight.

FIG. 156.



Distributions of colors by drops of 0.05 mm. radius.

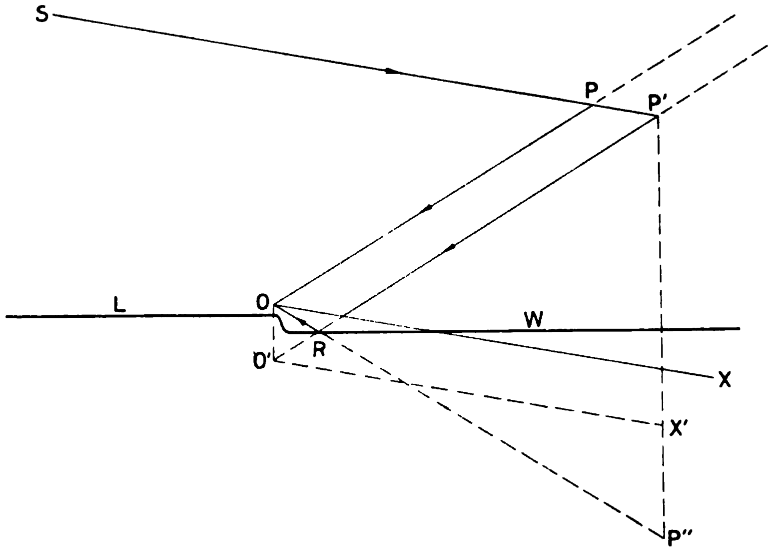
"Why is the rainbow so frequently seen during summer and so seldom during winter?" Its formation requires the coexistence of rain and sunshine, a condition that often occurs during local convectional showers but rarely during a general cyclonic storm, and as the former are characteristic of summer and the latter of winter, it follows that the occurrence of the rainbow correspondingly varies with the seasons.

"Why are rainbows so rarely seen at noon?" As above explained, the centre of the rainbow's circle is angularly as far below the level of the observer as the sun is above it, hence no portion of the bow can be seen (except from an elevation) when its angular radius is less than the elevation of the sun above the horizon. Now, during summer, the rainbow season, the elevation of the sun at noon is nearly everywhere greater than 42°, the angular radius of the primary bow, or even 51°, the radius of the

secondary bow. A rainbow at noon, therefore, is, except for very high latitudes, an impossible summer phenomenon, and, of course, a rare winter one, for reasons given above, even where possible.

"Do two people ever see the same rainbow?" Theory teaches, and ordinary experience shows, that as the observer remains stationary or moves, so also, other things being equal, does his rainbow. If then, two observers initially close together should move in opposite directions, each would find his rainbow responding in the same sense as his shadow, and presently the

FIG. 157.



Reflected rainbow.

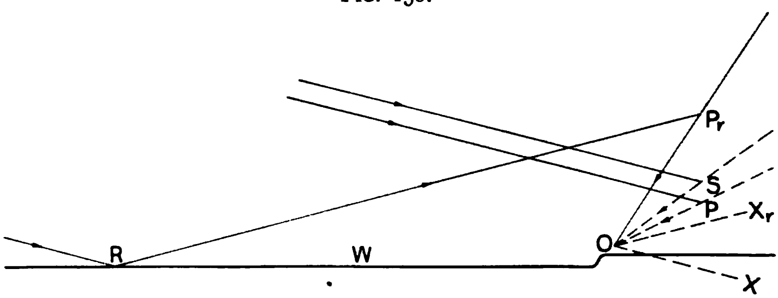
positions and, therefore, the identity of the two bows would become unquestionably different, from which it follows that as the eyes of two observers must always be separated by a greater or less distance their bows must also be correspondingly separated and different—different in the sense that they have different positions and are produced by different drops. In short, since the rainbow is a special distribution of colors (produced in a particular way) with reference to a definite point—the eye of the observer—and as no single distribution (other than uniform and infinite) can be the same for two separate points, it follows that two observers do not and cannot see the same rainbow.

"Can one see the same rainbow by reflection that he sees directly?" An object seen by reflection in a plane surface is seen by the same rays that but for the mirror would have focussed to a point on a line normal to it from the eye and as far back of it as the eye is in front. But, as just explained, the bows appropriate to two different points are produced by different drops; hence, a bow seen by reflection is not the same as the one seen directly.

Reflected Rainbows.—Since rainbows occasionally are seen reflected in smooth bodies of water they deserve, perhaps, a somewhat fuller explanation than that just given.

Let an observer be at *O* (Fig. 157). Under proper conditions of rain and sunshine he will see directly a primary bow due to drops on the surface of a cone formed by rotating *OP* about

FIG. 158.



Reflection rainbow.

OX, parallel to *SP,* keeping the angle, *POX,* roughly 42°, constant; and by reflection in the surface of the water, *W,* another primary bow due to drops on the surface of a different cone, one formed by rotating *O'P'* about *O'X',* also parallel to *SP,* keeping the angle *P'O'X'* constant. The bow seen by reflection necessarily will appear upside down, *P'* at *P''*, etc. The arc of the bow seen by reflection obviously (from the figure) will be less than that of the bow seen directly, and for that reason is likely to appear flatter.

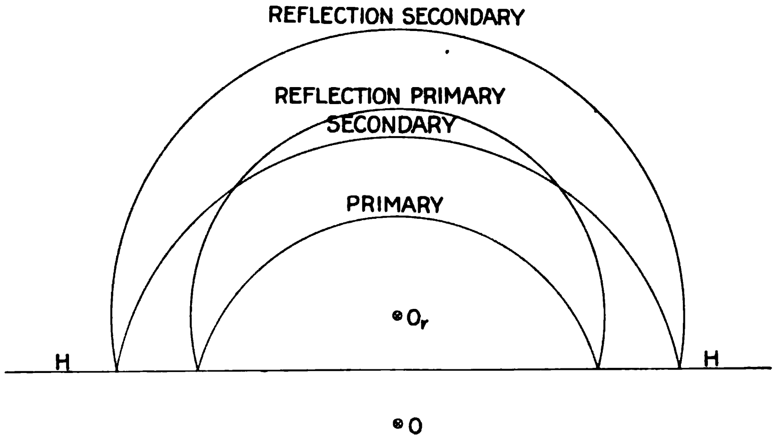
Reflection Rainbows.—A reflection rainbow is here defined as one due to reflection of the light source, the sun, usually, but itself seen directly.

Let the observer be at *O* (Fig. 158) with the sun and a smooth surface of water, *W,* at his back and illuminated rain in front. The direct sunshine will give a primary and a secondary bow

along the elements of cones formed by rotating OP and OS, respectively, about the common axis, OX, parallel to the incident rays, keeping the angles POX and SOX constant; while the reflected light will give a primary bow along the elements of a cone formed by the rotation of OP_r about the axis OX_r (OX_r parallel to RP_r), keeping the angle P_rOX_r constant, and equal to POX. Reflection bows of higher orders than that of the primary are likely to be too faint to be seen.

The angular elevation of the centre (axis really) of the reflection bows clearly is equal to the angular depression of the

FIG. 159.



Direct and reflection rainbows.

centre of the direct bows. Hence, the direct and the reflection bows of the same order intersect on the observer's level, as shown in Fig. 159.

Horizontal Rainbow.—A rainbow is occasionally formed by a sheet of drops resting on a smooth water surface, probably oily. Juday [185a], for instance, reports the simultaneous observance, October 23, 1914, of the primary bow, its first two supernumeraries, and also the secondary, in such a sheet on Lake Mendota, Wisconsin. The peculiar appearance of such bows is due entirely to the unusual distribution of the parent drops—there is nothing new in the theory of their formation.

[185a] *Monthly Weather Review*, **44**, p. 65, 1916.

Why There is No Visible Rainbow without Internal Reflection.—Since more light passes through the raindrop at the place of first internal reflection than anywhere else, it is reasonable to ask why this light, which is refracted as much as any other, instead of giving the brightest of all rainbows, shows none at all.

Clearly, and as is obvious from inspection of Fig. 149, the deviation of this non-reflected light varies from zero, in the case of that which is normal to the drop, to a maximum for the tangential. Its brightness, however, as seen by the observer, gradually decreases, with increase of deviation, to zero at $2(90°—\text{arc sin } 1/\mu)$ from the sun—μ being the air-water refractive index for the wave-length under consideration. Hence, owing to superposition of the several colors, all this refracted but non-reflected light is white, except a violet to bluish circular border about 84° from the sun, or other source, and far too faint ever to be seen.

CHAPTER IV.

REFRACTION PHENOMENA: REFRACTION BY ICE CRYSTALS.

Introduction.—The cirrus clouds and others formed at temperatures considerably below 0° C. usually consist of small but relatively thick snowflakes with flat bases, or ice spicules with flat or, rarely, pyramidal bases, always hexagonal in pattern and detail, as shown by Fig. 160 from Bentley's remarkable collection of photomicrographs of snow crystals.

Light from the sun, for instance, obviously takes many paths through such crystals and produces in each case a corresponding and peculiar optical phenomenon. Several of these phenomena, the halo of 22° radius, the halo of 46° radius, the circumzenithal arc, parhelia, etc., are quite familiar and their explanations definitely known. Others, however, have so rarely been seen and measured that the theories of their formation are still somewhat in doubt. Finally, many phenomena, theoretically possible, as results of refraction by ice crystals, appear so far to have escaped notice.

Prismatic Refraction.—Since the phenomena caused by the passage of light through ice crystals are numerous, it will be most convenient, in discussing them, first to obtain general equations for prismatic refraction, and then to substitute in these equations the numerical constants applicable to each particular case.

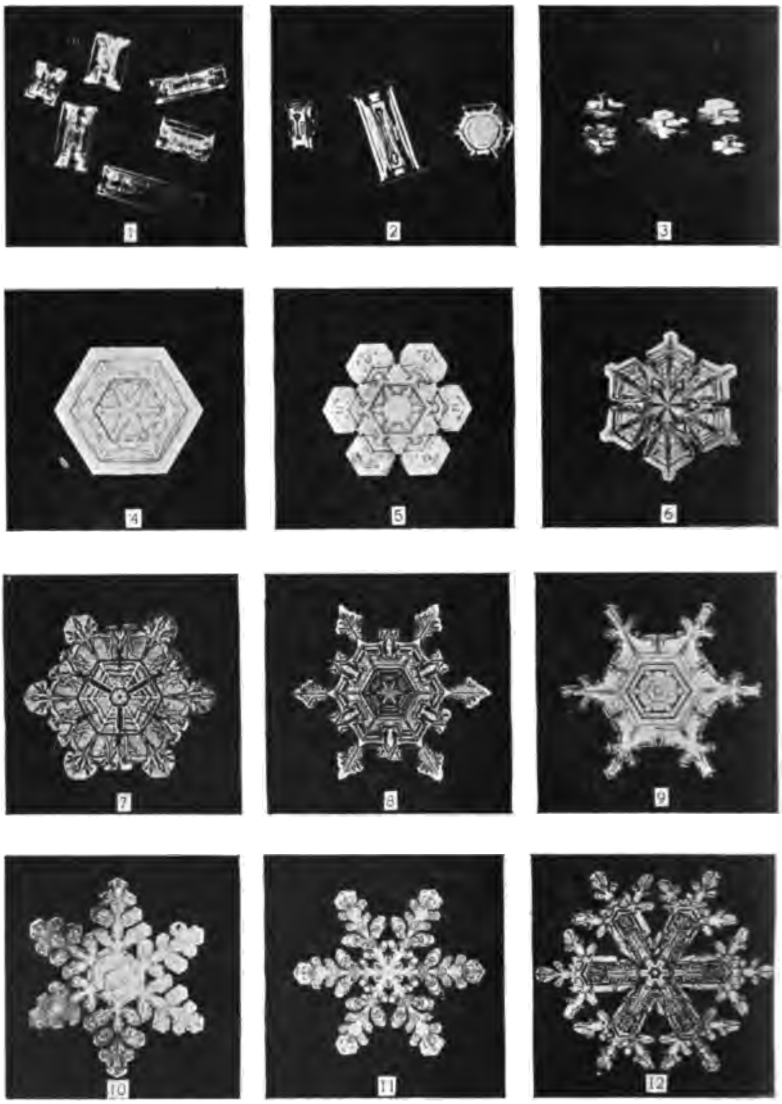
Deviation.—Let A (Fig. 161) be the angle between two adjacent faces of a prism; let CE be the path of a ray of light in a plane normal to their intersection (direction of travel immaterial): let i and i' be the angles of incidence and r and r' the corresponding angles of refraction. Then the change in direction, D, of the ray is given by the equation,

$$D = i + i' - (r + r') = i + i' - A \qquad (1)$$

Minimum Deviation.—Minimum deviation occurs when $\frac{dD}{di} = 0$ and $\frac{d^2D}{di^2} > 0$ But when

$$\frac{dD}{di} = 1 + \frac{di'}{di} = 0, \; di = - di'.$$

FIG. 160.



Snow crystals (Bentley).

and as

$$r + r' = A. \quad dr = - dr'.$$

Also, from the law of refraction,

$$\sin i = \mu \sin r.$$
$$\sin i' = \mu \sin r'$$

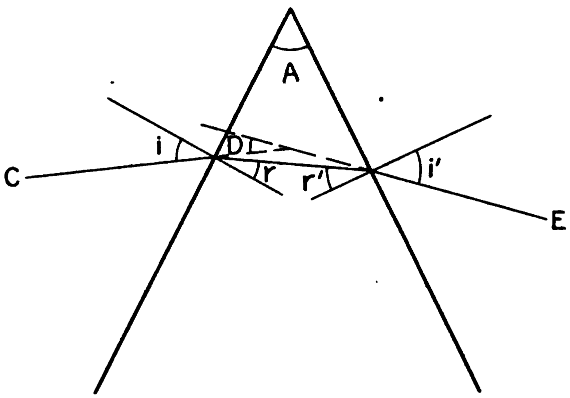
Hence, directly,

$$\frac{\sin i}{\sin i'} = \frac{\sin r}{\sin r'},$$

or

$$\sin i \sin r' = \sin i' \sin r$$

FIG. 161.



Deviation by refraction.

and, by differentiation, if $di = - di'$ and $dr = - dr'$,

$$\frac{\cos i}{\cos i'} = \frac{\cos r}{\cos r'}$$

or

$$\cos i \cos r' = \cos i' \cos r.$$

By addition, $\cos (i - r') = \cos (i' - r)$, or $i - r' = i' - r$.
By subtraction, $\cos (i + r') = \cos (i' + r)$, or $i + r' = i' + r$.

Hence, if, as assumed, $\frac{dD}{di} = 0$,

$$= i', \text{ and } r = r' = \frac{A}{2}.$$

From

$$\frac{dD}{di} = 1 + \frac{di'}{di} = 1 + \frac{\dfrac{\mu \cos r' \, dr'}{\cos i'}}{\dfrac{\mu \cos r \, dr}{\cos i}} = 1 - \frac{\cos i \cos r'}{\cos i' \cos r},$$

it follows, by a little reduction, that when $\frac{dD}{di} = 0$

$$\frac{d^2D}{di^2} = \frac{2\,\mu \cos^2 r \sin i - 2 \cos^2 i \sin r.}{\mu \cos i \cos^4 r}$$

But

$$\mu > 0, \cos^2 r > \cos^2 i, \sin i > \sin r, \text{ and } \mu \cos i \cos^2 r > 0.$$

Hence, when

$$\frac{dD}{di} = 0, \text{ that is, when } r = r', \frac{d^2D}{di^2} > 0$$

and the deviation has its minimum value.

Writing D_0 for the minimum deviation, it follows that

$$D_0 = 2\,i - A. \text{ - - - - - - - - - - - - - - - - } (2)$$

Hence, from $\sin i = \mu \sin r$, and $r = \frac{A}{2}$, we get

$$\sin \frac{D_0 + A}{2} = \mu \sin \frac{A}{2} \text{ - - - - - - - - - - - - - - } (3)$$

Maximum deviation, D_m, obviously occurs when

$$i = 90°,$$

or, for ice for which $\mu = 1.31$, when $r = 49° \, 46'$.

Since

$$D = i + i' - A$$

$$D_m = 90° + i' - A$$

and

$$\sin (D_m + A - 90°) = \mu \sin r' = \mu \sin (A - 49° \, 46'), \text{ for ice,}$$

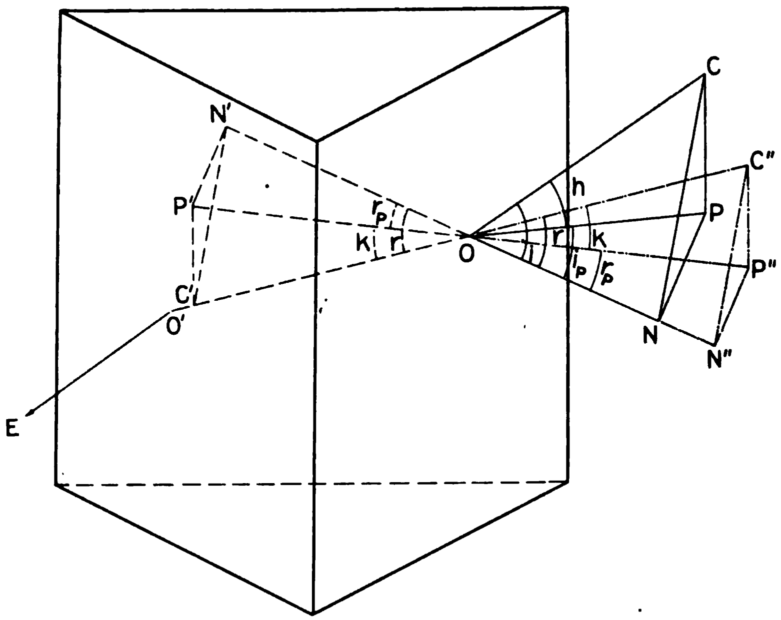
or

$$\cos \left[(180° - A) - D_m\right] = \mu \sin (A - 49° \, 46') \text{ - - - - - - } (4)$$

The above equations apply only to refraction in a plane normal to the intersection of the faces of the prism. When the incident ray is inclined to this plane, the effective angle of refraction is increased, and as such inclination usually occurs in the case of floating ice crystals it is necessary, in the study of halos, to evaluate its effect on the deviation.

Let the ray, CE (Fig. 162), enter a prism at O and leave it at O'. Let ON be the normal to the prism face at O, and let OP be the projection of CO onto a plane through O normal to the intersection of the refracting faces. Let P be the projection of C, and let the plane CNP be normal to ON. Similarly, let P' be the projection of C' on the principal plane, PON, and let the plane $C'P'N'$ be perpendicular to the normal ON extended; then, since

FIG. 162.



Refraction of rays inclined to principal plane.

the refracted ray OC' lies in the plane OCN, it follows that the triangles CNP and $C'N'P'$ are similar.

Therefore,

$$\frac{\sin h}{\sin k} = \frac{\sin i}{\sin r},$$

and

$$\sin h = \mu \sin k,$$

in which h and k are the angles between the ray and principal plane before and after refraction, respectively.

Similarly, if h' and $k'=k$ are the angles at O' corresponding to h and k at O,

$$\sin h' = \mu \sin k' = \mu \sin k.$$

Hence,

$$h = h'$$

That is, the inclination of the ray to a principal plane is the same after leaving a prism as before entering it.

Let i_p be the angle between the normal, ON, and the projection, OP, of the ray, CO, on the principal plane through O, and r_p the angle betwen the normal extended, ON', and the projection, OP' of the refracted ray, OC', onto the principal plane. Then (perhaps better seen by reversing $OC'N'P'$ to $OC''N''P''$),

$$\sin i_p = \frac{NP}{OP} = \frac{NP}{OC \cos h},$$

and

$$\sin r_p = \frac{N''\ P''}{O\ P''} = \frac{N''\ P''}{OC''\ \cos k}.$$

But $NP = \frac{OC}{OC''}\mu\ N''\ P'' = \mu\ N''\ P''$ if $OC = OC''$,

and

$$\sin i_p = \mu \frac{\cos k}{\cos h} \sin r_p\,.$$

Hence a ray inclined to the principal plane of a prism of refractive index μ is so bent that the projection of its path on this plane gives the index, μ' where

$$\mu' = \mu \frac{\cos k}{\cos h} = (\mu^2 - \sin^2 h)^{\frac{1}{2}}\ (1 - \sin^2 h)^{-\frac{1}{2}}$$ [186]

The minimum deviation, therefore, D'_0, measured in or projected onto the principal plane, of such rays is given by the equation,

$$\sin \frac{D'_0 + A}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{A}{2}\,. \quad (5)$$

and the maximum, D'_m, by the equation,

$$\cos\left[(180^\circ - A) - D'_m\right] = \mu \frac{\cos k}{\cos h} \sin (A - \alpha)\,, \quad (6)$$

[186] It may be interesting to note that this relation between the inclination of a ray to the principal plane of a prism and its deviation by that prism explains the curvature of spectrum lines as seen in an ordinary straight slit prism spectroscope.

in which α is the limiting value of the angle of refraction for the index μ', when

$$\mu' = \mu \frac{\cos k}{\cos h}.$$

The largest or limiting value of h at which light can still pass through the prism obviously is determined by the equation,

$$\frac{D + A}{2} = 90°,$$

in which D is the minimum deviation as projected on the principal plane.

In this case

$$\sin \frac{D + A}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{A}{2} = 1. \quad \text{(7)}$$

Therefore,

$$\frac{1}{\sin^2 \frac{A}{2}} = \mu^2 \frac{(1 - \sin^2 k)}{\cos^2 h} = \frac{\mu^2 - \sin^2 h}{\cos^2 h}$$

and

$$\cos h = \sqrt{\mu^2 - 1} \tan \frac{A}{2}.$$

Hence, when $A = 60°$, as between alternate sides of a hexagonal ice prism, or snowflake, the limiting value of h for $\mu = 1.31$, is 60° 45′, and when A is 90°, as between base and a side, 32° 12′.

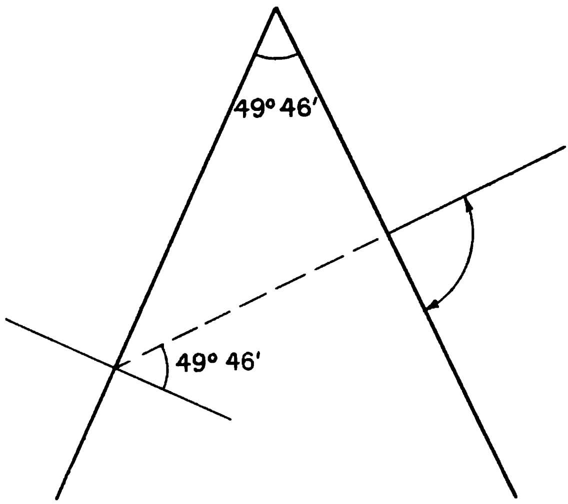
Internal Total Reflection and Its Effect on the Passage of Light Through Ice Crystals.—Since the limiting value of the "angle of incidence" is 90°, and the refractive index of ice 1.31, it follows that total reflection of an internal ray occurs at the angle α, given by the equation,

$$\sin 90° = 1.31 \sin \alpha = 1.31 \sin 49° \, 46'.$$

An internal ray, therefore, cannot leave an ice crystal if the angle it makes with the normal is greater than 49° 46′. Hence, as is clear from Fig. 163, a ray of light in the principal plane, and also most rays out of it, will pass through an ice crystal between faces whose inclination is not greater than 49° 46′ at all angles of incidence (measured on the base side of the normal) from 0° to 90°. On the other hand, no light can pass through an ice crystal at any angle of incidence between planes whose inclination is greater than 2 × 49° 46′. In proof of this latter

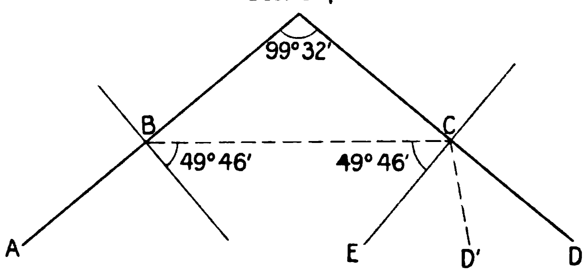
statement, let AB (Fig. 164) be a ray grazing the side of a crystal whose angle of refraction is 99° 32′ and entering at B. It will reach the opposite face at C, and either pass out in the direction CD or else suffer total reflection. But as CD lies along the face of the crystal, it is clear that any decrease of the angle of incidence at B from 90°, or increase of the inclination of the

FIG. 163.



Limiting angle of emission.

FIG. 164.



Limiting angle of transmission.

crystal faces to each other, each of which increases the angle BCE, causes total reflection at C, and thereby prevents transmission. Refracting angles intermediate between the above extremes obviously transmit light incident through an angular range less than 90° and greater than 0°.

The largest angle of incidence clearly is 90°, and the smallest i, as determined by the equation,

$$\sin i = \mu \sin r = 1.31 \sin (A - 49° 46')$$

If, then, $A = 60°$, $i = 13° 27'$, giving a transmission range of 76° 33'; if $A = 90°$, $i = 57° 48'$, range 32°. 12'; and similarly for other possible values of A.

General Illumination of the Sky Through Ice Crystals.—The deviation of a ray of light through refraction and n internal reflections obviously is given by the equation,

$$D = i + i' + n\pi - \Sigma A,$$

in which ΣA is the sum of the several angles passed by the ray in its course through the crystal. If these angles are all equal the equation becomes

$$D = i + i' + n\pi - (n + 1) A.$$

If, then, as frequently is the case, the ice crystal is a thick hexagonal disk floating horizontally, the position it oscillates about in falling, both angles passed by a once reflected ray will be 90°, provided the entering and emergent branches lie in the same plane, and the deviation will be

$$D = 2 i,$$

as readily seen in Fig. 165.

Hence, such crystals illuminate the sky at all distances from the sun out to 115° 36'. The effect, however, is not sufficiently striking ordinarily to arrest attention.

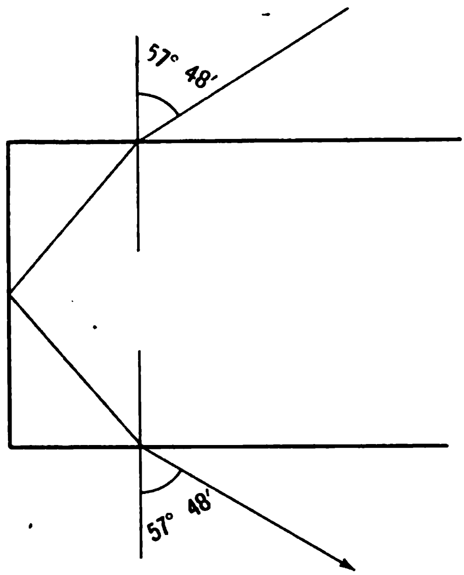
Parhelia of 22°.—Whenever the air through any depth or at any level contains innumerable hexagonal snow crystals with their sides vertical (the position about which relatively broad crystals oscillate) two colored bright spots, known as parhelia or sun dogs, appear at 22°, or more, from the sun, one to the right, the other to the left. Each bright spot is in the direction of maximum light or minimum refraction, and has the same altitude as the sun. When the refraction is in a principal plane, that is, as the sides of the crystal are vertical, when the sun is on the horizon, the angular distance, D_0, of each spot, also on the horizon, is given by the equation, derived from (3),

$$\sin \frac{D_0 + 60°}{2} = \mu \sin \frac{60°}{2}$$

For yellow light ($\mu = 1.31$), $D_0 = 21°\ 50'$; for red light ($\mu = 1.307$), $D_0 = 21°\ 34'$; and for violet ($\mu = 1.3117$), $D_0 = 22°\ 22'$. The order of the colors, therefore, counting from the sun, is red, yellow, etc., to violet, and the length or dispersion 48′ for a point source. For the sun, diameter 30′, the total length is 1° 18′, and width 30′.

Since the above are minimum angles, it follows that slight changes in either the inclination or orientation of the crystals causes rays of each color to come also from somewhat greater

FIG. 165.



Illumination of the sky by flat snow crystals.

distances from the sun. Hence, the only color thus produced that appears approximately pure is the darker portion of the red. Yellow and green are also moderately distinct, but blue, and especially violet, scarcely perceptible because of so much admixture of colors.

When the angular elevation of the sun is h, the distance, D', in azimuth, of each of these parhelia from the sun is given by the equation, derived from (5),

$$\sin\frac{D' + 60°}{2} = \mu\frac{\cos k}{\cos h}\sin\frac{60°}{2}.$$

The angular distance, Δ_0, measured on the arc of a great circle, between the sun and each of these parhelia, may be found from the right spherical triangle formed by the three sides: zenith to sun (complement of h); zenith to mid point between sun and a parhelion; " mid point " to sun, $\frac{\Delta_0}{2}$. The angle thus formed at the zenith is $\frac{D'}{2}$, and the angle at the " mid point " 90°. Hence,

$$\sin \frac{\Delta_0}{2} = \cos h \sin \frac{D'}{2}$$

For $\mu = 1.31$ the following relations[187] exist between h, D', and Δ_0.

h	D'	Δ_0
0°	$D_0 = \Delta_0 =$	21° 50′
5°	22° 2′	21° 56′
10°	22° 30′	22° 10′
15°	23° 20′	22° 32′
20°	24° 34′	23° 4′
25°	26° 22′	23° 50′
30°	28° 44′	24° 49′
35°	31° 56′	26° 3′
40°	36° 20′	27° 38′
45°	42° 30′	29° 42′
50°	51° 30′	32° 26′
55°	66° 2′	36° 26′
60°	98° 48′	44° 38′
60°45′	120° 0′	50° 4′

All the above values pertain, as explained, to minimum deviation. But as the orientation of the crystals is fortuitous, it follows that all possible deviations from minimum to maximum will occur, and the parhelia, therefore, be drawn out into streaks the lengths of which depend upon their angular altitude.

The maximum deviation for refraction in a principal plane (for sun on the horizon when the crystal edges are vertical) is given by the equation, derived from (4),

$$\cos (180° - 60° - D_m) = \mu \sin (60° - 49° 46').$$

The value of the maximum deviation in azimuth, D_m, corresponding to the solar altitude, h, is given by equation (6), and the actual maximum, Δ_m, by the equation,

$$\sin \frac{\Delta_m}{2} = \cos h \sin \frac{D'_m}{2}.$$

[187] Pernter-Exner, *Meteorologische Optik*, pp. 314, 315.

The following table [188] gives interesting relations between the quantities indicated:

h	D_m	Δ_m	$\Delta_m - \Delta_0$
0°	43° 28′	43° 28′	21° 38′
5°	43° 38′	43° 26′	
10°	44° 8′	43° 24′	21° 16′
15°	44° 59′	43° 20′	
20°	46° 15′	43° 18′	20° 18′
25°	48° 0′	43° 16′	
30°	50° 17′	43° 10′	18° 22′
35°	53° 15′	43° 4′	
40°	57° 9′	42° 58′	15° 22′
45°	62° 28′	43° 2′	
50°	68° 48′	43° 10′	10° 46′
55°	81° 3′	43° 44′	
60°	104° 54′	46° 44′	2° 6′
60°45′	120° 0′	50° 4′	0° 0′

From the computed values of $\Delta_m - \Delta_0$, fully supported by observations, it appears that when the angular elevation of a parhelion of 22° is moderate to small, 20° or less, it may extend over an arc, parallel to the horizon, of more than 20°. The end next the sun, produced by minimum deviation, is colored, beginning with red, through a short range. Similarly, the distal end, due to maximum deviation, is also colored, terminating with violet, though always too faint, perhaps, to be distinctly seen. Through the rest of its length the blending of the colors is quite complete, giving white, of course, as the result.

At greater altitudes the possible lengths of the parhelia of 22° become less and less, as shown by the table, though the color distribution remains the same.

Halo of 22°.—When the refracting edges of the ice crystals are vertical, as they tend to be in the case of relatively thin snowflakes falling through still air, parhelia are produced, as just explained. But, in general, these edges lie in all directions, especially at the windy cirrus level and when the crystals are of the short columnar type; and as refracted light reaches an observer in every plane through his eye and the sun (or moon) to which the refracting edges are approximately normal, it follows that the effect produced by fortuitously directed snow crystals must be more or less symmetrically distributed on all sides of the exciting luminary. There may, however, be a maximum brightness both

[188] Pernter-Exner, *Meteorologische Optik*, p. 317.

directly above and directly below the sun since ice needles tend to settle with their refracting edges horizontal.

As before, when the refracting angle is 60° and $\mu = 1.31$, corresponding to yellow light, $D_0 = 21° 50'$, and is independent of solar elevation. The inner portion of this, the most frequent and best known of all halos, is red, because light of that color is least refracted. Other colors follow, with increase of distance, in the regular spectral sequence, but with decrease of wave-length they so rapidly fade that even green is indistinct and blue seldom detected. This is owing to the variation in deviation caused by the tipping of the needles, as previously fully explained.

The brightest portion of the ring clearly is at the angle of minimum refraction from the sun. With increase of distance, light produced in this manner gradually fades (not all the crystals are ever simultaneously in position to give minimum refraction) until it ceases to be perceptible at 15° to 20° beyond the inner portion, or, say, 40° from the sun. On the other hand, no such light reaches the observer from places within the ring of maximum brightness, and, therefore, this portion of the sky is comparatively dark, except, and for an entirely different reason, to be explained later,[189] near the sun itself.

When the sun is within 10° of the horizon, the halo of 22°, and the parhelia of 22°, are practically superimposed. At greater altitudes they become distinctly separated, as per the accompanying table[190] for $\mu = 1.31$, in which h = solar elevation, Δ_0 = parhelic angular distance from the sun and D_0 = angular distance of halo from sun.

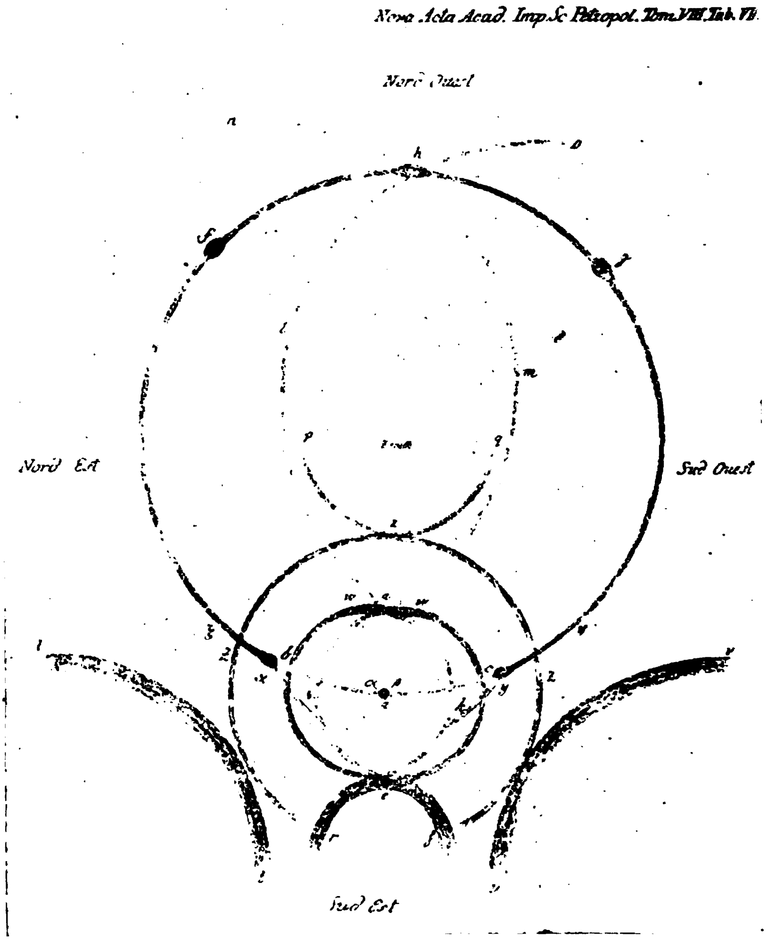
h	$\Delta_0 - D_0$
0°	0° 0′
10°	0° 20′
20°	1° 14′
30°	2° 59′
40°	5° 48′
50°	10° 36′
60°	22° 48′
60°45′	28° 14′

Arcs of Lowitz, or Vertical Arcs of the 22° Parhelia.—On rare occasions oblique extensions of the parhelia of 22°, concave towards the sun and with red inner borders, are seen, in addition to their horizontal tails, above described. These are known

[189] See chapter on diffraction.
[190] Pernter-Exner, *Meteorologische Optik*, p. 321.

as the arcs of Lowitz, after the astronomer who described them[191] as seen in the famous Petersburg halo complex (Fig. 166) of July 18, 1794. Their general explanation is simple, though exact

FIG. 166.



Petersburg halo of July 18, 1794.

computations of their outlines and of the relative intensities of their parts for different altitudes of the sun are rather tedious.

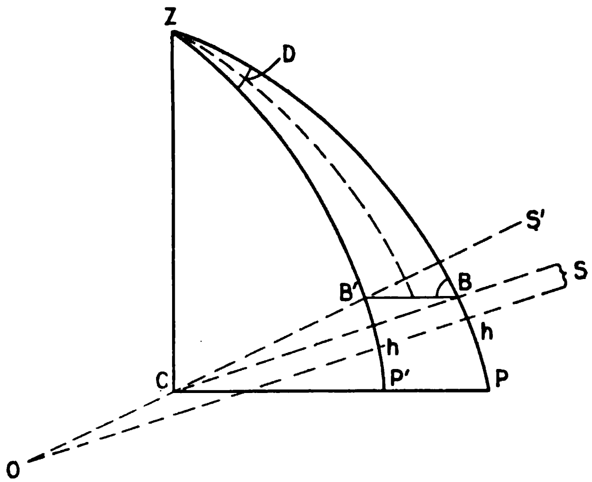
[191] *Nova. Acta Acad. Petropol.*, 8, 1794, p. 384.

As already explained, parhelia and their horizontal extensions are produced by ice crystals whose principal axes are vertical, the former by those set to minimum refraction, and the latter by crystals turned more or less from this unique position.

When, however, the principal axes oscillate about the vertical, as they obviously do in the case of snowflakes, the arcs of Lowitz, or obliquely vertical extensions of the parhelia of 22°, necessarily are produced, though rarely seen, because of the diffusion of their light in the midst of a general glare, as will be explained below.

Consider, first, for simplicity, the doubly special case in

FIG. 167.



Formation of the arc of Lowitz, crystals vibrating in solar vertical.

which the sun is on the horizon and the principal axes of the crystals oscillate in a vertical plane passing through the sun. Let C (Fig. 167) be the position of an ice crystal whose principal axis is in the direction CZ. Let an observer be at O and let the incident ray, SC, lying between Z and P make the angle h with the principal plane, CP. On emerging this ray, in its new direction, $S'O$, has the same inclination as formerly to the axis. Hence, SC and $S'C$ may be regarded as elements of a right cone of vertex C and axis CZ, and as the plane CZS is vertical, if S is on the horizon, as seen from O, the element CS (lowest element), being parallel to OS, owing to the great distance of S, will lie in

a plane tangent to the cone and parallel to the plane of the horizon, while every other element, such as CS', will lie above it. S', therefore, the apparent position of S due to refraction of the ray SC by the ice crystal at C is above this plane and, also, as seen from O at the same angular distance above the horizon. Similarly, when the axis of the crystal is tipped beyond the vertical in the opposite direction, S' drops below the horizon.

Further, let ZP and ZP' be arcs of great circles intersecting SC and $S'C$ at B and B', respectively. Then the projection of the deviation SCS', or SOS', on the principal plane is given by the angle D between these arcs, and on bisecting D, thus dividing ZBB' into two equal right spherical triangles, and putting $BB' = \Delta$, it is obvious that

$$\sin\frac{\Delta}{2} = \sin\frac{D}{2}\cos h \quad \text{- - - - - - - - - - - - -} \quad (1)$$

and

$$\cot B = \tan\frac{D}{2}\sin h \quad \text{- - - - - - - - - - - - -} \quad (2)$$

Clearly, then, the locus of S' is given when the arc BB' is known in terms of the angle B.

On eliminating h from the above equations, it appears that

$$\cot^2 B = \tan^2\frac{D}{2}\cos^2\frac{\Delta}{2} - \sin^2\frac{\Delta}{2} \quad \text{- - - - - - - - - - -} \quad (3)$$

Hence, that either Δ or B may be found when the other is given, it remains only to express $\tan^2\frac{D}{2}$ in terms of a function or functions of Δ.

But from (7), (p. 489),

$$\sin^2\frac{D+A}{2} = \mu^2\frac{\cos^2 k}{\cos^2 h}\sin^2\frac{A}{2}$$

$$= \left\{1 + (\mu^2 - 1)\sec^2 h\right\}\sin^2\frac{A}{2}.$$

Also from (3), (p. 486),

$$\mu^2 = \frac{\sin^2\dfrac{D_0 + A}{2}}{\sin^2\dfrac{A}{2}}$$

and from (1), (p. 498).

$$\sec^2 h = \frac{\sin^2 \frac{D}{2}}{\sin^2 \frac{\Delta}{2}}.$$

Hence,

$$\frac{\sin^2 \frac{D+A}{2} - \sin^2 \frac{A}{2}}{\sin^2 \frac{A}{2}} = \frac{\sin^2 \frac{D_0+A}{2} - \sin^2 \frac{A}{2}}{\sin^2 \frac{A}{2}} \times \frac{\sin^2 \frac{D}{2}}{\sin^2 \frac{\Delta}{2}}$$

or

$$\sin^2 \frac{\Delta}{2} \sin\left(\frac{D}{2} + A\right) \sin \frac{D}{2} = \sin^2 \frac{D}{2} \sin\left(\frac{D_0}{2} + A\right) \sin \frac{D_0}{2}.$$

Dividing by $\cos \frac{D}{2} \cos A$,

$$\sin^2 \frac{\Delta}{2} \tan \frac{D}{2} + \sin^2 \frac{\Delta}{2} \tan A = \frac{\sin\left(\frac{D_0}{2} + A\right) \sin \frac{D_0}{2} \tan \frac{D}{2}}{\cos A}$$

$$= \frac{1}{2}\left\{\left(1 - \frac{\cos (D_0 + A)}{\cos A}\right) \tan \frac{D}{2}\right\}:$$

Putting

$$\frac{\cos (D_0 + A)}{\cos A} = \cos \beta = \cos 73° \ 30'$$

$$\tan \frac{D}{2} = \frac{2 \tan A \sin^2 \frac{\Delta}{2}}{\cos\Delta - \cos\beta} \text{ - - - - - - - - - - - - - } (4)$$

On using this value of $\tan \frac{D}{2}$ (3) reduces to

$$\cot B = \frac{\sin \frac{\Delta}{2} \sqrt{4 \sin^2 \frac{\Delta}{2} \cos^2 \frac{\Delta}{2} \tan^2 A - (\cos\Delta - \cos\beta)^2}}{\cos\Delta - \cos\beta} \text{ - - } (5)$$

From (5) B is readily computed for any assumed value of Δ, as is also D from (4). Further, h can be found from (1) when Δ and D are known, or from (2) when B and D are known.

The following table, copied from Pernter-Exner,[192] as are

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[192] *l. c.*, p. 327.

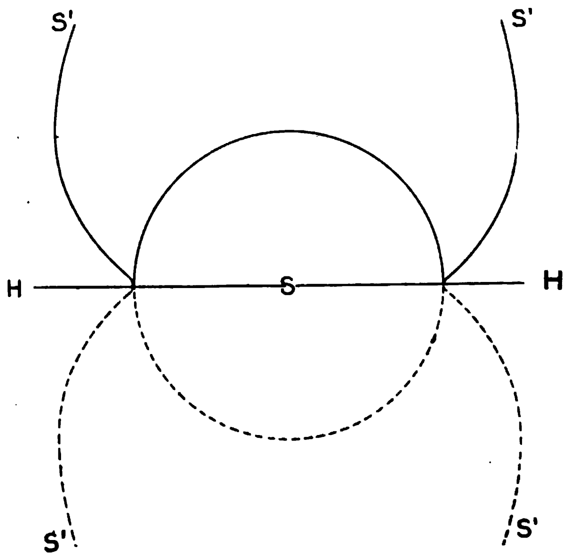
most of the above equations, gives data for drawing the locus of S' when the sun is on the horizon.

Since the value of Δ in this table increases with μ, other things being equal, it follows that these arcs must be colored and that their inner borders must be red, as stated.

h	Δ	B
0°	21° 50′	90° 0′
5°	21° 55′	89° 5′
10°	22° 10′	88° 1′
15°	22° 38′	86° 54′
20°	23° 04′	85° 45′
25°	23° 51′	84° 19′
30°	24° 49′	82° 38′
35°	26° 3′	80° 38′
40°	27° 38′	78° 5′
45°	29° 42′	74° 36′
50°	32° 26′	69° 43′
55°	36° 26′	61° 58′
60°	44° 38′	44° 41′
60°45′	50° 4′	33° 29′

This table is graphically represented by Fig. 168, in which the circle is the 22° halo, HH the horizon, S the sun and $S'S'$ the curve in question, dotted below the horizon where, of course,

FIG. 168.

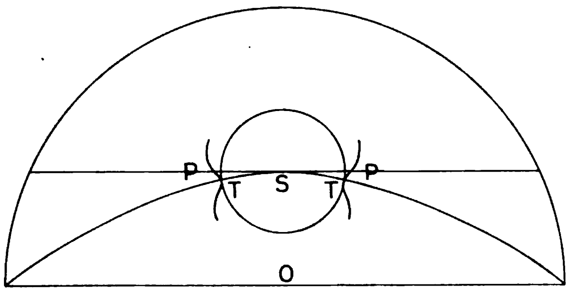


Arc of Lowitz, sun on horizon, crystals vibrating in solar vertical.

like the under portion of the circle, it is invisible, except from a suitable elevation.

Obviously, the optical effect is independent of the manner by which the inclination of the principal axis of the crystal to the incident ray is produced, and, therefore, crystals tilted in the vertical plane through the sun to an angle, $E + h$, to the plane of the horizon give the same result, if E is the solar elevation, as do crystals tilted to only the angle h in this plane when the sun is on the horizon. The points of contact with the halo of 22°, being due to crystals whose principal axes are normal to the incident rays, lie, therefore, at equal altitudes on opposite sides of the halo and in a plane that passes through the sun. Consequently, the angular altitude of these points is less than that of the sun, except when the latter is on the horizon. If, as before, E is the elevation of the sun and E' that of the points of contact in question, then, from the right spherical triangle formed by the radius

FIG. 169.



Arcs of Lowitz, elevation of sun 40°, crystals vibrating in solar vertical.

of the halo and the zenith distances of the sun and point of contact, respectively,

$$\cos (90° - E') = \cos (90° - E) \cos 21° \, 50'.$$

Fig. 169 represents, approximately, the outline of the bright band produced in this manner when the elevation of the sun is 40°; making that of the points of contact 36° 38′. O is the position of the observer, S the centre of the halo of 22°, PP parhelia of 22°, TT the points of tangency to this circle of the arcs of Lowitz, PT PT. In order that the arc may reach the halo, the tilt of the crystal must at least equal the elevation of the sun, and

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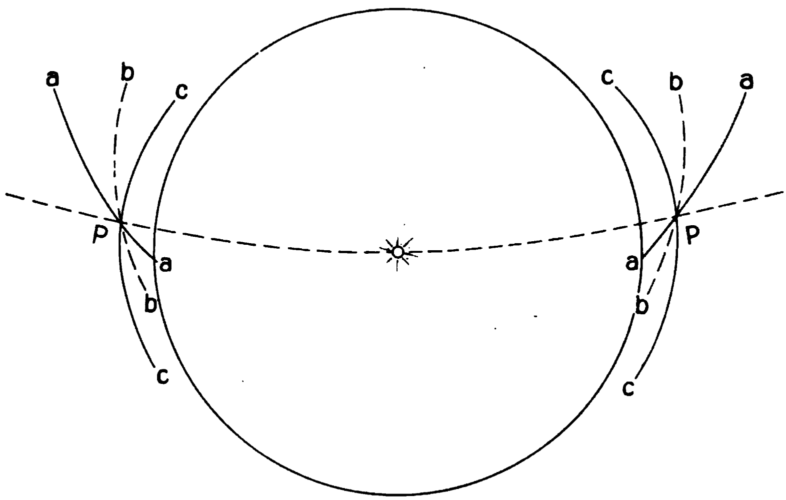
no portion of the lower branch (part below point of tangency) is given unless the tilt is greater than this elevation. Hence, if the extent of the tilting of snow crystals is less, in the great majority of cases, than 30°, as it probably is, only the upper branch is likely to be produced when the sun is 30° or more above the horizon.

Consider, now, the effect of the vibration of the principal axis in a vertical plane at right angles to the vertical plane through the sun. Let E be the elevation of the sun and i the inclination of the principal axis to the vertical, then the angle h between the incident ray and principal plane is given by the equation,

$$\sin h = \sin (90° - i) \sin E.$$

Let $E = 30°$, and $i = 20°$. Then the angular distance from the sun to a parhelion of 22° is 24° 49′, or say 3° from the halo

FIG. 170.



Arcs of Lowitz, crystals vibrating at random.

of 22°. Also $h = 28° \ 1'$, and the corresponding distance of image from halo is about 2° 40′, at approximately 20°, measured from centre of halo, above or below the parhelion, owing to direction of tip.

Vibrations of the principal axis in intermediate planes give images, of course, in intermediate positions, so that the total

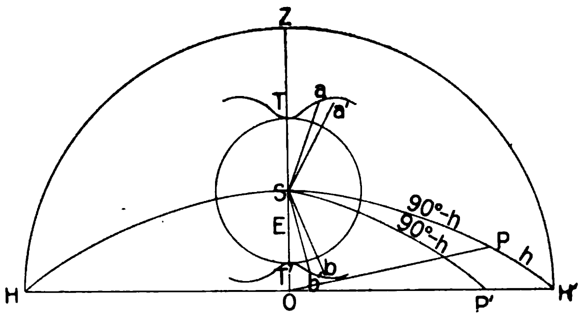
effect, when the elevation of the sun is 30°, may be somewhat as represented in Fig. 170, in which tilting is supposed to be restricted to 30° and less from the vertical.

In this Figure, *PP* are the parhelia of 22°, *aa, bb,* and *cc* the outlines of the images corresponding to minimum refraction when the principal axis is oscillated in a vertical plane through the sun, true tangent arcs; at 45°, roughly interpolated, to this sun plane; and at 90° to it, respectively.

It will be noticed that this light, nearly always too faint to be distinguished from the general glare, would be more concentrated, if the orientation of the ice crystals were fortuitous, which, presumably, often is the case, and consequently brighter below the parhelia than above them. Hence, because of limited tilting, as above explained, and because of the greater concentration of light in its lower branches, this halo, whenever seen at all, appears as short arcs including the parhelia and extending mainly below them.

Tangent Arcs of the Halo of 22°.—Obviously, when the sun is on the horizon, ice crystals whose principal axes lie or oscillate

FIG. 171.



Tangent arcs of the halo of 22°.

in horizontal planes must produce the same optical effects above and below (theoretically) the halo of 22° that are produced on its sides, as above explained, by crystals whose principal axes oscillate in the vertical plane through the sun. Each set of curves might properly be called tangent arcs of the halo of 22°, but as a matter of fact only those well-known and fairly common arcs that occur above and below the circular halo are so designated.

Similarly, when the elevation of the sun is *E*, arcs identical

with those just described and tangent to the halo at its highest and lowest points, as shown in Fig. 171, are formed by crystals whose principal axes oscillate in planes normal to the solar vertical and inclined at the angle E to the plane of the horizon.

But ice spicules or needles tend to float with their principal axes horizontal. Hence, it is necessary carefully to determine the optical effects of crystals in this particular position, as may be done by noting the transformations of the tangent arcs as the crystals are so turned as to carry their principal axes from the inclined to the horizontal plane.

Let, then, the principal axis of an ice needle lie parallel to OP (Fig. 171) in which O is the position of the observer, HSH' the inclined plane and S the sun at elevation E. Let h be the inclination of the principal axis to the incident radiation and let a or b be the position of the resulting image. Now, let the crystal, as suggested above, be so turned as to carry its axis from an inclined to a horizontal position, and in such manner as to keep *constant* the angle between the principal axis and the direction of the incident ray. That is, change the direction of the axis from parallel to OP to parallel to OP', with $SP = SP'$. Under these conditions the refracted ray will turn precisely as does the principal axis. Hence, if a' and b' are the new positions of a and b, the angle $aSa' = bSb' = PSP'$.

But from the right spherical triangle OSP'

$$\cos OSP' = \sin PSP' = \tan E \cot SP' = \tan E \tan h,$$

and

$$aSa' = bSb' \text{ arc} \sin \tan E \tan h.$$

Since the points of tangency of the " tangent arcs " under consideration are 90° from the corresponding points of the similar " arcs of Lowitz," it follows that the angle B of Fig. 167 and table on page 500 equals TSa (Fig. 171).

Hence,

$$\left.\begin{matrix} TSa' \\ T'Sb' \end{matrix}\right\} = S = B \pm \text{arc} \sin \tan E \tan h$$

The following table, adapted from Pernter-Exner, "Meteorologische Optik," pp. 338–339, gives the necessary data for accurately constructing tangent arcs corresponding to different solar elevations:

Values of Angle S.

E		5°		10°55′		15°		20°		25°2′	
h	Δ	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0°	21°50′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′
5	21 55	1 21	0 29	1 50	0 0	2 16	−0 26	2 45	−0 55	3 16	− 1 26
10	22 10	2 52	1 06	3 56	0 02	4 42	−0 44	5 40	−1 42	6 42	− 2 44
15	22 38	4 27	1 45	6 04	0 08	7 13	−1 01	8 42	−2 30	10 17	− 4 05
20	23 04	6 15	2 35	8 27	0 25	10 01	−1 11	12 02	−3 12	14 12	− 5 22
25	23 51	8 01	3 21	10 51	0 31	12 52	−1 30	15 27	−4 05	18 16	− 6 54
30	24 49	10 16	4 28	13 46	0 58	16 16	−1 32	19 30	−4 46	23 01	− 8 17
35	26 03	12 53	5 51	17 08	1 36	20 11	−1 27	24 08	−5 24	28 28	− 9 44
40	27 38	16 08	7 42	21 14	2 36	24 55	−1 05	29 42	−5 52	34 59	−11 19
45	29 42	20 25	10 23	26 31	4 17	30 57	−0 09	36 45	−5 57	43 16	−12 28
50	32 26	26 16	14 18	33 34	7 00	38 55	1 39	46 10	−5 26	54 07	−13 33
55	36 26	35 13	20 51	44 02	12 02	50 32	5 32	59 21	−3 17	69 53	−13 49
60	44 38	54 02	36 36	64 50	25 48	72 58	17 40	84 25	6 13	99 19	− 8 41
60°45′	50 04	65 30	47 32	76 40	36 22	85 06	27 56	97 04	15 28	113 02	0 0

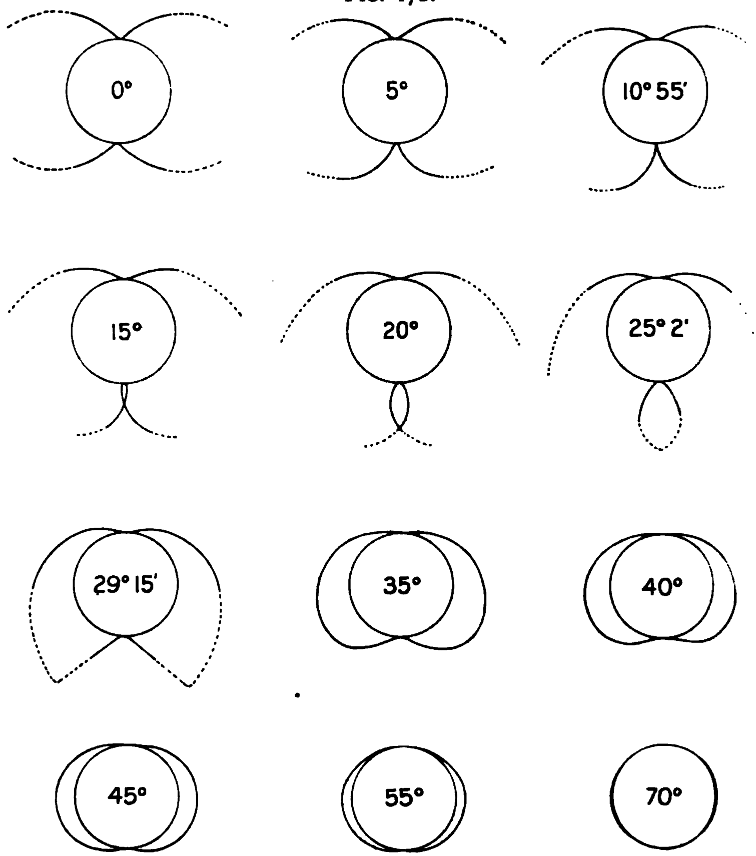
E		29°15′		35°		40°		45°		50°	
h	Δ	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0°	21°50′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′
5	21 55	3 44	− 1 54	4 26	− 2 36	5 08	− 3 18	5 56	− 4 06	6 54	− 5 04
10	22 10	7 39	− 3 41	9 04	− 5 06	10 30	− 6 32	12 09	− 8 11	14 07	−10 09
15	22 38	11 54	− 5 32	13 51	− 7 43	16 06	− 9 54	18 39	−12 27	21 44	−15 32
20	23 04	16 11	− 7 21	19 11	−10 21	22 12	−13 22	25 46	−16 56	20 08	−21 18
25	23 51	20 49	− 9 27	24 45	−13 23	28 43	−17 21	33 29	−22 07	39 26	−28 04
30	24 49	26 14	−11 30	31 12	−16 28	36 20	−21 36	42 38	−27 54	50 50	−36 06
35	26 03	32 27	−13 43	38 43	−19 59	45 21	−26 37	53 48	−35 04	65 55	−47 11
40	27 38	39 57	−16 07	47 54	−24 04	56 40	−32 50	68 57	−45 07	101 55	−78 05
45	29 42	49 27	−18 39	59 50	−29 02	72 26	−41 38	105 24	−74 36	.. ..	.. ..
50	32 26	62 09	−21 35	76 50	−36 16	110 17	−69 43	.. ..	.. ..	.. ..	.. ..
55	36 26	81 09	−25 07	118 02	−61 58	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
60	44 38	121 15	−30 37	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
60°45′	50 04	146 31	−33 29	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..

E		55°		60°		70°		80°	
h	Δ	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0°	21°50′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′
5	21 55	8 06	− 6 16	9 38	− 7 48	14 50	−13 00	30 40	−28 50
10	22 10	16 34	−12 36	19 46	−15 48	30 58	−27 00	91 59	−88 01
15	22 38	25 36	−19 28	30 45	−24 33	50 31	−44 19	.. ..	.. ..
20	23 04	35 45	−26 57	41 31	−34 41	94 25	−85 35	.. ..	.. ..
25	23 51	47 26	−36 01	59 34	−48 12	.. ..	.. ..	.. ..	.. ..
30	24 49	62 54	−48 07	97 22	−82 38	.. ..	.. ..	.. ..	.. ..
35	26 03	99 22	−80 38	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
40	27 38	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
45	29 42	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
50	32 26	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
55	36 26	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
60	44 38	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
60°45′	50 04	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..

The upper and lower tangent arcs change with elevation, *E*, of the sun, as indicated by the table and shown in Fig. 172, copied from Pernter-Exner, "Meteorologische Optik." Portions below the natural horizon can only be seen from sufficient heights.

With increase of elevation of the sun from the horizon the branches of the lower tangent arc draw closer together; become

FIG. 172.



Upper and lower tangent arcs at solar elevations indicated.

pointed; cross, forming a loop; then open out and turn up where they merge with the drooped branches of the upper tangent arc and thus form an enclosing curve, with red inner border, which, at first, when the elevation is 30° to 35°, is bagged below; then, when the elevation is 50° to 60°, approximately elliptical; and

finally, at elevations of 75° and more, indistinguishably merged with the circular halo. The brightest portions of these arcs are at and near the points of tangency. Hence, when the solar elevation is 35° to 45° the lower arc, if visible over only its brightest part, appears as an inferior tangent arc, which, indeed, it is, circular, perhaps, and limited to, say, 10° to 20°, which, in reality, it is not.

Frequency of Horizontal and Rarity of Vertical Tangent Arcs.—Since the theory of the formation of the horizontal (upper and lower) tangent arcs of the halo of 22° is identical with that of the vertical (arcs of Lowitz), it will be interesting to consider why, though the former is fairly common, the latter is extremely rare. Obviously this must somehow be connected with the attitudes, which radically do differ, of the principal axes of the ice crystals that produce them. The horizontal tangent arcs are produced, as already explained, by crystals whose principal axes are horizontal, and, therefore, by columnar crystals, since these and these alone tend to assume this as an attitude of maximum frequency and thus produce a corresponding concentration of light.

The vertical tangent arcs (arcs of Lowitz), on the other hand, being produced by crystals whose principal axes oscillate in that particular vertical plane that passes through the sun nearly always are too faint to be seen, because, in part, this unique attitude can only rarely be assumed by any considerable proportion of the crystals present. Even the combined effect of the crystals in all vertical planes is seldom noticed because, as explained, of its width and consequent faint nebulosity.

Another factor that probably contributes to the contrast between the frequencies of occurrence of the two types of tangent arcs is the fact that columnar crystals whose principal axes tend to lie horizontal probably are far more effective as refractors or halo producers, than are the tabular crystals whose principal axes tend to stand vertical. This is because the tabular crystals are so filled with air spaces or other heterogeneities, as shown by their photomicrographs, that anything like regular transmission of light through them from edge to edge is hardly possible. The columns, on the other hand, appear to be more nearly homogeneous and, consequently, much more effective as refractors. This is partially confirmed by the fact that halos often are

seen close at hand in polar regions when the air is filled with ice needles, and rarely if ever seen in ordinary snowstorms consisting essentially of tabular crystals.

The greater efficiency (presumably), then, of the columnar crystal, whose principal axis tends to lie horizontally, as a refractor, over that of the tabular crystal whose principal axis tends to stand vertical, together with the further fact that the orientation of the vertical plane of oscillation must nearly always be fortuitous, seems to explain why the horizontal tangent arc of the halo of 22° is so frequently seen and the vertical so rarely.

It should be remembered, however, that, in apparent contradiction of the above statements concerning the maximum frequency attitudes of the principal axes of ice crystals, there are two special types of columnar crystals whose principal axes tend to stand vertical; namely, those that are shorter than broad (tabular when much shorter than broad), and those that have tabular caps. Perhaps, therefore, all halo phenomena are due essentially to columnar and very little to tabular crystals.

Parhelia of 46°—Since the flat ends of columnar snow crystals and the flat sides of tabular ones both are at right angles to the sides, it follows that optical phenomena must occur that are produced by refraction at such angles, analogous to those already explained for the 60° angle.

Let, then, the 90° intersection be vertical, as it is, more or less, in the case especially of columnar crystals, and let the orientation be that of minimum refraction. If, now, the sun is on the horizon, the distance from it to either of its refraction images, also on the horizon, corresponding to the angle in question, are given by substitution in the general equation,

$$\sin \frac{D_0 + A}{2} = \mu \sin \frac{A}{2}.$$

On putting $\mu = 1.31$, this becomes

$$\sin \left(\frac{D_0}{2} + 45°\right) = 1.31 \sin 45°,$$

from which $D_0 = 45° 44'$.

Hence, these images are known as the parhelia of 46°.

With increase of elevation of the sun the inclination, h, of the incident ray to the vertical face of the crystal is equally increased, as is also the elevations of the images, as we know from previous

considerations. Hence the positions of the parhelia of 46° corresponding to different elevations of the sun may be found in the same manner as those of 22°. On substituting, then, in the equation,

$$\sin \frac{D' + A}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{A}{2},$$

in which $\sin h = \mu \sin k$, the proper values of μ and A, namely 1.31 and 90°, respectively, and also computing the corresponding values of Δ'_0, one obtains the following table:

h	D'_0	Δ'_0
0°	45° 44′	45° 44′
5°	46° 11′	46° 0′
10°	47° 36′	46° 50′
15°	50° 08′	48° 18′
20°	54° 08′	50° 38′
25°	60° 48′	54° 36′
30°	72° 48′	61° 52′
32°12′	90° 0′	73° 30′

These parhelia, like those of 22°, also trail off parallel to the horizon, for crystals whose attitudes differ somewhat from that of minimum refraction. Such trails, however, necessarily are very faint, and perhaps never observed. In fact, these parhelia themselves are only rarely seen.

Halo of 46°.—Since, as just explained, the image, S_1 (Fig. 173), of the sun produced in the principal plane of a 90° refracting angle of an ice crystal, as seen by the observer, O_1, is 45° 44′ from the sun, S, itself ($\mu = 1.31$), it follows that when such crystals are very abundant and set at random in all directions the innumerable images so produced must together assume the shape of a ring about the sun of radius 45° 44′. This is the well-known, though not very common, halo of 46°.

Whenever at all conspicuous, this halo also shows colors (red being nearest the sun) which, because of the greater dispersion produced by the angle of 90° than by the angle of 60°, are more widely separated than in the halo of 22°. Hence, it likewise has the greater width of the two—about 2° 36′, corresponding to the diameter of the sun, 30′, plus the dispersion, that is, to $30' + D_r$, $(\mu = 1.317) - D_r$, $(\mu = 1.307) = 30' + 47° \; 16' - 45° \; 10' = 2° \; 36'$.

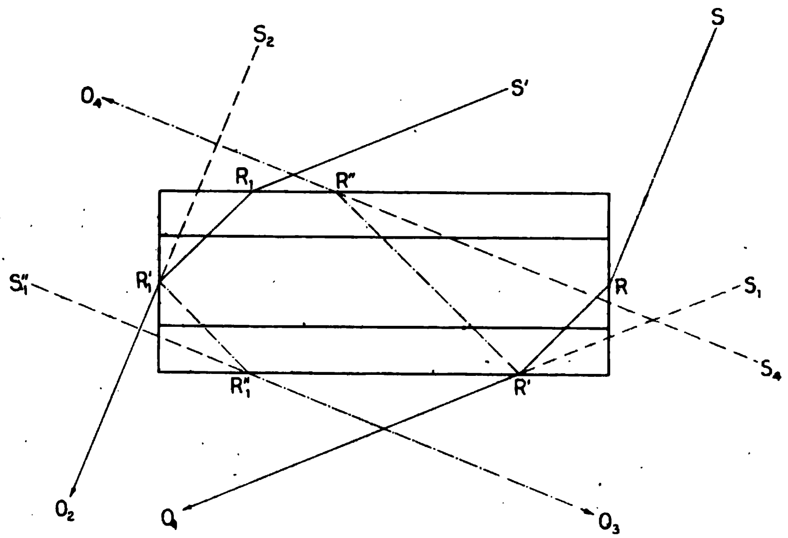
In addition to the colored ring, there is also a broad outer band of diffuse white light corresponding to all refractions other

than the minimum, but it is too faint and too uniformily distributed to be conspicuous, or, perhaps, unmistakably seen even when carefully looked for.

Halo of 90°.—Occasionally, a faint white halo, sometimes called the halo of Hevelius, is seen at 90° from the sun.

Several explanations of this halo have been suggested, but none gives it the right distance from the sun or is otherwise satisfactory. The following simple theory of its formation, therefore, is offered, based on the presence of fortuitously directed columnar crystals—short columns. Let a ray of whatever

FIG. 173.



Formation of halo of 46°.

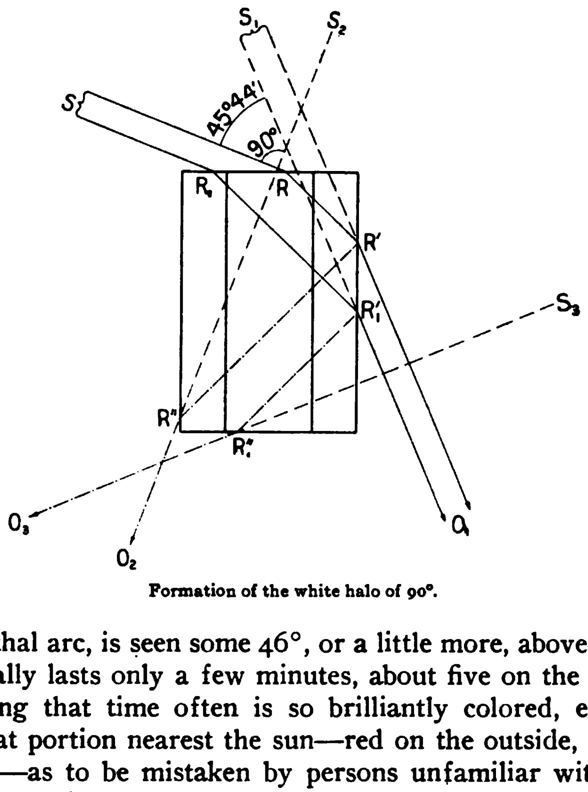
wave-length be incident at R, say (Fig. 174), on the flat end of a columnar crystal in the direction appropriate to ***minimum deviation***, and let most of that portion of it reflected at R' emerge from the opposite face at R''. The resulting image, S_2, seen by an observer at O_2, clearly is 90° from S. And since this direction holds for the more concentrated (minimum deviated) portion of every color, it follows that the resulting 90° halo must be white. It also must be more sharply defined on the side away from than on the side next to the sun.

Bouguer's Halo, White Halo of 136°.—Rays entering a hori-

zontal crystal face at R_1 and emerging at R''_1 (Figs. 173 and 174) clearly must produce a faint white image at about 135° 44′ ($\mu = 1.31$) from the sun, and a cloud of such horizontal crystals a white halo of outer radius 44° 16′ and inner radius much less about the antisolar point as a centre. Perhaps this is the Bouguer's halo, or false white rainbow, the theory of which has been considered obscure.

Circumzenithal Arc.—Occasionally, an arc of, perhaps, 90°, having its centre at the zenith, and, therefore, known as the cir-

FIG. 174.



Formation of the white halo of 90°.

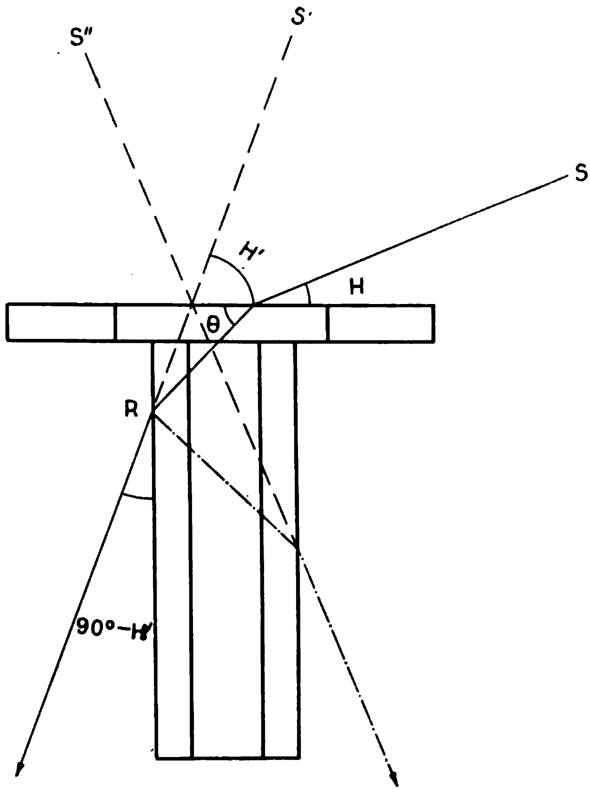
cumzenithal arc, is seen some 46°, or a little more, above the sun. It generally lasts only a few minutes, about five on the average, but during that time often is so brilliantly colored, especially along that portion nearest the sun—red on the outside, to violet inclusive—as to be mistaken by persons unfamiliar with it for an exceptionally bright rainbow. It occurs most frequently when the altitude of the sun is about 20° and at times when the parhelia of 22° are conspicuous. Presumably, therefore, when the principal axes of a large portion of the crystals are practically vertical; that is, when the snow crystals are largely columnar

with tabular caps, a moderately common form,[193] as outlined in Fig. 175, or, more likely perhaps, merely tabular.

The explanation of this halo, as of many others, was first given by Bravais,[194] and is very simple.

In still or steadily flowing air the type of crystals assumed

FIG. 175.



Formation of circumzenithal arc.

will keep their principal axes substantially vertical. Let, then, H (Fig. 175) be the altitude of the sun above the horizon, and also the angle between the incident ray and the upper horizontal surface of the crystal; let θ be the angle between this surface and

[193] Bentley, *Monthly Weather Review,* 30, p. 609, 1902.

[194] Mémoire sur les halos, *Journal de l'École polytechnique,* 31me cahier. 1845.

the refracted ray, and let H' be the altitude of the solar image produced by the 90° prism.

Hence,

$$\cos H = \mu \cos \theta,$$

and

$$\sin H' = \mu \sin \theta = \sqrt{\mu^2 - \cos^2 H}.$$

As previously explained, the angles between a principal plane (plane normal to the refracting edge) and incident and emergent rays are equal. Therefore, since the position of the image is altered by rotation of the crystal about its principal axis while its altitude remains unchanged, it follows that the halo so produced is a limited circular arc whose centre is the zenith.

From the equation for minimum refraction,

$$\sin \frac{D_0 + A}{2} = \mu \sin \frac{A}{2},$$

it appears that in the present case, and for $\mu = 1.31$,

$$D_0 = 45° \; 44'.$$

This corresponds to $H = 22° \; 8'$, and to $H' = 67° \; 52'$. In this case

$$H' - H = 45° \; 44',$$

which is the radius of the 46° halo.

With increase or decrease of the altitude of the sun from 22° 8′, the solar distance of the circumzenithal arc increases, but so slowly at first that the gain amounts to only about 1° when the sun has sunk to 16°, or risen to 27°. Hence, this arc is also, though erroneously, called the upper tangent arc of the halo of 46°.

When the sun is on the horizon the solar distance of the circumzenithal arc is 57° 48′, and the interval between it and the 46° halo 12° 4′. On the other hand, the arc rapidly converges on the zenith as the altitude of the sun approaches 32°, and is theoretically impossible for solar altitudes greater than 32° 12′.

Kern's Arc.—Kern's arc, so designated from the name of the first observer to report [195] it, occurs exactly opposite the corre-

[195] *Koninklijk nederlandsch meteorologisch Instituut. Onweders, optische verschijn., enz., in Nederland,* 1895, O. 66.

sponding circumzenithal arc and on the same circle. It might, therefore, also be called the anticircumzenithal arc.

No explanation of this arc is at hand. Its theory, however, appears to be identical with that of the circumzenithal, with the single exception that while the latter is produced by light transmitted at R (Fig. 175) the former, which is much the fainter of the two,[196] is due to that portion of the light which is there reflected, thus producing the image S''.

Circumhorizontal Arc.—A colored arc, red on the upper side, of perhaps 90° in extent, is occasionally seen parallel to the horizon and about 46°, or a little more, below the sun. This arc is produced by light entering snow crystals through vertical sides and passing out through horizontal bases, and, therefore, the theory of its formation is identical with that of the circumzenithal arc. On merely substituting "zenith distance" for "elevation" all the numerical values of the one become those of the other. Hence, the circumhorizontal arc cannot appear when the zenith distance of the sun is greater than 32° 12′. Similarly, when this distance is 22° 8′, corresponding to minimum deviation, the solar distance of the circumhorizontal arc is 45° 44′, the radius of the halo of 46°. Further, for all values of the zenith distance of the sun from 16° to 27° the circumhorizontal arc is within 1° of tangency to the halo of 46°. Hence, it is also, though incorrectly, called the lower tangent arc of the halo of 46°.

Lateral Tangent Arcs of the Halo of 46°.—Just as flat-topped crystals with vertical sides produce a circumzenithal arc when the altitude of the sun is between 0° and 32° 12′, so, too, similar crystals whose axes are horizontal and directed towards any point whose solar distance is between 90° and 57° 48′, or between 0° and 32° 12′, produce a colored arc—red next the sun—about this directive point as a centre. And as there are two such points corresponding to each solar distance, one to the right, the other to the left, of the solar vertical, it follows that arcs formed in the above manner are symmetrically situated with respect to this vertical. Further, when the solar distance of the directive point is 67° 52′ or 22° 8′, the resulting arc is tangent to the halo of 46°, and as always some, at least, of the innumerable crystals are turned towards this point, except when the altitude of the sun is greater than these values, respectively, it follows, with the

[196] *Monthly Weather Review,* 34, p. 124, 1906.

same exceptions, that the blend of the numerous arcs produced by the various directed crystals is always tangent to the halo of 46°, and also that except near the point of tangency only the red of these blends is reasonably pure.

Obviously, there are two classes of lateral tangent arcs, namely, lower, as seen at S_1 by an observer at O_1 (Fig. 173), and upper, as seen at S_2 by an observer at O_2. These will next be considered separately as infralateral and supralateral arcs.

Infralateral Tangent Arcs of the Halo of 46°.—Let the circle about S (Fig. 176) be the halo of 46°; let the altitude, H, of the sun be less than 67° 52′, and let the principal axes of the columnar crystals be horizontal and directed towards the point, P, on the horizon distant 67° 52′ from S. As previously explained, the infralateral tangent arcs will be tangent to this halo at the point T, where it is intersected by the arc SP. The position of T may easily be determined from the value of the angle A, between the vertical SB and the arc SP.

Obviously, from the right spherical triangle, SBP,

$$\cos A = \tan H \cot 67° \; 52'.$$

Since refraction by the crystal is limited to solar distances of P between 57° 48′ and 90°, it follows that A_1 and A_2, corresponding to the lower and upper ends of the tangent arc, are given by the equations,

$$\cos A_1 = \tan H \cot 57_{\circ} \; 48'$$

and

$$A_2 = 90°,$$

respectively.

When the altitude of the sun is 57° 48′, or a little greater, the two tangent arcs, springing from a common point on the solar vertical, form a wide V.

When the solar altitude equals 67° 52′, the two arcs, now merged into a smooth continuous curve, are tangent to the halo at its lowest point.

Finally, for altitudes of the sun greater than 67° 52′, the arcs, still appearing as a single curve, are slightly separated from the circular halo even at its lowest and closest point.

Supralateral Tangent Arcs.—When the altitude of the sun is less than 22° 8′ supralateral tangent arcs similar to the infralateral are produced.

The point of tangency of the supralateral tangent arc with the halo of 46° is given in terms of the arc, A, on this halo from its upper point.

When the solar altitude, H, is less than 22° 8′,

$$\cos A = \tan H \cot 22° \, 8'.$$

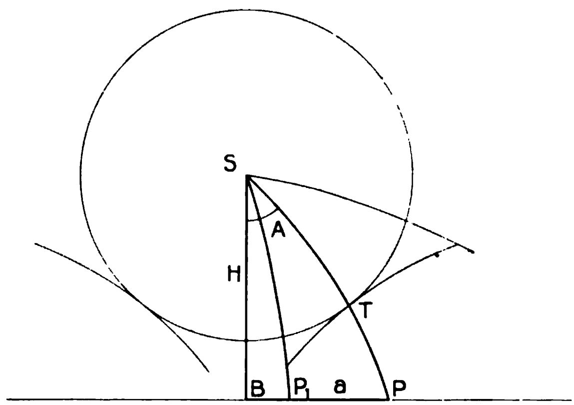
Similarly, the possible end of the arc is given by the equation,

$$\cos A' = \tan H \cot 32° \, 12'.$$

When the altitude of the sun is 22° 8′, both arcs, forming a continuous curve, are tangent to the halo at its highest point.

Finally, for altitudes between 22° 8′ and the extreme limit, 32° 12′, these arcs are more or less above the halo of 46°.

FIG. 176.



Formation of the lateral tangent arcs of the halo of 46°.

The following table gives the value of A for different altitudes of the sun.

Infralateral arcs.

H	A
0°	90° 0′
10°	85° 54′
20°	81° 35′
30°	76° 25′
40°	70° 03′
50°	61° 00′
60°	45° 13′
67° 52′	0° 0′

Supralateral arcs.

H	A
0°	90° 0′
5°	77° 35′
10°	64° 18′
15°	48° 47′
20°	26° 30′
22° 8′	0° 0′
25°	0° 0′
30°	0° 0′
32° 12′	0° 0′

Halos of Unusual Radii.—Halos of various radii other than those already given have occasionally been reported. They can readily be accounted for on the assumption that the columnar ice crystals have certain pyramidal bases that afford the appropriate refraction angles.

Secondary Halos.—Obviously, each bright spot of the primary halo phenomena, especially the upper and lower points of the 22° circle and its parhelia, must in turn be the source of secondary halos. Doubtless, the 22° halos of the lateral parhelia contribute much to the flaring vertical column through the sun that occasionally has been seen; and, perhaps, the brilliant upper and lower points of the halo of 22° may produce faint secondary parhelic circles. In general, however, very few of the secondary halos are ever bright enough to be seen even when carefully looked for.

Singular Halos.—A few halos not included in any of the above classes have been *once* reported. No satisfactory explanations of them have been offered. Clearly, though, since the ice crystal appears in many modified forms—with flat tabular, and pyramidal ends, for instance—and even in orderly clusters, it is obvious that although only a few halos are well known, a great many are possible.

CHAPTER V.

REFLECTION PHENOMENA.

A FEW optical phenomena of the atmosphere, usually classed as halos, are produced by simple reflection.

Parhelic Circle.—Very occasionally, a white circle, perhaps faint and tending to be diffuse, passes through the sun parallel to the horizon and, therefore, crosses the positions of the parhelia, anthelion, paranthelia, etc. This circle is produced by simple reflection (hence it is white) from vertical faces of ice crystals, as may easily be demonstrated.

To this end, let PP (Fig. 177) be a plane parallel to the horizon and normal to a vertical face of an ice crystal at C. Let SC be an incident ray reflected to the observer at O. Let CN be normal to the reflecting face, and lie in the plane PP. By the laws of reflection, CS, CN, and CO lie in a common plane, and the angle SCN = the angle OCN. If, now, $CO' = CO$, and $O'M$ and OM' be drawn normal to the plane PP, it is obvious that the triangle NMO' = the triangle $NM'O$, and that $MO' = M'O$. Hence, as the angles CMO' and $CM'O$ are right angles, and $CO' = CO$, the angle MCO', that is, the elevation of the sun above the plane PP, or above the plane of the horizon to which PP is parallel = the angle $M'CO$, the angle of depression at C of O below the plane PP, or angle of elevation of the crystal, C, above the observer's horizon.

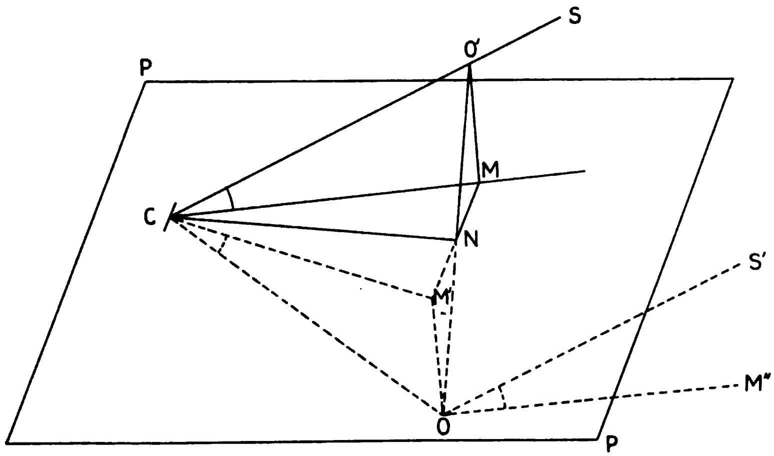
But C is the position of any *vertical* face that reflects light to the observer at O. Hence, the parhelic circle passes through the sun and is parallel to the horizon. Hence, also, it is superimposed upon such parhelia and parhelic tails as may coexist. Occasionally, therefore, one may be unable to distinguish between a portion of a parhelic tail and a segment of the parhelic circle. There can be no doubt, however, in regard to any portion that occurs nearer the sun than the appropriate positions of the parhelia of 22°. Such portion cannot be produced by refraction, and, consequently, must belong to the parhelic circle.

Since the formation of this circle requires a predominance of vertical faces, and since a simple columnar hexagonal prism tends not only to keep its major axis horizontal but also one of

its faces down, it follows that the required vertical surfaces must be either the flat ends of such crystals, the near end acting by direct reflection and the far one by internal reflection; or the hexagonal faces of short columnar and tabular crystals whose near and far sides also act by direct and internal reflection, respectively.

Anthelion.—On rare occasions a bright white spot, known as the anthelion, is seen on the parhelic circle opposite the sun. Of course, all crystals that contribute to the production of the parhelic circle also add to the brightness of the anthelion, but

FIG. 177.



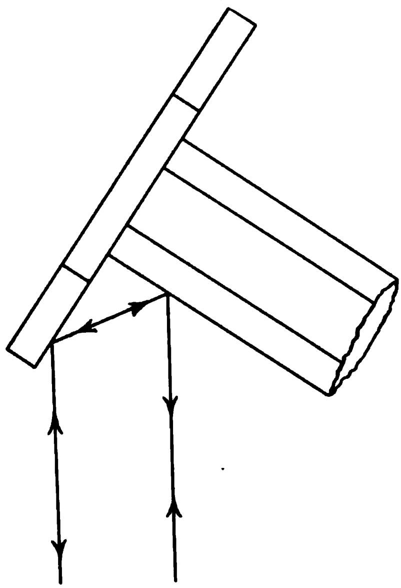
Formation of the parhelic circle.

as the latter sometimes occurs when the circle is inconspicuous, or not even seen, it follows that the supplementary light must be accounted for in some other way. A possible explanation is found in the action of columnar crystals with tabular caps at each end (Fig. 160). When both caps are the same size, or nearly so, and the columnar portion rather long the principal axes of such crystals must tend to remain horizontal, whatever the amplitudes of their oscillations. Now, whenever a crystal of this kind is horizontal, and one pair of its hexagonal faces vertical, light incident at whatever angle on either of these faces and finally reflected by the cap, or the reverse (see Fig. 178), will take such a course that the inclination of the reflected ray to the plane of the horizon will be the same as that of the incident

ray, and their projected directions on this plane exactly opposite to each other. Hence, this type of crystal will send much light to an observer from that point of the parhelic circle directly opposite the sun, enough, perhaps, when such crystals are particularly abundant, to produce a noticeably bright anthelion. Light refracted in and, after internal reflections, back out by the same vertical side also adds to the anthelion.

Oblique Arcs of the Anthelion.—On several occasions from

FIG. 178.



Formation of the anthelion.

one to four oblique white arcs have been seen to pass through the anthelion. Usually, when they occur, there are two such arcs symmetrically placed on either side of the vertical. The cause of these arcs is not definitely known, but as they are white (only once reported to show colors) it appears that presumably they are owing to reflection, possibly as follows:

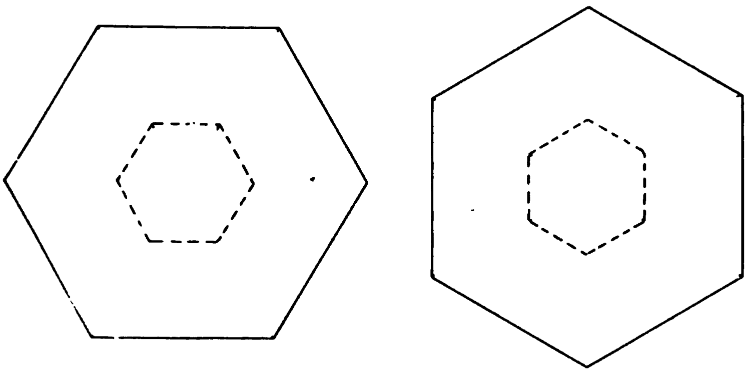
Let *A* and *B* (Fig. 179), be end-on views of horizontal columnar crystals with tabular caps. *A's* position is the stablest; *B's* probably, one of secondary stability. Such crystals return much light parallel, as just explained, to the line of sight to the anthelion. This light, when reflected by the inclined sides of ice

needles with or without caps, in either of the two positions and oriented at random, produces white arcs through the anthelion and symmetrical about its vertical; the more divergent pair being due to the *A* position.

Similar arcs also pass through the sun, due to the corresponding reflection of direct radiation, and, though independent, appear to combine with the anthelic arcs into a single set of curves.

Parhelia of 120°.—Bright spots that show no trace of color are occasionally seen on the parhelic circle 120° from the sun, in azimuth, or 60° from the anthelion. These are known as the parhelia of 120°, and while their origin is not certain, the fact

FIG. 179.



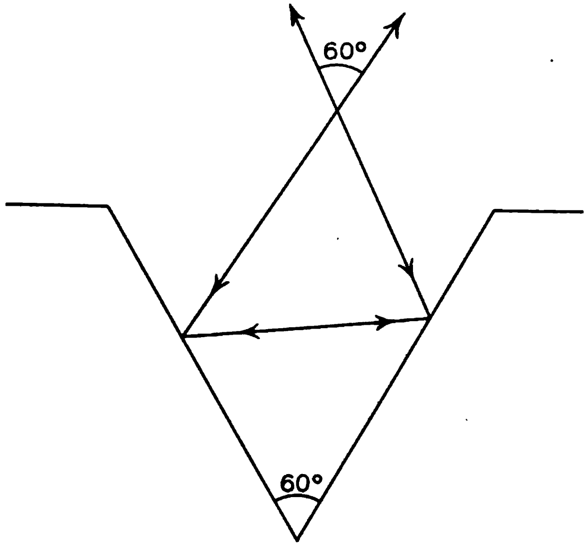
A B
End views of columnar crystals with tabular caps.

that they are white is well nigh conclusive evidence that they are somehow produced by simple reflection, possibly from the faces of reëntrant angles.

These angles, which are of frequent occurrence in tabular crystals, are of two values, 60° and 120°, respectively, as shown in Figs. 180 and 181. And since such crystals fall more or less flatwise, the polygonal faces tend to remain vertical. Hence, when the crystals are rather thick and the faces of the reëntrant angles, therefore, sufficiently broad, light incident on one such face may be reflected to its companion and then from it in turn in such direction that the projections of the initial and final rays onto the plane of the horizon will differ in direction by 120°, as is obvious from Figs. 180 and 181. Both angles produce the

same effect, though, owing to the value of the angles of incidence involved, the 120° one presumably is the most effective. Similarly, the internal angles of 120° also contribute to these parhelia.

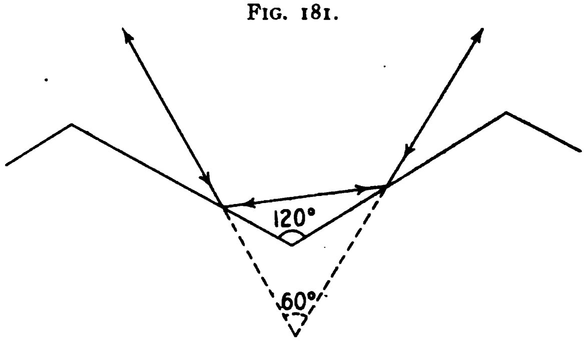
FIG. 180.



Formation of parhelia of 120°.

Parhelia of 90°.—The brightest spots that on rare occasions are on the parhelic circle midway or more from sun to anthelion, that is, the parhelia of 90°, probably are owing merely to

FIG. 181.



Formation of parhelia of 120°.

the intersection of this circle with the halo of 90°. Both circles are white, and, therefore, the brighter spots produced by their intersection are also white.

Pillars.—During very cold weather vertical pillars of white light are often seen extending above and below the sun, when its elevation is small, or merely rising above it, when it is on the horizon.

The upper and lower portions of these pillars, counting from the sun, are owing, as has long been known, merely to reflection by the under and the upper surfaces, respectively, of tabular ice crystals.

Crosses.—On very rare occasions strips of white light have been seen to intersect over the sun at right angles. This rare phenomenon is owing, presumably, merely to the simultaneous occurrence of a parhelic circle, or segment of it next the sun, and a light pillar. Possibly it might also be produced by the intersection of the secondary halos of 22°, or even by some combination of "pillar," parhelic circle, and secondary halos. A competent observer, however, could easily distinguish between the several possible causes of a light "cross," and thus determine the actual origin of any particular instance of this phenomenon he might happen to see.

Recent Halo Complexes.

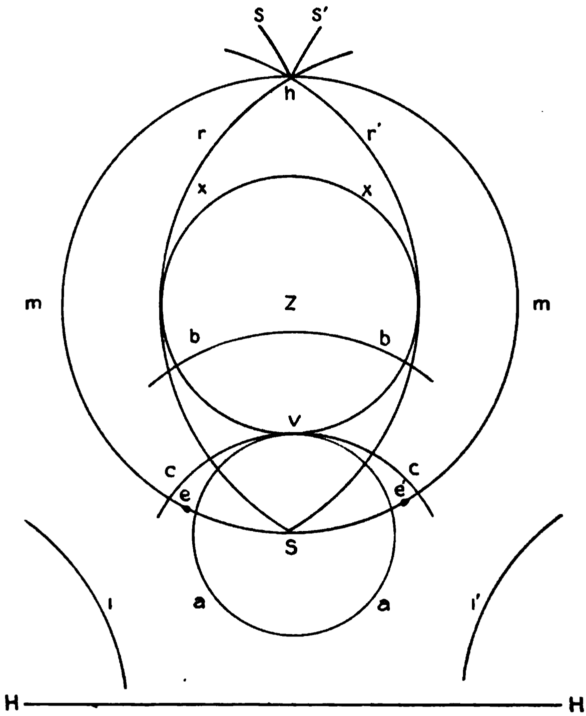
Two unusual halo complexes have recently been reported that together show nearly all the well-known phenomena of that kind. They therefore are given here, both because of their individual worth and also as a convenient summary of this subject. They are:

I. An exceptional combination, including both the refraction and reflection (primary and secondary) types, observed and independently measured with theodolites on March 8, 1920, by F. J. Bavendick and W. H. Brunkow [196a] at the Ellendale, North Dakota, aerological station. Its appearance at 1.30 P.M., 90th meridian time, is indicated by Fig. 182 in which the seemingly secondary parhelic circle *xx* is especially interesting. The elevation of the sun, about 38°15′, made the appearance of the circumzenithal arc impossible; nor was there an opportunity later to see it at Ellendale, since in an hour or two the entire halo faded away in a gradually thickening cloud. It was, however, seen in the same cloud sheet over northwestern Iowa, but with a much lower sun and as a portion of a simpler complex.

[196a] *Monthly Weather Review*, 48, p. 330, 1920.

II. An equally rare combination observed by Mr. E. W. Woolard [196b] and others, at Boulder, Colorado, on January 10, 1918; temperature, –20° C. Its appearance at 10 A.M., 105th meridian time, when the altitude of the sun was 19°50′, is indicated by Fig. 183. The most noteworthy features of this complex are, (1) the curvature of the sun-pillar *p*, obviously due to a

FIG. 182.



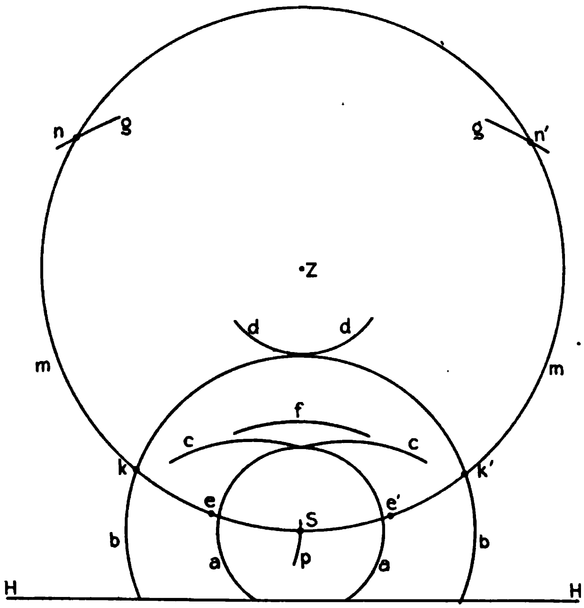
Ellendale (N. Dak.) halo of March 8, 1920. *H H* horizon; *S* sun; *Z* zenith; *a a* halo of 22°; *e*, *e′* parhelia of 22°; *c c* upper tangent arc of halo of 22°; *b b* portion of halo of 46°; *m m* parhelic circle; *h* anthelion; *r*, *r′* wide-angle oblique arcs of the anthelion; *s*, *s′* narrow-angle oblique arcs of the anthelion; *v* so-called vertical parhelion of 22°; *x x* apparently secondary parhelic circle due to *v*.

prevailing tip (eastern edge up) of the reflecting faces, caused, presumably, by gentle surface winds incident to the onset of a cold wave; and (2) the flat arc *f* between the halos of 22° and 46°, and symmetrical about the solar vertical. This latter arc,

[196b] *Monthly Weather Review*, **48**, p. 331, 1920.

previously reported only by Parry[196c] and Ferguson,[196d] clearly is produced, as Hastings[196d] explains, by refraction through randomly oriented ice needles in their most stable position—that is, with a pair of the side faces horizontal. Under the given circumstances, the computed Parry arc, with its centre 28°47′ above the sun, is as indicated, and agrees fully with the original sketch.

FIG. 183.



Boulder (Colo.) halo of January 10, 1918. *H H* horizon; *S* sun; *Z* zenith; *aa* halo of 22°; *e e′* parhelia of 22°; *p* sun-pillar, curved owing to prevailing tip of crystals due, presumably, to light surface winds; *cc* upper tangent arc of halo of 22°; *f* "Parry arc"; *bb* halo of 46°; *k, k′* parhelia of 46°; *d d* circumzenithal arc; *m m* parhelic circle; *n, n′* paranthelia; *g, g* portions of the paranthelic arc.

It may here be helpful, perhaps, to recall the fact that points on the Parry and other similar arcs are found through the laws of oblique refraction (pp. 487 and 488):

1. Incident and emerging rays make equal angles with a principal plane.

2. When μ is the index of refraction and h the angle between

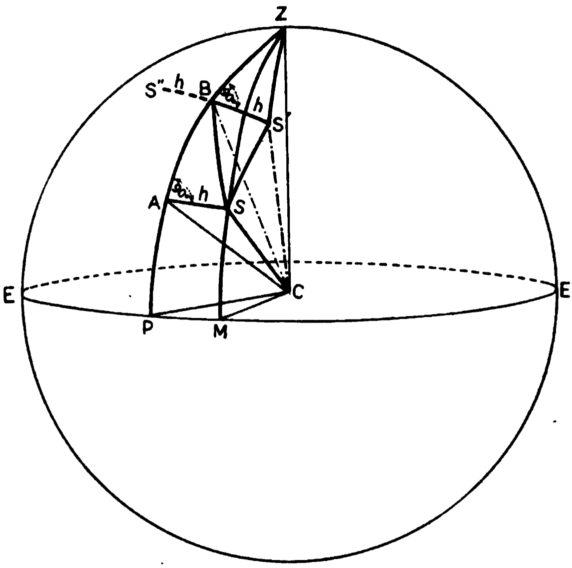
196c *First Voyage*, p. 164; 196d *Monthly Weather Review*, **34**, p. 123, 1906.
196d *Monthly Weather Review*, **48**, p. 322, 1920.

the incident ray and a principal plane, the "index" of the projection of the ray onto this plane is

$$\left(\frac{\mu^2 - \sin^2 h}{1 - \sin^2 h}\right)^{\frac{1}{2}}$$

Hence, a straight-forward method of locating the Parry arc is as follows: Let an ice needle (hexagonal rod with flat ends) be at the centre C, Fig. 184, of the celestial sphere; let EE be the intersection of this sphere by the top face, say, of the crystal;

FIG. 184.



Locating points on the Parry and similar halo arcs.

let S be the position of the sun as viewed from C, and S' its apparent position as given by the emerging ray; let MZ be the solar vertical; ZP the intersection of the sphere by a principal plane, and SCA any given angle h between the incident ray and this plane; and, finally, ZS the known zenith distance of the sun, or angle of incidence of the ray SC onto the top surface of the crystal. Then, from the right spherical triangle ZAS the corresponding "angle of incidence" ZA of the projection AC of the ray SC onto the principal plane can be computed for any value of h; and with this "angle of incidence" and the "projection

index," the "projection deviation" AB can be found, CB being the backward extension of the projection of the emerging ray onto the principal plane. The difference between these two values gives ZB of the right spherical triangle ZBS', whose side BS', according to the first law of oblique refraction, is equal to AS, which is known. Hence, through the triangles indicated, the exact position of S' with reference to S can be found for any assumed value of h ($o \leqq h \leqq ZS$), and by taking a sufficient number of such values the Parry arc traced with any desired degree of accuracy.

Much of such entering light as may be reflected by a vertical end face of the crystal will pass out through the same side face as that which is not so reflected, and produce an image S'' which, obviously, is symmetrical with S' about the principal plane ZP. The arc thus produced clearly is tangent to the Parry arc at the intersection with the solar vertical, but rarely bright enough to be seen.

CHAPTER VI.

DIFFRACTION PHENOMENA.

Coronas.—Coronas consist of one or more sets of rainbow-colored rings, usually of only a few degrees radius, concentrically surrounding the sun, moon, or other bright object when covered by a thin cloud veil. They differ from halos in having smaller (except in rare cases) and variable radii, and in having the reverse order of colors; that is, blue nearest the sun, say, and red farthest away.

Clearly, then, coronas are caused by diffraction, or the distribution of effective (non-neutralizing) quantities of light off the primary path, resulting from the action of cloud particles on radiation incident from a distant source.

Consider, then, the diffractive action of a layer of innumerable water droplets on a parallel beam of monochromatic light.

In this case the wave front, or continuous locus of any given phase, is flat—pits and pimples on it would quickly be smoothed out by dispersion—and everywhere normal to the line of travel. Also the droplets, because of their very short focal lengths and consequent great dispersive power, affect the parallel beam substantially as would so many opaque disks each of the size of a great circle of the corresponding droplet and normal to the line of travel. Furthermore, as the incident light is parallel the centres of the droplets may be regarded as lying in a common plane, each being located where the line of sight to its actual position intersects the plane in question.

The problem, then, reduces to that of finding the diffraction pattern produced in an isotropic transparent medium by a great many irregularly distributed opaque disks on a plane wave front of monochromatic light.

For the solution of this problem, it is convenient to make use of the fact, first explained by Huygens, and later developed by Fresnel, Stokes, Rayleigh and others (see any good work on optics), that each point in a wave front may itself be regarded as a secondary source of light of the same color.

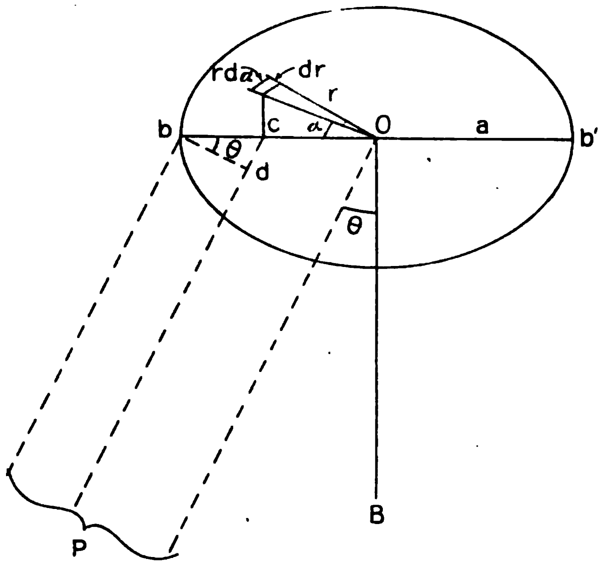
According to Stokes,[197] the intensity in different directions of the secondary radiation is proportional to $1 + \cos\theta$, in which

[197] *Math. and Phys. Papers*, vol. ii, p. 243; *Camb. Phil. Trans.*, 9, 1, 1849.

θ is the angle of deviation from the course of the parent light. That is, it is symmetrically distributed about the normal to the wave front at the secondary source, with its maximum value straight forward and minimum (zero) directly back.

A further aid to the solution of this problem is furnished by Babinet's principle, which may be explained as follows: When parallel light passes through a large opening in an opaque screen, but little illumination occurs outside the primary beam, and that little owing to rays from the edge of the opening whose angle of deviation from the original direction is very small. If,

FIG. 185.



Diffraction by a circular opening.

now, this large opening should be partially covered by a great many opaque disks, as assumed in the problem under consideration, and if illumination should result, as it does, at places where formerly there was complete shadow, then, if the opaque disks should become transparent and the transparent interspaces opaque, precisely the same illumination in the "shadow region" would obtain as before, but in exactly the opposite phase, as is obvious from the fact that the two illuminations acting together produce darkness.

Babinet's principle, therefore, enables one to use circular

openings and opaque disks interchangeably in the solution of diffraction problems. And as it is easier to discuss the circle mathematically than an irregular area the above problem will be substituted by its physical equivalent. That is, circular openings in an opaque screen will be substituted for opaque disks on a wave front.

Let O (Fig. 185) be the centre of a small circular opening of radius a in an opaque screen, and let parallel light pass through this opening, normal to its plane, in the direction OB. Let the plane fixed by P, the point whose illumination is to be determined, and the line OB intersect the circle in the diameter bb', and let the angle $BOP = \theta$. Finally, let r be the distance of any element of the circle from the centre, O, c, the foot of the perpendicular from this element onto the diameter bb', and α the angle between r and the radius Ob.

Clearly, then, the difference of phase at P between light from an element of the wave front at b and from any other element is given by the expression

$$2\pi \frac{cd}{\lambda}, \text{ or } \frac{2\pi}{\lambda}(a - r\cos\alpha)\sin\theta,$$

in which λ is the wave-length. Also, since the displacement at P owing to the element at b is given by the expression

$$\sin 2\pi \frac{t}{T} r\, d\alpha\, dr.$$

in which t is the time of travel from b to P and T the period, the total displacement, X, at P is given by the equation,

$$X = \int_0^{2\pi}\int_0^a \sin 2\pi \left[\left(\frac{t}{T} - \frac{a\sin\theta}{\lambda}\right) + \frac{r\sin\theta}{\lambda}\cos\alpha\right] r\, d\alpha\, dr.$$

Hence, developing, and putting

$$\int_0^{2\pi}\int_0^a \cos 2\pi\left(\frac{r}{\lambda}\sin\theta\cos\alpha\right) r\, d\alpha\, dr = A$$

and

$$\int_0^{2\pi}\int_0^a \sin 2\pi\left(\frac{r}{\lambda}\sin\theta\cos\alpha\right) r\, d\alpha\, dr = B$$

$$X = A\sin 2\pi\left(\frac{T}{t} - \frac{a\sin\theta}{\lambda}\right) + B\cos 2\pi\left(\frac{t}{T} - \frac{a\sin\theta}{\lambda}\right).$$

Therefore, A and B are components at right angles to each other of the resultant amplitude. Hence, the intensity, I, is given by the equation,

$$I = A^2 + B^2.$$

Putting

$$\frac{\pi r \sin \theta}{\lambda} = \beta r$$

$$A = \int_0^{2\pi} \int_0^a \cos (2\beta\, r \cos \alpha)\, d\alpha\, r\, dr$$

$$= \int_0^{2\pi} \int_0^a \left[1 - \frac{(2\beta\, r \cos \alpha)^2}{1.2} + \frac{(2\beta\, r \cos \alpha)^4}{1 \cdot 2 \cdot 3 \cdot 4} - - - - \cdot \right] d\alpha r dr$$

But

$$\int_0^{2\pi} \cos^{2n} \alpha d\alpha = \frac{1 \cdot 3 \cdot 5 \cdot \ldots\ldots 2n - 1}{2 \cdot 4 \cdot 6 \cdot \ldots\ldots 2n} 2\pi$$

Hence,

$$A = \pi \int_0^a \left[2r\, dr - \frac{2\beta^2 r^3\, dr}{1} + \frac{2\beta^4\, r^5\, dr}{(1 \cdot 2)^2} - \frac{2\beta^6\, r^7\, dr}{(1 \cdot 2 \cdot 3)^2} + \cdots \right]$$

$$= \pi \left[a^2 - \tfrac{1}{2} \frac{\beta^2\, a^4}{1} + \frac{1}{3} \frac{\beta^4\, a^6}{(1 \cdot 2)^2} - \frac{1}{4} \frac{\beta^6\, a^8}{(1 \cdot 2 \cdot 3)^2} + \cdots\cdots \right]$$

and, putting

$$\beta\, a = \frac{\pi a \sin \theta}{\lambda} = m.$$

$$A^2 = \pi^2 a^4 \left[1 - \tfrac{1}{2} \frac{m^2}{1} + \frac{1}{3} \frac{m^4}{(1 \cdot 2)^2} - \frac{1}{4} \frac{m^6}{(1 \cdot 2 \cdot 3)^2} + \cdots \right]^2$$

A similar development gives the other component in terms of a series of odd valued sines of α. Hence, as the elements are symmetrically distributed on either side of the diagonal bb', $B = o$, and

$$I = A^2.$$

On giving m various values, tables and curves of intensity may be constructed. The following table by Airy, copied from Mascart's "Traité d'optique," v. I, p. 310, is restricted to that portion of the expression within the brackets.

The following table of diffraction maxima and minima is also copied from Mascart, *l.c.*, p. 312.

m	*A*	*I*	*m*	*A*	*I*
0.0	1.0000	1.0000	3.0	−0.0922	0.0085
0.1	0.9950	0.9900	3.1	−0.0751	0.0056
0.2	0.9801	0.9606	3.2	−0.0568	0.0032
0.3	0.9557	0.9134	3.3	−0.0379	0.0014
0.4	0.9221	0.8503	3.4	−0.0192	0.0004
0.5	0.8801	0.7746	3.5	−0.0013	0.0000
0.6	0.8305	0.6897	3.6	0.0151	0.0002
0.7	0.7742	0.5994	3.7	0.0296	0.0009
0.8	0.7124	0.5075	3.8	0.0419	0.0017
0.9	0.6461	0.4174	3.9	0.0516	0.0027
1.0	0.5767	0.3326	4.0	0.0587	0.0035
1.1	0.5054	0.2554	4.1	0.0629	0.0040
1.2	0.4335	0.1879	4.2	0.0645	0.0042
1.3	0.3622	0.1312	4.3	0.0634	0.0040
1.4	0.2927	0.0857	4.4	0.0600	0.0036
1.5	0.2261	0.0511	4.5	0.0545	0.0030
1.6	0.1633	0.0267	4.6	0.0473	0.0022
1.7	0.1054	0.0111	4.7	0.0387	0.0015
1.8	0.0530	0.0028	4.8	0.0291	0.0008
1.9	0.0067	0.0000	4.9	0.0190	0.0004
2.0	−0.0330	0.0011	5.0	0.0087	0.0001
2.1	−0.0660	0.0044	5.1	−0.0013	0.0000
2.2	−0.0922	0.0085	5.2	−0.0107	0.0001
2.3	−0.1116	0.0125	5.3	−0.0191	0.0004
2.4	−0.1244	0.0155	5.4	−0.0263	0.0007
2.5	−0.1310	0.0172	5.5	−0.0321	0.0010
2.6	−0.1320	0.0174	5.6	−0.0364	0.0013
2.7	−0.1279	0.0164	5.7	−0.0390	0.0015
2.8	−0.1194	0.0143	5.8	−0.0400	0.0016
2.9	−0.1073	0.0115	5.9	−0.0394	0.0016
3.0	−0.0922	0.0085	6.0	−0.0372	0.0014

Diffraction Maxima and Minima.

	Maxima $\frac{m}{\pi}$	Maxima Diff.	Maxima *I*	Minima $\frac{m}{\pi}$	Minima Diff.	Minima *I*
1	0.000		1.00000	0.610		0
		0.819			0.506	
2	0.819		0.01745	1.116		0
		0.527			0.503	
3	1.346		0.00415	1.619		0
		0.512			0.502	
4	1.858		0.00165	2.121		0
		0.504			0.501	
5	2.362		0.00078	2.622		0
		0.500			0.500	
6	2.862		0.00043	2.122		0
		0.500			0.500	
7	3.362		0.00027	3.622		0
		0.500			0.501	
8	3.862		0.00018	4.123		0
		0.500			0.500	
9	4.362		0.00012	5.623		0

It will be noticed that the decrease of intensity from maximum to minimum, though large at first, quickly becomes very small.

From the values of $\frac{m}{\pi}$, corresponding to diffraction minima, it is evident that

$$\sin\theta = (n + 0.22)\frac{\lambda}{2a}, \text{ very nearly,}$$

in which n is the order of the minimum, counting from the centre.

This important equation gives the angular distance from the light source at which the successive diffraction minima occur for any particular wave-length and size of drop or disk. It also gives the diameter of the drop when the wave-length, angular distance from the centre, and order of the minimum are known. Furthermore, it shows that the larger the wave-length and the smaller the droplet the larger the diffraction circle, or halo.

The above discussion applies to a single circular disk on the wave front. An exact duplicate disk obviously would produce an exact duplicate diffraction pattern. If, then, two such disks occur close together, and if the distance between their centres, or other homologous points is b, and ϕ the angle between the line connecting these points and the line connecting the farthest to the point of observation, then the difference in phase between the two lights at the latter place is $2\pi\frac{b\sin\phi}{\lambda}$, and a secondary diffraction pattern, in addition to the two primary circular ones, is produced, directed at right angles to the line connecting the centres of the disks. Similarly, conspicuous diffraction patterns are produced by any regular geometric distribution of many disks.

Let, however, the disks, or droplets, be numerous, irregularly distributed, and all of the same size (if of various sizes their effects cannot easily be summed up). Let each produce at a given point a disturbance whose amplitude is A, but let the phases be $\epsilon_1, \epsilon_2 - - - - - \epsilon_n$, and let R be the resultant amplitude.

Then,

$$R^2 = \Sigma\,(A\cos\epsilon)^2 + \Sigma\,(A\sin\epsilon)^2,$$

$$= A^2\left[(\cos\epsilon_1 + \cos\epsilon_2 + --- + \cos\epsilon_n)^2 + (\sin\epsilon_1 + \sin\epsilon_2 + .\,.\,.\,. + \sin\epsilon_n)^2\right]$$

$$R^2 = A^2\,(n\cos^2\epsilon + n\sin^2\epsilon + 2n\cos\epsilon\cos\epsilon' + 2n\sin\epsilon\sin\epsilon'),$$

$$= A^2 n + 2A^2 n\cos(\epsilon - \epsilon').$$

But as n is large and the disks irregularly scattered, it is clear that the phase difference, $\epsilon - \epsilon'$, between the innumerable pairs will have all manner of values with, on the whole, the positive and negative well balanced. Hence, as close as can be detected,

$$R^2 = nA^2.$$

That is, the diffraction rings, corona, for instance, produced by a large number, n, of irregularly distributed neighboring droplets are the same as those produced by any one of them, but n times as bright.

When the incident light is complex, the diffraction pattern produced by the several wave-lengths necessarily overlap and produce correspondingly colored rings—red, if present, being the outermost, and blue the innermost.

Size of Cloud Particles.—Since the diffraction pattern produced by a great many irregularly distributed droplets of uniform size is the same as that due to a single one, it is clear that the size of the cloud particles producing coronas may be determined by the equation,

$$\sin \theta = (n + 0.22) \frac{\lambda}{2a},$$

in which, as already explained, θ is the angular distance from the centre of the corona to the nth minimum corresponding to light of the wave-length, λ, and a, the radius to be determined.

Measurements made in this manner have shown that the radii of corona-producing cloud droplets, though varying over a considerable range, commonly average about .007 mm. to .010 mm.

It may be interesting to note in this connection that a contracting or decreasing corona implies growing droplets and, perhaps, the approach of rain; and that an expanding corona implies, on the other hand, decreasing or evaporating droplets and, presumably, the approach of fair weather.

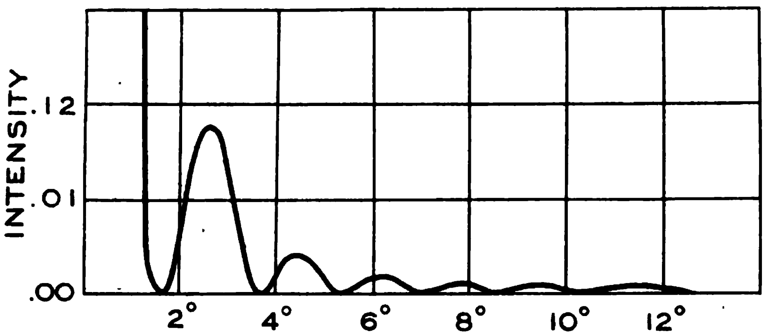
Fig. 186, copied from an instructive article by Simpson,[198] gives the angular and intensity distribution of the monochromatic light $\lambda = .000571$ mm. in a corona produced by droplets of .01 mm. radius.

Droplets versus Ice Needles as Producers of Coronas.—When

[198] Q. *Jr. Roy. Met. Soc.*, **38**, p. 291, 1912.

coronas are seen in clouds whose temperature is above 0° C., or in which halos do not form, it is certain that they are due to droplets. It is well known, however, that the most brilliant coronas—those of multiple rings and large diameter—usually are formed by very high clouds whose temperature often must be far below freezing. Naturally, then, it has been inferred that these coronas are produced by the diffractive action of ice needles. Simpson,[199] however, appears to have disproved the probability that they are formed in this manner. "On no occasion," he says, referring to his stay in the Antarctic, "were a corona and halo seen at the same time on the same cloud." Furthermore, he explains, as the axes of the needles are essentially horizontal, this being their stable position, only those

FIG. 186.



Distribution of intensity by droplets, radius = 0.01 mm., $\lambda = .571\ \mu$.

at right angles to radii from the sun, or other luminary, could produce coronas of the kind observed, while the equally numerous crystals of every other orientation would produce such different patterns that the total effect probably could be but little more than white light—certainly nothing approaching the pure brilliant colors often seen in these coronas.

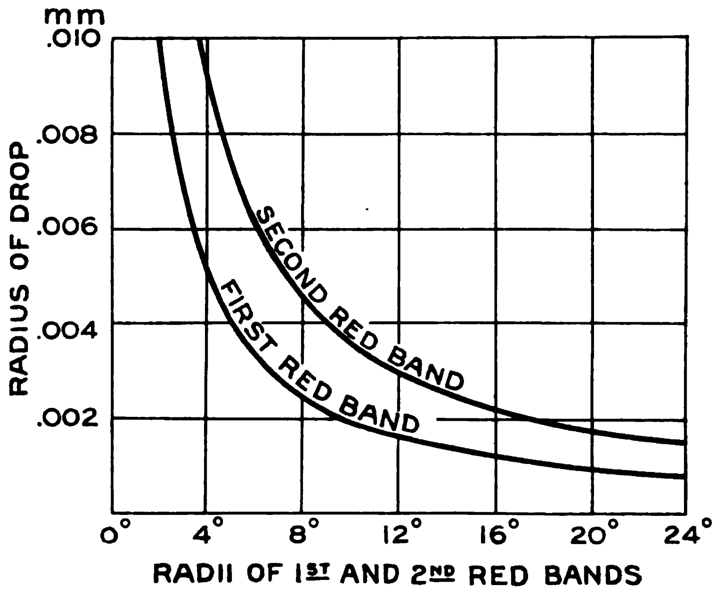
Presumably, therefore, the brilliant coronas of high clouds are due to very small undercooled water droplets of approximately uniform size, and not, as has generally been supposed, to ice needles.

Iridescent Clouds.—Thin and perhaps slowly evaporating

[199] *l. c.*

cirro-stratus and cirro-cumulus clouds occasionally develop numerous iridescent borders and patches of irregular shape, especially of red and green, at various distances from the sun up to 30° or more. A brilliantly colored iridescent cloud of considerable area is justly regarded as one of the most beautiful of sky phenomena, but one of which until recently there was no satisfactory explanation. Simpson,[200] however, has shown that the colored patches in question, presumably, are only fragments of coronas formed by exceedingly small droplets of very approximately

FIG. 187.



Relation between size of drop and size of corona.

uniform size. The relation between the radius of droplet and angular distances from the centre to the first and second red bands is shown in Fig. 187, also copied from the paper cited, from which it appears that coronas of the requisite size may occur, and, therefore, that the assumption that iridescent clouds are only fragments of unusually large and exceptionally brilliant coronas presumably is correct.

Bishop's Ring.—After the eruption of Krakatoa in 1883, of

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[200] *l. c.*

Mont Pelé in 1902, and of Katmai in 1912, a faint reddish-brown corona was often seen, under favorable circumstances, around the sun. This is known as Bishop's ring, after Mr. Bishop of Honolulu, who first described it.

The width of this ring, as seen after the eruption of Krakatoa, was about 10°, and the distance from the sun to its outer edge, that is, to the first minimum, 22° to 23°. Substituting this value of the angular radius of the first minimum in the equation, explained above,

$$\sin \theta = (n + 0.22) \frac{\lambda}{2a}$$

and letting $\lambda = .000571$ mm., it appears that the diameter of the dust particles, assumed either spheres or circular disks of approximately uniform size, that produced this peculiar corona is given by the equation,

$$2a = 1.22 \frac{.000571\text{mm.}}{\sin 22° 30'}$$
$$= .00182 \text{ mm. about.}$$

Glory or Brocken-Bow.—When favorably situated, one occasionally may see rings of colored light around the shadow of his own head as cast upon a neighboring fog bank or cloud. This phenomenon, to which several names have been given—glory, Brocken-bow, Brocken-spectre, mountain-spectre—is produced by the diffraction by particles comparatively near the surface of light reflected from deeper portions of the fog or cloud.

The reflected light obviously emerges in every direction, but the nearer one looks along the path of incidence the larger the ratio of illuminated to non-illuminated particles in his line of sight. Indeed, at any appreciable angle from this special direction a considerable proportion of the droplets in one's vision evidently must lie in the shadows of others nearer the surface. Hence, not only will the shadow of one's head be surrounded by the brightest reflected light, like the " heiligenschein " one may see around the shadow of his head on a bedewed lawn, but it will also be the centre of the brightest and only perceptible glory or reflection halo, and that for the simple reason that the more intense the initial light the more brilliant its diffraction effects.

Chapter VII.

PHENOMENA DUE TO SCATTERING: COLOR OF THE SKY.

The color of the cloudless sky, though generally blue, may, according to circumstances, be anything within the range of the entire spectrum. At great altitudes the zenithal portions are distinctly violet, but at moderate elevations often a clear blue. With increase of the angular distance from the vertical, however, an admixture of white light soon becomes perceptible that often merges into a grayish horizon. Just after sunset and also before sunrise portions of the sky often are distinctly green, yellow, orange, or even dark red, according especially to location and to the humidity and dust content of the atmosphere. Hence, these colors and the general appearance of the sky have rightly been used immemorially as more or less trustworthy signs of the coming weather.

Early Ideas.—Many attempts have been made to account for the blue of the sky [201]—the other colors being comparatively ignored. Some have held that it is just the nature of the atmosphere, or of particles in it, to reflect the blue of sunlight and to transmit the other colors. But as they did not explain how the atmosphere, or these particles, happened to have such nature the mystery actually remained as profound as ever. Another interesting hypothesis, suggested by Leonardo da Vinci, was to the effect that the blue is the resultant of a mixture of more or less white light, reflected by the atmosphere, with the black of space. But the futility of this idea is immediately obvious from the fact that gray alone could be produced by any such mixture.

The first logical attempt to explain (as that term is now understood) why the sky is blue was made by Newton,[202] who supposed it to be due to the same sort of interference between the rays reflected from the front and rear surfaces of transparent objects (in this case minute water drops) that produce the colors of soap bubbles. In fact, he thought that the "blue of the first order," the blue nearest the black central spot of the "Newton's rings," is of the same color as the blue of the sky, and that they were produced in the same way. This explanation, though er-

[201] See summary and bibliography by Dorsey, *Monthly Weather Review* 28, p. 382, 1900.

[202] *Optics*, book ii.

roneous, and based only on analogy, was accepted without modification for nearly 175 years. At about the end of this period, however, Clausius [203] demonstrated analytically that a cloud of droplets of the small size assumed by Newton would cause the stars and other celestial objects to appear enormously magnified. He, therefore, modified Newton's theory by assuming that the droplets are larger but vesicular with very thin walls. In this way the magnification trouble is avoided, but the theory is not improved. First, because water droplets are not hollow; and, second, because, as shown by Brücke,[204] the color of the sky differs radically from the blue of the "first order."

Although the above appears to have been the first serious criticism of the Newtonian theory of sky colors, observational and experimental data sufficient to render it untenable had long been known. This consisted of (*a*) Arago's [205] discovery in 1811 that sky light is partially polarized and that this polarization is a maximum along a circle about 90° from the sun; and (*b*) Brewster's [206] discovery, shortly thereafter, that polarization by reflection is a maximum when the tangent of the angle of incidence is equal to the refractive index of the reflector divided by that of the adjacent medium.

If, then, sky light is the result of simple reflection, the angle of polarization (angle of incidence corresponding to maximum polarization) of the reflecting medium must be 45°—since the arc of maximum polarization is 90° from the sun. But the angle of polarization of water in air is about 74°. Hence, the color of the sky cannot be due to reflection from water droplets, as Newton and many others assumed.

Modern Theory.—The real origin of the blue of the sky, scattering of light by particles far too small to reflect specularly, appears to have been first indicated by Brücke's [207] experiments, which showed (*a*) that a transparent medium, rendered turbulent by sufficiently small particles, appears blue when illuminated with white light; and (*b*) that objects may be seen through such medium clearly and distinctly. A few years later, Tyndall [208] made a large

[203] *Crell's Jr.*, **34**, p. 122, 1847; p. 185, 1848; *Pogg. Ann.*, **72**, p. 294, 1847.
[204] *Pogg. Ann.*, **88**, p. 363, 1853.
[205] *Oeuvres*, **7**, p. 394, and p. 430.
[206] *Phil. Trans.*, **33**, p. 125, 1815.
[207] *l. c.*
[208] *Phil. Mag.*, **37**, p. 384, 1869; **38**, p. 156, 1869.

number of experiments on the action of chemically formed "clouds" on incident white light, and found that not only did they scatter blue light when their particles were very small, but also that this light was completely polarized at right angles to the incident beam. Here, then, was the experimental solution of the problem of the blue of the sky and its polarization. About two years later, Lord Rayleigh [209] supplied the necessary theory, and thus, at last, one of the oldest and most difficult of the many problems of meteorological optics became completely solved. In a later paper Lord Rayleigh [210] showed that in the absence of dust of all kinds "the light scattered from the molecules [of air] would suffice to give us a blue sky, not so very greatly darker than that actually enjoyed." And still later, King [211] concluded that "The analysis of the present paper seems to support the view that at levels about Mount Wilson [1730 metres] molecular scattering is sufficient to account completely both for attenuation of solar radiation and for the intensity and quality of sky radiation." However, whether the scattering be by fine dust or by individual molecules the theory is the same, and, as developed in Rayleigh's first paper, substantially as follows:

Let a beam of light of wave-length λ be incident, say, to be definite, from the zenith. There will be little or no scattering from that portion of the beam in free ether, as is obvious from the facts (*a*) that extremely distant stars are still visible, and (*b*) that interstellar spaces are nearly black. From the portion in the atmosphere, however, there is abundant lateral scattering by the innumerable particles of dust and molecules of air, each of which is optically denser than the ether and so small in comparison to λ^3 that the applied force is practically constant throughout its volume. Each such particle merely increases the local inertia of the ether, and, thereby, since the rigidity is not affected, correspondingly reduces the amplitude of a passing light wave. If, then, a force should be applied to each particle, such as to counterbalance the increasing inertia, the light would pass on exactly as in empty space and, therefore, without scattering. On the other hand, precisely the same force, but reversed in direction, if acting alone on free ether would produce the same

[209] *Phil. Mag.*, **41**, pp. 107, 274, 447, 1871; **12**, p. 81, 1881.
[210] *Phil. Mag.*, **47**, p. 375, 1899.
[211] *Phil. Trans., A.*, **212**, 375, 1913.

effect that the disturbing particle produces. This force obviously must have the same period and direction as the undisturbed luminous vibrations and be proportional to the difference in optical density between the particle and the ether.

The only factors that conceivably can affect the ratio of the amplitude of scattered to incident light are: direction, or, rather, angle between directions of force and point of observation; ratio between the optical densities of the disturbing particle and the ether; volume of particle; distance from particle; wave-length; and velocity of light. Hence, in comparing the extents to which lights of different colors are scattered, the first two factors may be neglected, since they apply in equal measure to all. Furthermore, as the ratio in question, like all ratios, is a mere number and, therefore, dimensionless, the last factor must be omitted, since it and it alone involves time. There remain, then, only the volume of the particle, distance from it, and the wave-length to consider. But from the dynamics of the problem it appears that the ratio of the two amplitudes must vary directly as the volume of the particle and inversely as the distance from it. That is,

$$N = \frac{L^3}{L} f\ (\lambda),$$

in which N is some number, L a unit of length, and $f(\lambda)$ that function of λ that renders the equation dimensionless. Hence,

$$f\ (\lambda) = \lambda^{-2},$$

and, therefore, the ratio of the two intensities is proportional to λ^{-4}. Obviously, then, light from a serene sky should belong essentially to the blue or short wave-length end of the spectrum.

If, as commonly expressed, the displacement in the incident wave is $A \cos\left(\frac{2\pi\ v\ t}{\lambda}\right)$, in which A is the amplitude; v the velocity of light, λ the wave-length; and t the time since any convenient instant when the displacement was A, then the corresponding acceleration is

$$\frac{d^2}{dt^2} A \cos \frac{2\pi\ v\ t}{\lambda} = -A \left(\frac{2\pi}{\lambda}\ v\right)^2 \cos \frac{2\pi}{\lambda}\ vt.$$

Hence, the force that would have to be applied to a sufficiently minute particle in order that the wave might pass over it undisturbed is

$$-(D' - D)\ T\ A \left(\frac{2\pi\ v}{\lambda}\right)^2 \cos \frac{2\pi}{\lambda}\ vt.$$

in which D' and D are the optical densities of the particle and ether, respectively; and T the volume of the particle. And this, as explained, is also the expression for the force which, if operating alone on the ether, would produce the same light effects that actually are induced by the particle in question.

Now it has been shown by Stokes,[212] and also by Lord Rayleigh,[213] that the displacement X produced by the force $F \cos \frac{2\pi vt}{\lambda}$ is given by the expression,

$$X = \frac{F \sin\alpha}{4\pi v^2 D r} \cos \frac{2\pi}{\lambda} (vt - r)$$

in which α is the angle between the direction of the force and the radius vector, r, that connects the centre of the force with the point at which the displacement is observed.

On substituting for the force $F \cos \frac{2\pi v t}{\lambda}$ its value, one finds that

$$X = A \frac{D' - D}{D} \frac{\pi T}{r \lambda^2} \sin\alpha \cos \frac{2\pi}{\lambda} (v t - r)$$

Hence, the intensity of the light scattered by a single particle is

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 T^2}{r^2 \lambda^4} \sin^2\alpha.$$

and for a cloud

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 \sin^2\alpha}{\lambda^4} \Sigma \frac{T^2}{r^2},$$

in which $\Sigma \frac{T^2}{r^2}$ is the sum of the values of $\frac{T^2}{r^2}$ for all the particles in the line of sight, or

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 \sin^2\alpha}{\lambda^4} N \left(\frac{T}{r}\right)^2_m$$

in which N is the total number of particles in the line of sight, and $\left(\frac{T}{r}\right)^2_m$ the mean of the several values of $\left(\frac{T}{r}\right)^2$.

The above equations are based on the assumption that the displacements in the incident wave are all in the same plane—that the incident light is plane polarized. If, however, they lie in parallel planes, passing through the axis of propagation, that is, if the incident light is unpolarized, we may resolve each dis-

[212] *Camb. Phil. Trans.*, **9**, p. 1, 1849; *Math. and Phys. Papers II*, pp. 243-328.

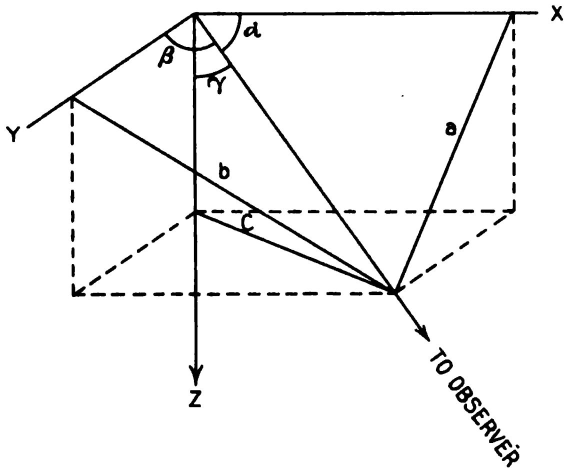
[213] *Phil. Mag.*, **41**, p. 107, 1871.

placement, always normal to the line of travel, into two components at right angles to each other and obtain their joint effect in any given direction. Let the line from the centre of the force to the point of observation make the angles α, β, and γ with these components and the direction of travel, respectively. Then, since $\sin^2\alpha + \sin^2\beta = 1 + \cos^2\gamma$ (see Fig. 188), the intensity of the scattered light at the angle γ from the direction of travel of a non-polarized beam is,

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 (1 + \cos^2\gamma)}{\lambda^4} N \left(\frac{T}{r}\right)^2_m$$

According to this equation, the maximum amount of scattered

FIG. 188.



Intensity of scattered light in a given direction.

light is along the path—forward and back—of the incident beam, and least at right angles to it. Also, the intensity is directly proportional to the square of the volume of the disturbing particle, provided it is sufficiently small.

The effect of the size of particle on scattering as it approaches λ^3 is not well known. However, Lord Rayleigh[214] has shown that the intensity of the light scattered by relatively large spherical particles varies as the inverse 8th power of the wave-length.

Extinction Coefficient.—The intensity or brightness of solar or other radiation is decreased with increase of air path by (*a*) scattering, (*b*) selective absorption, (*c*) diffraction, (*d*) reflec-

[214] *Phil. Mag.*, 12, p. 81, 1881.

tion, and (*e*) refraction. When the sky is clear, however, only (*a*) is particularly effective in the visual region, but here it is quite effective, since each disturbing particle evidently scatters energy from incident radiation in proportion to the expression,

$$\frac{\pi^2}{r^2}\frac{T^2}{\lambda^4}\left(\frac{D' - D}{D}\right)^2 \int_0^\pi \sin^2\alpha 2\pi\, r^2 \sin\alpha d\alpha = \frac{8\pi^3}{3}\frac{T^2}{\lambda^4}\left(\frac{D' - D}{D}\right)^2.$$

Let E be the energy delivered per unit cross-section of the incident beam in any interval of time, and let n be the number of disturbing particles (all alike) per unit volume. Then the energy gain (negative) during the same time per unit cross-section, and penetration, dx, is given by the equation,

$$dE = - E\, n\, dx\, \frac{8\pi^3 T^2}{3\lambda^4}\left(\frac{D' - D}{D}\right)^2.$$

If, then, E_0 is the energy in the beam before any scattering took place, and E the energy remaining after penetrating the distance x into the turbulent medium in question,

$$E = E_0\, e^{-\epsilon x}$$

in which the extinction coefficient

$$\epsilon = \frac{8\pi n}{3}\frac{T^2}{\lambda^4}\left(\frac{D' - D}{D}\right)^2.$$

But $D = \mu^2$ (from the equations, $v = \sqrt{\frac{\text{Elasticity}}{\text{Density}}} = \frac{1}{\mu}$, density of ether $= 1$, μ for either $= 1$), $\mu =$ refractive index. Also, in the case of scattering by air molecules, or by any small particles in a medium whose refractive idex is 1,

$$n\, T\left(\frac{D' - D}{D}\right) = D'' - 1$$

in which $D'' =$ average optical density of the turbulent space. If μ is the refractive index of the medium, air, say,

$$n\, T\left(\frac{D' - D}{D}\right) = \mu^2 - 1 = (\mu + 1)(\mu - 1) = 2(\mu - 1), \text{ nearly,}$$

since μ differs but little from unity.

Hence, substituting,

$$\epsilon = \frac{32}{3}\frac{\pi^3}{n\lambda^4}(\mu - 1)^2, \text{ approximately.}$$

Clearly, then, the intensity of atmospheric and dust haze rapidly decreases with increase of wave-length, a fact that justifies the use, on aeroplanes, for instance, of "haze cutters" (filters that transmit only the longer waves) for both visual and photographic work. A fog haze, however, cannot be much cut.

This is because the extinction it produces, being due, owing to the relatively large size of the fog droplets, chiefly to diffraction and reflection, is nearly equally effective for all colors.

The scattering of light by the molecules of the atmosphere and the suspended fine dust particles decrease the intensity of both the direct insolation and the scattered radiation, but at the same time gives to all portions of the sky, other than that occupied by the sun, a luminosity that otherwise would not exist—without scattering there would be no sky light at all. The residual sunlight and the total sky light vary greatly with time of day, latitude, altitude, season, weather, and dustiness of the atmosphere. Kimball,[215] for instance, finds "that photometric measurements made at Mount Weather, Va. [Lat. 39° 4′ N., Long. 77° 54′ W., altitude 526 m.], show that with a clear sky the total mid-day illumination on a horizontal surface varied from 10,000 foot-candles in June to 3600 foot-candles in January. It is less than the direct solar illumination on a normal surface from September to February, inclusive, but exceeds the latter from May to August, inclusive, for a period of from four to eight hours in the middle of the day.

"The illumination on a horizontal surface from a completely overcast sky may be half as great as the total illumination with a clear sky, and is frequently one-third as great. On the other hand, during severe thunderstorms at noon in midsummer, the illumination may be reduced to less than 1 per cent. of the illumination with a clear sky.

"The ratio of sky-light illumination to total illumination on a horizontal surface at noon in midsummer varies from one-third to one-tenth. In midwinter it varies from one-half to one-fifth.

"When the sky is clear, the twilight illumination on a horizontal surface falls to 1 foot-candle about half an hour after sunset, or when the sun is about 6° below the horizon."

Prevailing Color.—If I_0 is the initial intensity and I_1 the remaining intensity after penetrating the uniformly turbulent medium, the distance x, then,

$$I_1 = I_0 e^{-\frac{kx}{\lambda^4}}$$

where

$$k = \frac{32\pi^3}{3n}(\mu - 1)^2.$$

This residual light, in turn, is scattered, and if I_2 is the in-

[215] *Monthly Weather Review*, 42, p. 650, 1914.

tensity of the light scattered by a single particle at the angle α from the direction of displacement

$$I_2 = I_1 \frac{k'}{\lambda^4}$$

where

$$k' = \frac{\pi^2 T^2}{r^2}\left(\frac{D' - D}{D}\right)^2 \sin^2\alpha,$$

and

$$I_2 = I_0 \frac{k'}{\lambda^4} e^{-\frac{kx}{\lambda^4}}.$$

Hence, the ratio of intensity received to initial intensity, or I_2/I_0, is small both for very long and very short wave-lengths. Its maximum value occurs at $\lambda^4_m = kx$, where

$$\left(\frac{I_2}{I_0}\right) = \left(\frac{I_2}{I_0}\right)_m \left(\frac{\lambda_m}{\lambda}\right)^4 e^{1-\left(\frac{\lambda_m}{\lambda}\right)^4},$$

or, if I_0 is uniform throughout the spectrum,

$$I_2 = I_m \left(\frac{\lambda_m}{\lambda}\right)^4 e^{1-\left(\frac{\lambda_m}{\lambda}\right)^4}.$$

According to Abbot, Fowle, and Aldrich,[216] the mean energy intensities of I_0, in arbitrary units, are:

$\lambda = 0.39$	0.42	0.43	0.45	0.47	0.50	0.55	0.60	0.70
$I_0 = 3614$	5251	5321	6027	6240	6062	5623	5042	3644

The luminous intensities would show greater contrasts, since the eye is more sensitive to the mid-region of the visible spectrum than to either end.

From these values of I_0, and the equation for the intensity of I_2, it can be shown that the prevailing color of the clear sky, except when the sun is on or below the horizon, is neither violet nor red, but some intermediate color, generally blue, as we know by observation.

Twilight Colors.—As the sun sinks to and below the horizon during clear weather, a number of color changes occur over large portions of the sky, especially the eastern and western. The phenomena that actually occur vary greatly, but the following may be regarded as typical, especially for arid and semi-arid regions:

(*a*) A whitish, yellowish, or even bronze glow of 5° or 6° radius that concentrically encircles the sun as it approaches the horizon, and whose upper segment remains visible for perhaps 20 minutes after sundown.

[216] *Annals Astrophys. Obsy.*, Smithsonian Institution, 3, p. 197, 1913.

The chief contributing factors to this glow appear to be (1) scattering, which is a maximum in the direction, forward and back, of the initial radiation, and (2) diffraction by the dust particles of the lower atmosphere. In both cases blue and violet are practically excluded, owing to the very long air paths.

(*b*) A grayish blue circle that rises above the eastern horizon as the sun sinks below the western. This is merely the shadow of the earth.

(*c*) A purplish arch that rests on the earth shadow and gradually merges into the blue of the sky at a distance of perhaps 10°, and also fades away as the arch rises.

Obviously, any direct sunlight in the lower dusty atmosphere to the east must have penetrated long distances through the denser air, and thus have become prevailing red, while that reaching the higher atmosphere is still rich in blue and violet. Hence, the observer sees red light scattered from the first of these layers and blue to violet from the other, and thereby gets the effect of the superposition of the opposite ends of the visible spectrum, that is, purple. The effect is most pronounced when the luminous layers are seen more or less " end on." Hence, the light is brightest at the border of the earth shadow. The fact that the red component of the purple is from the lower atmosphere and the others from the higher is evident from the bluish crepuscular rays that often radiate, apparently, from the antisolar point—shadow streaks cast through the lower dust-laden air by western clouds or mountain peaks, often below the horizon.

(*d*) A bright segment only a few degrees deep but many in extent that rests on the western horizon just after sundown. The lowest portion often is red and the upper yellowish. A product essentially of scattered light by the lower and dustier portions of the atmosphere, where the light before being scattered is already reduced essentially to the colors seen.

(*e*) A purple glow covering much of the western sky, reaching its maximum intensity when the sun is about 4° below the horizon and disappearing when it is about 6° below. The explanation of this purple glow in the western sky presumably is the same as that in the eastern sky as given above under (*c*). The crepuscular rays of this region, apparently radiating from the sun, often are greenish-blue.

(*f*) A faint purple glow covering the entire sky when the sun is 6° or more below the horizon, and gradually disappearing

in the west when the sun is 16° to 18° below the horizon. This appears to be due to secondary scattering of light from the illuminated atmosphere far to the west.

The foregoing descriptions, which, of course, apply equally to dawn, are by no means universally applicable. Indeed, the sky very commonly is greenish instead of purple, probably when the atmosphere is but moderately dust-laden. Furthermore, the explanations are only qualitative. A rigid analysis, even if the distribution of the atmosphere and its dust and moisture content were known—which they are not, nor are they constant—would be at least difficult and tedious.

Duration of Astronomical Twilight. (*Interval Between Times When the Upper Edge of the Sun is on and the True Position of Its Centre 18° Below the Horizon.*)

Date		North latitude														
		0°	10°	20°	25°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
		h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.
January	1	1 14	1 15	1 18	1 21	1 26	1 28	1 29	1 31	1 34	1 37	1 41	1 45	1 49	1 53	1 59
	11	1 14	1 14	1 18	1 21	1 25	1 27	1 29	1 31	1 33	1 36	1 39	1 43	1 37	1 52	1 57
	21	1 13	1 13	1 17	1 20	1 23	1 25	1 28	1 30	1 32	1 34	1 38	1 41	1 45	1 49	1 54
February	1	1 12	1 12	1 15	1 18	1 22	1 24	1 26	1 28	1 30	1 33	1 36	1 39	1 43	1 47	1 52
	11	1 11	1 12	1 14	1 17	1 21	1 23	1 25	1 27	1 29	1 32	1 34	1 37	1 41	1 45	1 49
	21	1 10	1 11	1 13	1 16	1 20	1 22	1 24	1 26	1 28	1 31	1 33	1 36	1 40	1 44	1 48
March	1	1 10	1 11	1 13	1 16	1 20	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	11	1 09	1 10	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	21	1 09	1 10	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 34	1 37	1 41	1 45	1 50
April	1	1 09	1 11	1 14	1 17	1 21	1 23	1 25	1 27	1 30	1 33	1 36	1 40	1 44	1 49	1 54
	11	1 10	1 11	1 15	1 18	1 22	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 00
	21	1 11	1 12	1 16	1 20	1 24	1 27	1 29	1 32	1 36	1 39	1 43	1 48	1 54	2 01	2 08
May	1	1 12	1 13	1 18	1 22	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 20
	11	1 13	1 14	1 19	1 24	1 30	1 33	1 36	1 40	1 43	1 48	1 54	2 01	2 10	2 20	2 35
	21	1 13	1 15	1 21	1 26	1 32	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 20	2 35	2 58
June	1	1 14	1 16	1 23	1 28	1 35	1 38	1 41	1 46	1 52	1 59	2 07	2 18	2 31	2 54	....
	11	1 15	1 17	1 24	1 29	1 36	1 40	1 44	1 49	1 55	2 02	2 12	2 23	2 40	3 11	....
	21	1 15	1 18	1 24	1 29	1 37	1 41	1 45	1 50	1 56	2 03	2 13	2 25	2 44	3 19	....
July	1	1 15	1 17	1 24	1 29	1 36	1 40	1 44	1 49	1 55	2 02	2 12	2 23	2 40	3 10	....
	11	1 14	1 16	1 23	1 28	1 35	1 38	1 41	1 46	1 52	1 59	2 07	2 18	2 31	2 54	....
	21	1 13	1 15	1 21	1 26	1 32	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 21	2 36	3 00
August	1	1 13	1 14	1 19	1 24	1 30	1 33	1 36	1 40	1 44	1 48	1 54	2 02	2 10	2 20	2 35
	11	1 12	1 13	1 18	1 22	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 20
	21	1 11	1 12	1 16	1 20	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 01	2 09
September	1	1 10	1 11	1 14	1 18	1 22	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 53	2 00
	11	1 09	1 11	1 13	1 17	1 21	1 23	1 25	1 27	1 30	1 33	1 36	1 39	1 44	1 49	1 54
	21	1 09	1 10	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 34	1 37	1 41	1 45	1 50
October	1	1 09	1 10	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	11	1 10	1 11	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	21	1 10	1 11	1 13	1 16	1 20	1 22	1 24	1 26	1 28	1 31	1 33	1 36	1 40	1 44	1 48
November	1	1 11	1 12	1 14	1 17	1 21	1 23	1 25	1 27	1 29	1 32	1 34	1 38	1 41	1 46	1 49
	11	1 12	1 12	1 16	1 18	1 22	1 24	1 26	1 28	1 30	1 33	1 36	1 40	1 43	1 47	1 52
	21	1 13	1 13	1 17	1 20	1 24	1 26	1 28	1 30	1 32	1 35	1 38	1 42	1 46	1 49	1 55
December	1	1 14	1 14	1 18	1 21	1 25	1 27	1 29	1 31	1 33	1 36	1 40	1 44	1 47	1 52	1 57
	11	1 14	1 15	1 18	1 22	1 26	1 28	1 30	1 32	1 34	1 37	1 41	1 45	1 49	1 53	1 59
	21	1 15	1 16	1 19	1 22	1 26	1 28	1 30	1 32	1 35	1 38	1 41	1 45	1 49	1 54	1 59

Duration of Civil Twilight. (Interval Between Times When the Upper Edge of the Sun is on and the True Position of Its Centre 6° Below the Horizon.)

Date		North latitude														
		0°	10°	20°	25°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
		m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
January	1	22	22	24	25	27	27	27	28	29	30	32	33	34	36	39
	11	22	22	24	25	26	27	28	28	29	30	31	32	33	35	38
	21	22	22	23	24	26	26	27	27	28	29	30	32	33	34	37
February	1	22	22	23	24	25	26	27	27	27	28	29	31	32	34	35
	11	22	22	22	23	25	26	26	27	27	28	29	31	32	34	35
	21	21	22	22	23	24	25	25	26	27	28	28	29	30	32	33
March	1	21	22	22	23	24	24	25	26	27	28	28	29	30	31	33
	11	21	21	22	23	24	24	25	26	26	27	27	28	30	31	33
	21	21	21	22	23	24	24	25	26	26	27	27	28	30	31	33
April	1	21	21	22	23	24	25	25	26	27	28	28	29	30	32	33
	11	21	22	22	23	24	25	26	26	27	28	28	29	31	32	34
	21	22	22	22	23	25	25	26	27	28	28	29	30	32	34	35
May	1	22	22	23	24	25	26	27	28	28	29	30	32	33	35	36
	11	22	22	23	24	26	27	28	29	29	30	31	33	35	36	39
	21	22	22	24	25	27	28	28	29	30	31	33	35	36	38	41
June	1	22	22	24	25	27	28	28	29	31	32	34	36	37	40	43
	11	22	23	24	26	28	28	29	30	31	33	34	36	38	41	44
	21	22	23	25	26	28	29	29	30	31	33	34	36	38	42	44
July	1	22	23	24	26	28	28	29	30	31	33	34	36	38	41	44
	11	22	22	24	25	27	28	28	29	31	32	34	36	37	40	43
	21	22	22	24	25	27	28	28	29	30	31	33	35	36	38	41
August	1	22	22	23	24	26	27	28	29	29	30	31	33	35	36	39
	11	22	22	23	24	25	26	27	28	28	29	30	32	33	35	36
	21	22	22	22	23	25	25	26	27	28	28	29	30	32	34	35
September	1	21	22	22	23	24	25	26	26	27	28	28	29	31	32	34
	11	21	21	22	23	24	25	25	26	27	28	28	29	30	31	33
	21	21	21	22	23	24	24	25	26	27	27	27	28	30	31	33
October	1	21	21	22	23	24	24	25	26	26	27	27	29	30	31	32
	11	21	22	22	23	24	24	25	26	27	28	28	29	30	31	33
	21	21	22	22	23	24	25	25	26	27	28	28	29	30	32	33
November	1	22	22	22	23	25	25	26	27	28	28	29	30	31	33	34
	11	22	22	23	24	25	26	27	28	28	29	30	31	32	33	35
	21	22	22	23	24	26	26	27	28	28	29	30	32	33	34	37
December	1	22	22	24	25	26	27	28	28	29	30	31	33	34	35	38
	11	22	22	24	25	27	27	28	28	29	30	32	33	34	36	39
	21	22	23	24	25	27	27	28	28	29	31	32	33	34	37	39

Relative Illumination Intensities.

Source of illumination	Intensity	Ratio to zenithal full moon
	Foot-candles	
Zenithal sun	9,600.0	465,000.0
Twilight at sunset or sunrise	33.0	1,598.0
Twilight centre of sun 1° below horizon	30.0	1,453.0
Twilight centre of sun 2° below horizon	15.0	727.0
Twilight centre of sun 3° below horizon	7.4	358.0
Twilight centre of sun 4° below horizon	3.1	150.0
Twilight centre of sun 5° below horizon	1.1	53.0
Twilight centre of sun 6° below horizon (End of civil)	0.40	19.0
Twilight centre of sun 7° below horizon	0.10	5.0
Twilight centre of sun 8° below horizon	0.04	2.0
Twilight centre of sun 8°40′ below horizon	0.02	1.0
Zenithal full moon	0.02	1.0
Twilight centre of sun 9° below horizon	0.015	0.75
Twilight centre of sun 10° below horizon	0.008	0.40
Starlight	0.00008	0.004

Duration of Twilight.—The duration of twilight, whether civil, that is, the time after sunset or before sunrise during which there is sufficient light for outdoor occupations, or astronomical, the time until or after complete darkness, varies with the amount of cloudiness and inclination of the ecliptic to the horizon. In the case of clear skies, civil twilight ends, or begins, when the true position of the sun (centre) is about 6° below the horizon, and astronomical twilight when it is about 18° below.

The tables of twilight duration (pages 667 and 668) were computed by Kimball [217] from the equation,

$$h = \frac{\sin \alpha - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

in which h is the sun's hour angle from the meridian, α the sun's altitude (negative below the horizon), δ the solar declination, and ϕ the latitude.

Twilight Illumination.—The brightness of twilight changes slowly or rapidly, according as the sun is less or more, respectively, than about 4° below the horizon. The last table above, based on photometric measurements by Kimball and Thiessen,[218] gives the approximate value of a number of clear-sky, twilight and other natural illumination intensities on a fully exposed horizontal surface.

[217] *Monthly Weather Review*, **44**, p. 614, 1916.
[218] *Monthly Weather Review*, **44**, p. 614, 1916.

Chapter VIII.

PHENOMENA DUE TO SCATTERING: SKY POLARIZATION

The polarization of sky light, discovered in 1811 by Arago,[219] often is more or less modified by specular reflection from relatively large particles—cloud droplets, coarse dust, etc.—but in general it results from the combination of primarily and secondarily scattered radiation.

Condition of Primarily Scattered Light.—As explained by Lord Rayleigh,[220] the light scattered from an incident beam by a gas molecule or other sufficiently small object is symmetrically distributed about the line of enforced motion of that particle as an axis, and completely polarized in the plane at right angles to this line. This follows directly from the fact that plane polarized light is merely light whose vibrations are all normal to the same plane—the plane of polarization. If, then, the incident beam is non-polarized, ordinary sunlight, for instance, the scattered light, therefore, will be completely polarized at right angles to the direction of incidence, and partially polarized in other directions. And, since the plane of polarization is fixed by the sun, observer, and point observed, it follows from Fig. 188 that the ratio

$$\frac{\text{polarized light}}{\text{total light}} = \frac{\sin^2 \gamma}{1 + \cos^2 \gamma},$$

where γ = the angular distance of the point observed from the sun. That is, the polarization increases from zero in the direction both of the sun and the antisolar point to a maximum (complete) midway between them, or normal to the incident rays.

Condition of Secondarily Scattered Light.—While primary scattering of ordinary light by gas molecules and fine dust particles accounts for a large part of the observed polarization and other phenomena of sky light, the non-polarized light that always exists in a greater or less amount at 90° from the sun; the luminosity—partially polarized—of shaded air masses; and the existence of neutral points (small regions whose light is not

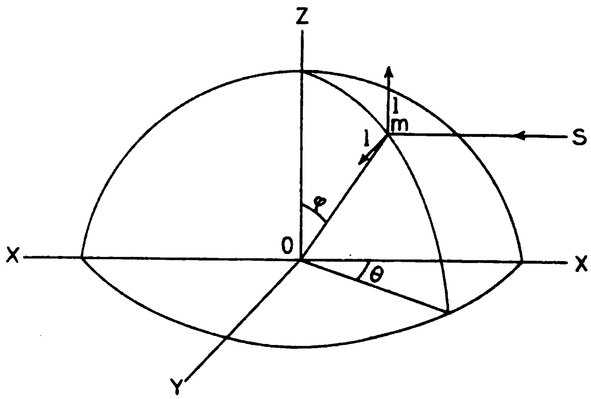
219 *Astronomie Populaire*, **2**, p. 99.

220 *Phil. Mag.*, **41**, p. 107, 1871.

polarized) are all due, as Soret[221] has shown, to secondary scattering. Tertiary and indefinitely higher scattering obviously also exist, but their effects are too small to justify consideration.

To determine the nature and magnitude of secondary scattering, let O (Fig. 189) be the position of a particle shielded from direct insolation but otherwise exposed, and consider its effect on the total incoming sky light. Let the sun be on the horizon; let OX be parallel to the solar radiation, OZ vertical, and OY normal to the plane ZX; let m be any particle a unit distance from O; and let Om make the angle ϕ with the vertical, and its projection on the plane XY the angle θ with OX.

FIG. 189.



Intensity of secondarily scattered light.

As the solar rays are non-polarized they may be treated as consisting of two parts of equal amplitude, l, say, polarized at right angles to each other. For convenience, let the displacements be parallel to OZ and OY, corresponding to polarization in the horizontal and vertical planes, respectively.

On resolving the vertical amplitude into two components, one normal, the other parallel, to Om and the former (which alone is operative on the particle at O) in turn into components parallel to the X, Y, and Z axes, respectively, one finds that,

$$l'_X = -l \sin\phi \cos\phi \cos\theta$$
$$l'_Y = -l \sin\phi \cos\phi \sin\theta$$
$$l'_Z = l \sin^2\phi$$

[221] *Archives de Sci. Phys. et Nat.*, **20**, p. 439, 1888.

Similarly, on resolving the horizontal amplitude into components normal and parallel to the plane OZm, and these in turn parallel to the three axes, one obtains,

$$
\begin{aligned}
l''_X &= l\cos^2\phi\sin\theta\cos\theta - l\sin\theta\cos\theta = -l\sin^2\phi\sin\theta\cos\theta \\
l''_Y &= l\cos^2\phi\sin^2\theta + l\cos^2\theta \\
l''_Z &= -l\sin\phi\cos\phi\sin\theta
\end{aligned}
$$

As a crude first approximation let the distribution of the atmosphere about O be equal in all upward directions and assume all parts to be equally illuminated. Further, let a per unit area be the number of particles that, if distributed over the hemispherical shell, would produce at O the same optical effect that actually obtains. Then, the total intensity components at O (found by squaring the amplitudes and integrating over the hemisphere) are given by the equations,

$$I_X = 2\,a\,l^2\int_0^{\frac{\pi}{2}}\int_0^{\pi}(\sin^2\phi\cos^2\phi\cos^2\theta + \sin^4\phi\sin^2\theta\cos^2\theta)\sin\phi\,d\phi\,d\theta$$

$$I_Y = 2al^2\int_0^{\frac{\pi}{2}}\int_0^{\pi}(\sin^2\phi\cos^2\phi\sin^2\theta + \cos^4\phi\sin^4\theta + 2\cos^2\phi\sin^2\theta\cos^2\theta + \cos^4\theta)$$

$$\sin\phi\,d\phi\,d\theta$$

$$I_Z = 2\,a\,l^2\int_0^{\frac{\pi}{2}}\int_0^{\pi}(\sin^4\phi + \sin^2\phi\cos^2\phi\sin^2\theta)\sin\phi\,d\phi\,d\theta$$

Or,

$$
\begin{aligned}
I_X &= 2\pi\,a\,l^2 \times 2/15 \\
I_Y &= 2\pi\,a\,l^2 \times 3/5 \\
I_Z &= 2\pi\,a\,l^2 \times 3/5
\end{aligned}
$$

The intensity components of secondary diffusion at the centre of a sphere of uniformly distributed particles would be just twice the above, which, as stated, applies to the centre of a hemisphere.

Since there is an appreciable amplitude along all three of the rectangular axes, it follows that secondary scattering sends

more or less non-polarized light in all directions, and, therefore, prevents sky light from being completely polarized even at right angles to the direction of insolation—the direction of complete polarization by primary scattering.

The above assumption that the light-scattering particles are distributed equally along any upward radius from O obviously is not in close agreement with the actual distribution of the atmosphere and its dust content as visible from any given point in it. Let this distribution be $a(n+1) - a\,n \cos\phi$ particles per unit area of the hemisphere instead of a, as previously assumed. Then,

$$I_X = 2\pi a\, l^2 \left[2/15\,(n+1) - 1/16\, n\right]$$
$$I_Y = 2\pi a\, l^2 \left[3/5\,(n+1) - 17/48\, n\right]$$
$$I_Z = 2\pi a\, l^2 \left[3/5\,(n+1) - 5/24\, n\right].$$

If $n = 12$, that is, if the horizon is 13 times brighter than the zenith—a common condition,

$$I_X = 2\pi\, a\, l^2 \times 0.983$$
$$I_Y = 2\pi\, a\, l^2 \times 3.55$$
$$I_Z = 2\pi\, a\, l^2 \times 5.3$$

This distribution of intensities still gives non-polarized light in all directions. It also gives a preponderant amount of polarization, I_z, in the horizontal plane, which neutralizes at certain places the polarization in the vertical plane due to primary scattering.

The combination, then, of primarily and secondarily scattered light must produce a variety of polarization and other phenomena which necessarily vary with the altitude of the sun, dust content of the atmosphere, and state of the weather. Many observational studies have been made of sky polarization and the facts found to agree with the above theoretical considerations. The principal facts are:

(1) Part of the light from nearly all points in a clear sky is plane polarized, whatever the season, location, altitude of the sun, or other conditions.

(2) The polarized portion of sky light, in turn, is divisible into two parts: (a) the positive, due to the first or primary scattering, in which the plane of polarization (plane normal to the

vibrations) is given by the source (sun), point of observation, and eye of the observer; and (*b*) the negative, due to secondary scattering, in which the plane of polarization is normal to that of the primary, and, therefore, because of the ring-like distribution of the atmosphere about any point on the earth's surface, essentially horizontal.

(3) Generally speaking, the percentage of polarized light along any great circle connecting the sun and the antisolar point increases from zero near either to a maximum midway between them, which, in turn, increases with the altitude of the point in question.

(4) The point of absolute maximum polarization is in the solar vertical and ordinarily about 90°, as stated, from the sun.

(5) In general, the percentage of polarization decreases with the amount of light reflected through the sky, whether from the surface or from relatively large particles in suspension. It therefore decreases with (*a*) percentage of snow covering; (*b*) percentage of cloudiness; (*c*) dustiness, or anything that itself leads to an increase of dustiness, such as high winds, especially over arid regions, but everywhere during dry weather, strong vertical convection—hence, generally less during summer than winter—volcanic explosions of the Krakatoa type, etc.

(6) The percentage of polarization generally increases with the wave-length of the light examined.

(7) Even shaded masses of air, if exposed to sky radiation, emit perceptible amounts of polarized light.

(8) Three small regions of unpolarized light, Babinet's, Brewster's and Arago's neutral points, occur on the solar vertical; the first some 15° to 20° above the sun, the second about the same distance below it, and the third 20°, roughly, above the antisolar point.

(9) As the sun rises above or sinks below the horizon the antisolar distance of Arago's point increases from about 20° to, roughly, 23°; while the solar distance of Babinet's point decreases from a maximum of, approximately, 20° to, perhaps, 18°, for a solar depression of 5° or 6°, and to 0°, as does also Brewster's point, as the zenith is approached.

(10) When the upper atmosphere is greatly turbid, as it has often been after violent volcanic explosions, other neutral points, in addition to those above mentioned, are occasionally observed.

PART IV.

FACTORS OF CLIMATIC CONTROL.

CHAPTER I.

GENERAL SUMMARY.

INTRODUCTION.

THE following is a discussion of the principal factors, and the effects of their possible changes, that determine what the climate —the various averages and extremes of weather—of any given place shall be; a discussion of the physics of climate and not of its geographic distribution.

Many people, relying on their memories alone, insist that our climates are now very different from what they used to be. Their fathers made similar statements about the climates of still earlier times, as did also their fathers' fathers, as their several writings show, and so on through the ages; and the bulk of this testimony is to the effect that our climates are getting worse—evidence, perhaps, that flesh has always been heir to ills. The records, however, of the past 100 years show that while there have been several slight and short-period (2 to 3 or 4 years) climatic changes during that time, that will be explained later, there have been no long-period ones. There is, though, much evidence that appreciable climatic changes of many years' duration have occurred within historic times. This evidence, which many do not accept as conclusive, is found in the growth rings of old trees; the known changes in the areas and depths of several inland seas; the records in regard to the breaking up of ice in rivers and the opening of navigation; and in a variety of other more or less significant facts.

But whatever the truth in regard to historic climates may be, nothing is more certain than that during the geologic past there have been many and important climatic changes of great duration. Innumerable fossil remains both in the Arctic and Antarctic regions tell of long ages when genial or, at least, temperate climates extended well among the higher latitudes, while deep scor-

ings and ancient moraines, hundreds and even thousands of miles from the nearest existing glacier, tell quite as positively of other ages when vast ice sheets spread far into the zones we now call temperate; and this in spite of the fact (there is no good evidence to the contrary) that from the beginning of geologic records the surface temperatures have been distributed in the same general sense as at present—highest in equatorial regions and lowest about the poles.

It must be remembered, of course, that the previous existence of comparatively mild climates in limited high latitude regions does not prove that the average temperature of the world as a whole was then much if any higher than it is now, but only that at those places the growing seasons were long enough to permit the then indigenous vegetation to mature its seeds (a much more rapid process in high latitudes owing to the greater length of the summer days than in low). and that the temperature of the littoral waters at the same places was such as to foster the local marine life. Both conditions conceivably might have been met by a free and therefore abundant oceanic circulation; or, perhaps, locally by protection from cold currents and drifting ice. Similarly, local glaciation doubtless often was produced by local causes. But, on the other hand, such extensive glaciation as several times obtained must have required a world-wide lowering of temperature. Indeed, no escape seems possible from the conclusion that the world has experienced many a profound climatic change of both types, local and universal.

When this series of climatic changes began there is no sure means of knowing, for the records, especially those of glacial origin, grow gradually fainter and more scanty with increase of geologic age, so scanty indeed as to force the belief that the effects of many of the earlier changes may long since have been completely obliterated. But, however this may be, it is well-nigh certain that from the time of the earliest known of these changes down to the very present the series has been irregularly continuous, and the end, one might reasonably assume, is not yet. Change after change of climate in an almost endless succession, and even additional ice ages, may still be experienced, though when they

shall begin (except in the case of the small and fleeting changes to be noted below), how intense they may be, or how long they shall last, no one can form the slightest idea.

Clearly, then, a matter so fundamental as this, namely, the profound modification of those agencies that not only fashion the face of the earth, but also control its flora and govern its fauna, challenges and deserves every contribution that science can give to its complete or even partial elucidation. Hence it is that during the past fifty years, or more, numerous attempts, some of them invoking purely terrestrial and others extra-terrestrial or cosmical conditions, have been made to find a probable and at the same time an adequate physical basis for, or cause of, the known climatic changes of the distant past, and especially for those disastrous changes that brought about the extensive glaciations that prevailed during the so-called ice ages. But nearly all the older suggestions and working hypotheses as to the cause of the ice ages have been definitely and finally abandoned, either because of inconsistency with known physical laws, or abandoned because they were found inadequate to meet the conditions imposed upon them by the results of the very investigations which, in many cases, they themselves had helped to inspire.

FACTS OF CLIMATIC CHANGES.

Among the more important facts with respect to climatic changes that appear to have been established and which presumably, therefore, must be met by any theory that would account for such changes, or explain specifically the origin of ice ages, are the following:

(*a*) The number of larger climatic changes were at least several, the smaller many.

(*b*) The greater changes and doubtless many of the smaller also were simultaneous over the entire earth (there is accumulating evidence in favor of this conclusion), and in the same sense; that is, the world became colder everywhere at the same time (climatically speaking) or warmer everywhere.

(*c*) They were of unequal intensity.

(*d*) They were of irregular occurrence and of unequal duration.

(*e*) They, at least one or more, progressed with secondary variations of intensity, or with advances and retreats of the ice front.

(*f*) There often were centres of maximum intensity—certainly of ice accumulation and, doubtless, of other effects.

(*g*) There were numerous local changes, suggestive of local causes.

(*h*) They have occurred from early, probably from the earliest, geological ages down to the present, and presumably will continue irregularly to recur for many ages yet to come.

EXISTING FACTORS OF CLIMATIC CONTROL.

Before attempting to find the probable cause or causes of climatic changes it will be convenient first to consider the present factors of climatic control, since the variations of some of these undoubtedly have produced such changes, even, presumably, some if not all of those great changes that brought on maxima and minima of glaciation. It is possible, of course, that neither singly nor collectively were the factors in question largely productive of the known changes in geologic climates, but as climate to-day is subject to a complex control, all terms of which are more or less variable, it is certain that the climates of that portion of the geologic past (the only portion that will here be considered) during which the earth had an atmosphere and a hydrosphere, were also subject to a similar complex control consisting certainly of all the factors that now are effective, and probably of no others. Hence, while it is conceivable that some one dominant cause such as marked and age-long changes in the solar constant, the passage of the solar system through a vast nebula, and the like, may have produced all the great changes of geologic climates, it seems far safer to assume that climate was then controlled essentially as climate is now controlled, and, therefore, that the climatic changes of the past, whatever their nature, intensity, or duration, were due to changes in those factors of climatic control which are now operative and known to be appreciably variable.

The following list includes the principal factors of climatic control as they exist to-day:

Chief Factors of Climatic Control.

Name	Character
1. LATITUDE.	Invariable to within negligible amounts.
2. BRIGHTNESS OF MOON AND PLANETS.	Widely variable, but of no climatic significance, since they jointly produce a temperature variation of only 0.0001° C., roughly.
3. SOLAR "CONSTANT" AT A FIXED DISTANCE.	Slightly variable. There are small irregular variations of, roughly, a seven to ten day period and probably also a small variation coincident with the eleven year sun spot period. Other changes are not known, but may exist.
4. SOLAR DISTANCE.	Slightly variable, with a geologically negligible annual period due to eccentricity of the earth's orbit; and also, for the same reason, both a 100,000 year, roughly (now about 80,000 year), secular period; and a much longer pseudo period. The larger of these eccentricity changes undoubtedly are of climatic importance, but, as presently explained in the discussion of Croll's theory, there is strong evidence against the assumption that they were the chief or even an important factor in the production of glaciation. There also are slight monthly changes in the solar distance due to perturbations by the moon; and other slight changes owing to perturbations by the planets. In any case, however, the climatic effect due to perturbations is negligible—a maximum temperature change (computed) of, roughly, 0.01° C.
5. OBLIQUITY OF ECLIPTIC.	Slightly variable. According to Sir John Herschel this variation never exceeds 1° 20′ on either side of the mean; and according to Newcomb, while the limit of variation is still unknown, the amount does not exceed 2° or 3° in a million years. In either case recent geologic climates, including that of the last ice age, could not have been much influenced by this factor.
6. PERIHELION PHASE.	Variable through a period of, roughly, 21,000 years. By virtue of this variation the winter of the southern hemisphere, say, may at one time occur, as it now does, at aphelion, and therefore be long and cold; and again at perihelion, when it must be relatively short and mild. However, while this is a climatic factor which varies with the eccentricity of the earth's

Chief Factors of Climatic Control.—Continued.

Name	Character
	orbit, the period is too short to permit of its being considered as of great influence in the production of either the glacial or interglacial climates.
7. Extent and Composition of the Atmosphere.	Probably somewhat variable through geological periods, otherwise relatively fixed.
8. Vulcanism.	Irregularly variable.
9. Sun Spots.	Greatly variable, with an 11-year period and probably other periods also, both longer and shorter.
10. Land Elevation.	Greatly variable through geological periods, otherwise relatively fixed.
11. Land and Water Distribution.	Greatly variable through geological periods, otherwise relatively fixed.
12. Atmospheric Circulation.	Largely dependent upon the distribution of land and water, upon land elevation and upon oceanic circulation, and, therefore, in many regions radically variable through geological periods.
13. Ocean Circulation.	Greatly variable through geological periods, otherwise relatively fixed.
14. Surface Covering.	Greatly variable, in many places, from season to season; and also irregularly so from age to age.

Since these are the factors that now control climate, it seems probable, as already stated, that even those profound climatic changes with which the geologist is concerned were also caused by variations in one or more of these same factors. Indeed, certain of these factors—vulcanism, land elevation, and oceanic circulation—are known to have varied greatly during the several geologic periods, while the extent and composition of the atmosphere are suspected also to have changed. It will be well, therefore, to consider what effects such variations probably could have—in some cases surely have had—on our climates. This will constitute the first step in the problem of geologic climates. The next step must be taken by the geologist himself, for he must say whether the climatic changes possible through the supposed causes would be sufficient to account for the observed results, and especially whether the known climatic changes and the known variations in

the factors here considered occurred at such times and places as to permit of the assumption that they were actually related in the sense of cause and effect.

These several factors will be considered in the same order as above listed.

1. Since the wandering of the pole is limited to only a few metres, it is obvious that the resulting changes in the latitude produce no appreciable climatic effects.

2. The brightness of the moon, and also that of each of the several planets, is known in terms of that of the sun. On the assumption that the heat they supply is in proportion to their light it appears that at most their variations in phase and distance can alter the temperature of the surface of the earth by no more than 0.0001° C., an amount that obviously is wholly negligible.

Chapter II.

PRINCIPAL ICE-AGE THEORIES.

FACTORS 3, 4, 5, 6, 7.

It would be easy to catalogue perhaps a score of more or less rational hypotheses in regard to the origin of the ice ages, the subject under which the greater climatic changes generally are discussed, and doubtless even a larger number that are quite too absurd ever to have received serious consideration, and to point out in each case the known and the suspected elements of weakness. But this would only be a repetition of what, in part at least, has often been done before and, therefore, could serve no good purpose.

As already stated, only a few of these hypotheses still survive, nor do all of even these few really merit the following they have. Indeed, the only ones which still claim a large number of adherents are, respectively:

3. (*a*) *The Solar Variation Theory.*—This is based on the assumption that the solar radiation (the only solar influence that by any known process can affect terrestrial temperatures and terrestrial climates) has waxed and waned, either cyclically or irregularly, through considerable ranges and over long intervals of time.

This theory is seductively attractive—it looks so simple, so sufficient, and so safe from attack. There are, however, two criticisms of it that should be mentioned: (1) A change of the solar constant obviously alters all surface temperatures by a roughly constant percentage. Hence a decrease of the heat from the sun would, in general, cause a decrease of the interzonal temperature gradients; and this in turn a less vigorous atmospheric circulation, and a less copious rain or snowfall—exactly the reverse of the condition, namely, abundant precipitation, most favorable to extensive glaciation. (2) If the solar variation theory is true it follows, as will be shown later, that great solar changes and extensive mountain building must usually, if not always, have been coepochal—a seemingly complete *reductio ad absurdum.*

4, 5, 6. (*b*) *Croll's Eccentricity Theory.*[2]—To make this theory clear, it is necessary to recall two important facts in regard to the earth's movement about the sun: (1) That the orbital position of the earth at any season, that of midsummer, say, progressively changes at such rate as to describe a complete circuit in about 21,000 years. This necessarily produces a cyclic change of the same period in the length, temperature and contrast of the seasons, and also in the contrast between the climates of the two hemispheres, northern and southern. Thus, when aphelion is attained near midsummer of either hemisphere as it now is for the northern, that part of the earth enjoys comparatively long, temperate summers, and short, mild winters; while the opposite hemisphere, the southern at present, is exposed to short hot summers, and long, cold winters. Hence, on such occasions, the climatic contrast between the two hemispheres is at a maximum, provided, of course, that their ratios of land to water areas and other factors are the same. After about 10,500 years another maximum contrast occurs, but with the climates of the two hemispheres interchanged, and so on indefinitely. (2) That the eccentricity of the earth's orbit, never greater than 0.07, and at rare intervals dropping to nearly or even quite zero, undergoes irregular but always slow and long cyclic changes. In addition to a change usually, though not always, relatively small whose average period is roughly 100,000 years (now about 80,000), the eccentricity has also a far more irregular, and generally much larger change whose average period, if a thing so irregular may be said to have a period, is three or four times as great. That is, as a rule, the eccentricity of the earth's orbit is continuously large, within the limit 0.07, or continuously small, for a period of 200,000 years, more or less; but in each case unequally so, because of the shorter period and more regular changes.

The first of these phenomena, the continuous change of the perihelion phase, varies, as explained, the relative lengths and intensities of the summers and winters of the northern and southern hemispheres; while the second, or the change of eccentricity of the earth's orbit, varies the magnitudes of these contrasts.

Now Croll's theory of the ice ages assumes that when the earth's orbit is very eccentric, or when the earth's maximum solar distance differs largely from its minimum solar distance, ice will

[2] *Phil. Mag.*, **28**, p. 121, 1864, and elsewhere.

accumulate to a great extent over that half of the globe which has its winter during aphelion.

For some time this theory was very generally accepted, and it seems still to have many adherents, despite the destructive criticisms of Newcomb[3] and Culverwell.[4]

The chief objections to Croll's theory are:

1. That the assumption that midwinter and midsummer temperatures are directly proportional to the sun's heat at these times is not at all in accord with observed facts.

2. That each ice-age (within a glacial epoch, when eccentricity is large) would be limited to a fraction of the secular perihelion period, 21,000 years, which, according to most geologists, is too short a time.

3. That the successive ice ages would have occurred alternately in the northern and southern hemispheres instead of, as is generally believed to have been the case, in both hemispheres simultaneously.

4. That during the past 3,000,000 years there would have been fully 100 extensive glacial advances and retreats in each hemisphere (eccentricity having been rather large through much the greater portion of this time), a deduction unsupported by confirmatory geological evidence.

5. That the last extensive ice sheet in either hemisphere must have retracted roughly to its present limits some 80,000 years ago (eccentricity having become small about that time and remained small ever since) instead of less than 9000 as Gerald de Geer[5] has well-nigh conclusively demonstrated.

As W. B. Wright[6] puts it: "An almost fatal objection to Croll's famous theory is the date it assigns to the end of the last Ice Age, which it places at some 80,000 years back. If, as De Geer seems to have clearly established, the ice-margin retreated north past Stockholm only about 9000 years ago, this practically excludes any possibility of a connection between glaciation and changes in the eccentricity of the earth's orbit."

That changes in the maximum and minimum distances of the earth from the sun have affected our climates and that they will

[3] *Amer. Jr. Sci.*, 11, p. 263, 1876; *Phil. Mag.*, 17, p. 142, 1884.
[4] *Phil. Mag.*, 38, p. 541, 1894.
[5] *Geolog. Congress*, Stockholm, 1910.
[6] *The Quaternary Ice Age*, p. 451, Macmillan and Co., 1914.

continue to affect them seem too obvious to admit of doubt, but that such changes ever were, or ever will be, of sufficient magnitude to be the sole, or even the chief, cause of an ice-age appears to be flatly contradicted both by rigid deductions from the laws of physics and meteorology and by close observations of geological records.

7. (*c*) *The Carbon Dioxide Theory.*—This theory, advocated by Tyndall,[7] Arrhenius,[8] Chamberlin[9] and others, is based on the selective absorption of carbon dioxide for radiation of different wave lengths, and on its assumed variation in amount.

It is true that carbon dioxide is more absorptive of terrestrial than of solar radiations, and that it therefore produces a greenhouse or blanketing effect, and it is also probably true that its amount in the atmosphere has varied through appreciable ranges, as a result of volcanic and other additions on the one hand, and of oceanic absorption and chemical combination on the other. But it is not possible to say exactly how great an effect a given change in the amount of carbon dioxide in the atmosphere would have on the temperature of the earth. However, by bringing a number of known facts to bear on the subject it seems feasible to determine its approximate value. Thus the experiments of Schaefer[10] show that, at atmospheric pressure, a column of carbon dioxide 50 centimetres long is ample for maximum absorption, since one of this length absorbs quite as completely as does a column 200 centimetres long at the same density. Also the experiments of Ångström,[11] and those of E. v. Bahr,[12] show that the absorption of radiation by carbon dioxide or other gas increases with increase of pressure, and, what is of great importance, that both qualitatively and quantitatively this increase of absorption is exactly the same whether the given higher pressure be obtained by compression of the pure gas to a column of shorter length, or, leaving the column unchanged, by the simple addition of an inert gas.

According to these experiments, if a given column or quantity of carbon dioxide at a pressure of 50 mm. absorbs 20 per

[7] *Phil. Mag.*, 22. p. 277, 1861.
[8] *Phil. Mag.*, 41. p. 237, 1896.
[9] *Jr. Geol.*, 7, p. 545, 1899.
[10] *Ann. der Phys.*, vol. xvi, p. 93, 1905.
[11] *Arkiv för Matematik, Astron. och Fysik,* vol. iv, No. 30, 1908.
[12] *Ann. der Phys.*, vol. xxix, p. 780, 1909.

cent. of the incident selective radiation, then, at 100 mm. it will absorb 25 per cent., at 200 mm. 30 per cent., at 400 mm. 35 per cent., and at 800 mm. about 38.5 per cent.

Now, the amount of carbon dioxide in the atmosphere is equivalent to a column of the pure gas, at ordinary room temperature and atmospheric pressure, of roughly 250 centimetres in length. Hence, as a little calculation proves, using the coefficients of absorption at different pressures given by the experiments of Ångström and E. v. Bahr, just described, the carbon dioxide now in the atmosphere must, under its present vertical distribution, absorb radiation very approximately as would a column 475 centimetres long of the pure gas at the barometric pressure of 400 millimetres. But Schaefer's experiments above referred to show that such a column would be just as effective an absorber as a cylinder two or three times this length, and, on the other hand, no more effective than a column one-half or one-fourth as long; in each case the absorption would be complete in the selective regions of the gas in question.

Hence, finally, doubling or halving the amount of carbon dioxide now in the atmosphere, since this would make but little difference in the pressure, would not appreciably affect the total amount of radiation actually absorbed by it, whether of terrestrial or of solar origin, though it would affect the vertical distribution or location of the absorption.

Again, as explained by Abbot and Fowle,[13] the water vapor always present in the atmosphere, because of its high coefficients of absorption in substantially the same regions where carbon dioxide is effective, leaves but little radiation for the latter to take up. Hence, for this reason, as well as for the one given above, either doubling or halving the present amount of carbon dioxide could alter but little the total amount of radiation actually absorbed by the atmosphere, and, therefore, seemingly, could not appreciably change the average temperature of the earth, or be at all effective in the production of marked climatic changes.

Nevertheless, in spite of the above objections, there appears to be at least one way (variation in absorption at levels above the water vapor) by which a change, especially if a decrease, in the

[13] *Annals of the Astrophysical Observatory*, Smithsonian Institution, vol. ii, p. 172, 1908.

amount of carbon dioxide in the atmosphere might affect temperatures at the surface of the earth. Hence, the above arguments do not perhaps fully warrant the idea that no such change was ever an appreciable factor in the production of an ice age.

Further consideration of this particular point will be taken up later, after the discussion of certain other questions essential to a clear understanding of the subject.

These three theories, then, of the origin of the ice ages, namely: The solar variation theory, the eccentricity theory, and the carbon dioxide theory, are the only ones that at present appear to have many adherents, and even these few seem more likely to lose than to gain in number and ardency of defenders. The first is strong only as, and to the extent that, other theories are disproved or shown to be improbable; the second has failed utterly under searching criticism; while the third has been sadly impaired.

Chapter III.

8. VULCANISM: THEORY.

GASEOUS CONTRIBUTION TO THE ATMOSPHERE.

ALTHOUGH a variety of gases, vapors and fumes are given off by active volcanoes, probably only one of them, carbon dioxide, is of sufficient volume and of such nature as to produce any effect on climate. Indeed, besides carbon dioxide, the only atmospheric constituents that are especially effective in modifying the average temperature of the earth are water vapor and, probably, ozone. The former of these, or water vapor, except as locally modified by temperature and topography, including location and extent of land and sea, presumably has varied but little in amount since the formation of the earliest oceans, while a practically continuous series of animal fossils from beyond the earliest paleozoic age to the present is abundant proof of an equally continuous supply of free oxygen. Hence, in an effort roughly to determine what climatic changes might have been caused by variations in the atmosphere, whether produced by vulcanism or otherwise, it would appear that only the amount of carbon dioxide need be considered.

But this has been discussed above, to some extent, and will be taken up again in its proper order. Suffice it to anticipate here the general conclusion that while variations in the amounts of carbon dioxide in the atmosphere may have somewhat modified our climates, it probably never was the controlling or even an important factor in the production of any one of the great climatic changes of the past, nor can be of any great climatic change the future possibly may bring.

CHANGE IN SURFACE COVERING.

The effect of volcanic ejecta, whether in the nature of ash, or lava flow, is to convert the region so covered into a temporary desert, even where rain may be abundant, and, therefore, to subject it to an increased range of temperature extremes, and at the same time, if in a previously vegetated region, slightly to increase its average temperature, owing to decrease of evaporation. How-

ever, it seems highly probable that the areas so deprived of vegetation were never at any one time sufficiently large to produce marked effects upon the climate of the world as a whole, nor indeed anywhere except over themselves and within their own immediate neighborhoods. Hence, in considering universal climatic changes, it seems safe to neglect this special effect of volcanic activity.

DUST IN THE UPPER ATMOSPHERE.

It was suggested a number of years ago by the cousins P. and F. Sarasin[14] that the low temperature essential to the glaciation of ice ages was caused by the absorption of solar radiation by high volcanic dust-clouds. But the idea that dust of this nature, when scattered through the atmosphere, may lower the temperature of the surface of the earth was already old, having been advanced at a much earlier date, in fact, long before even the existence of ice ages had been suspected, much less attempts made to find their cause. Thus, in May, 1784, Benjamin Franklin (and he may not have been the first) wrote as follows:

> During several of the summer months of the year 1783, when the effects of the sun's rays to heat the earth in these northern regions should have been the greatest, there existed a constant fog over all Europe, and great part of North America. This fog was of a permanent nature; it was dry, and the rays of the sun seemed to have little effect toward dissipating it, as they easily do a moist fog arising from the water. They were indeed rendered so faint in passing through it that, when collected in the focus of a burning-glass, they would scarce kindle brown paper. Of course, their summer effect in heating the earth was exceedingly diminished.
>
> Hence the surface was early frozen.
>
> Hence the first snows remained on it unmelted, and received continual additions.
>
> Hence perhaps the winter of 1783-4 was more severe than any that happened for many years.
>
> The cause of this universal fog is not yet ascertained. Whether it was adventitious to this earth, and merely a smoke proceeding from the consumption by fire of some of those great burning balls or globes which we happen to meet with in our course round the sun, and which are sometimes seen to kindle and be destroyed in passing our atmosphere, and whose smoke might be attracted and retained by our earth; or whether it was the vast quantity of smoke, long continuing to issue during the summer from Hecla, in Iceland, and that other volcano which arose out of the sea near that island, which

[14] *Verhandlungen der Naturforschenden Gesellschaft in Basel,* vol. xiii, p. 603, 1901.

smoke might be spread by various winds over the northern part of the world, is yet uncertain.

It seems, however, worthy the inquiry, whether other hard winters, recorded in history, were preceded by similar permanent and widely-extended summer fogs. Because, if found to be so, men might from such fogs conjecture the probability of a succeeding hard winter, and of the damage to be expected by the breaking up of frozen rivers in the spring; and take such measures as are possible and practicable to secure themselves and effects from the mischiefs that attend the last.[15]

The idea, then, that volcanic dust may be an important factor in the production of climatic changes is not new, though by what physical process it could produce this result apparently has not formerly been explained, nor has the idea previously been specifically supported by a long series of direct observations. This is not to be taken as a criticism of the above-mentioned pioneer paper by the Sarasin cousins, for indeed the arguments, now easy, necessary to show that it must be a factor, were at that time impossible, because the observations upon which these arguments largely are based had not then been made. In fact, the absorption of radiation by volcanic dust, by which they supposed the earth's temperature to be lowered, can now be shown to be, of itself alone, not only insufficient, but even productive, in all probability, of the opposite effect—of a warming instead of a cooling of the earth's surface.

To make this point clear: Consider a thin shell of dust about the earth and let I be the average intensity of the normal component of solar radiation on it, and I_e the intensity of the radiation reaching the earth. Further, let a be the average coefficient of absorption of the dust shell for solar radiation, a coefficient independent, presumably, of intensity, and b its coefficient of absorption for terrestrial radiation, also independent of intensity. Obviously, in the case of equilibrium, all the energy absorbed by the dust is radiated away; half of it, very approximately, to the earth and half of it to space. Hence, starting with I as the intensity of the solar radiation normally incident per unit area and unit time, upon the dust layer, we have,

$$I_e = I \left\{ 1 + k\,(b - a) \right\} \cdots\cdots\cdots\cdots \quad (A.)$$

[15] See Sparks's "Life of Benjamin Franklin," vol. vi, 455–457 (cited in *Proceedings of the Amer. Phil. Soc.*, vol. xlv, p. 127, 1906.)

in which

$$k = \tfrac{1}{2}\left\{1 + \frac{b}{2} + \left(\frac{b}{2}\right)^2 + \cdots\cdots + \left(\frac{b}{2}\right)^\infty\right\}.$$

Now b is positive, and, therefore, k is also positive. Hence

$$I\left\{1 + k(b-a)\right\} \gtreqless I, \text{ according as } b \gtreqless a.$$

The conclusion, therefore, is: *The total amount of radiation reaching the earth is increased, unchanged, or decreased owing to absorption by the surrounding dust layer according as the dust's coefficient of absorption of terrestrial radiation is greater than, equal to, or less than its coefficient of absorption of solar radiation.*

Actually nearly all, both of the incoming and of the outgoing radiation, is oblique, but as equal portions of each pass through equal thickness of the shell it follows that the conclusion reached for normal radiation applies also for the oblique radiation.

While this general conclusion is self-evident, and, therefore, might have been stated without the use of symbols, nevertheless equation (A), to be used later on, will be found convenient in attempts to obtain quantitative values.

Now, in the case of many, if not all, rocky materials, such as make up the particles of volcanic dust, the coefficient of absorption is much greater for terrestrial radiation than for solar radiation,[16] or, in terms of the above symbols, in the case of volcanic dust, b is greater than a. Hence, so far as mere *absorption* of radiation is concerned, the only action mentioned by the cousins Sarasin, a veil of volcanic dust, in all probability, would slightly increase and not, as they supposed, decrease the average temperature of the earth.

But, then, absorption is not the only effect of a dust veil on radiation; *reflection* and *scattering* both are important and must be fully considered.

These actions, however, reflection and scattering, depend fundamentally upon the ratio of the linear dimensions of the particles concerned to the wave length of the incident radiation, and, therefore, before undertaking to discuss them in this connection, it will be essential to determine the approximate size of the individual grains of floating volcanic dust, and also the

[16] Coblentz, Publications of Carnegie Institution of Washington, Nos. 65 and 97.

average wave lengths in the regions of the respective maximum intensities of solar and terrestrial radiation. It will be desirable, also to consider whether or not, and, if so, how, dust of any kind can remain long suspended in the atmosphere. And this point, involving the structure of the atmosphere, will be examined first, since, obviously, the longer the dust can float the more important, climatically, it may have been in the past and in the future may again become.

Physical Structure of the Atmosphere.—The atmosphere is divisible into the stratosphere and the troposphere; or the isothermal region and the convective region; or, in other words, that region, in middle latitudes at and beyond about 11 kilometres above sea level, where, because of freedom from vertical convection, ordinary clouds never form, and that other, or turbulent, stormy region below this level, which is frequently swept by clouds and washed by snow and rain. The physical reason for or cause of the existence of the isothermal region is well known (see Chapter III, Part I) and is such that it is certain that ever since the earth was warmed by solar radiation, as at present, rather than by internal heat, the temperature of its atmosphere beyond a certain level, whatever its composition, must have varied but little, as it now varies but little, with change of altitude, and therefore that this region must then have been free, as it now is free, from clouds and condensation. Obviously, then, this peculiar physical structure of the atmosphere is of great importance in determining the duration of dust suspension for, clearly, any volcanic or other dust, that by whatever process is gotten into and distributed through the isothermal region where there are no clouds or other condensation to wash it out, must drift about until gravity, overcoming the viscosity of the atmosphere, by slow degrees shall have pulled it down to the region of clouds and storms, where it becomes moisture laden and quickly brought to the earth. How long such process must take depends, of course, upon a number of things, among which the size of the particles is vitally important.

Size of Volcanic Dust Particles.—For two or three years after the eruption of Krakatoa, in 1883, also after the eruption of Mont Pelé and Santâ Maria, in 1902, and again after the eruption of Katmai, in 1912, a sort of reddish-brown corona was

often, under favorable conditions, observed around the sun. It was from 10 to 12 degrees wide, and had, to the outer edge, an angular radius of from 22 to 23 degrees. This phenomenon, known as Bishop's ring, clearly was a result of diffraction of sunlight by the particles of volcanic dust in the upper atmosphere, and therefore it furnished a satisfactory means for determining the approximate size of the particles themselves. The subject has been rather fully discussed by Pernter,[17] who finds the diameter of the particles, assuming them spherical, to be approximately 185×10^{-6} cm., or 1.85 microns. The equation used has the form, page 531.

$$r = \frac{m}{\pi} \frac{\lambda}{\sin \theta}$$

in which r is the radius of the dust particle, λ the wave length of the diffracted light (here taken as 571×10^{-7} cm., or 0.571 micron), θ the angular radius of the ring, and m a numerical term which for the outer edge of the ring, and successive minima of brightness, has the approximate values, see page 532.

$$\frac{\pi}{2}(n + 0.22)$$

in which $n = 1, 2, 3, \ldots$, respectively.

Now, since the width and angular dimensions of Bishop's ring, as seen at different times and different places, have varied but little, the above value, 1.85 microns, may provisionally be assumed to be the average diameter of those particles of volcanic dust that remain long suspended in the atmosphere.

Time of Fall.—The steady or terminal velocity of a minute sphere falling in a fluid, assuming no slip between fluid and sphere is given by Stokes's [18] equation.

$$V = \frac{2}{9} g r^2 \left(\frac{\sigma - \rho}{\mu}\right)$$

in which V is the velocity of the fall, g the acceleration of gravity, r the radius of the sphere, σ the density of the sphere, ρ the density of the fluid, and μ its viscosity.

However, there always is slip, so that the actual velocity of fall is, according to Cunningham,[19]

$$V = \frac{2}{9} g r^2 \left(\frac{\sigma - \rho}{\mu}\right)\left(1 + A\frac{l}{r}\right)$$

[17] *Met. Zeit.*, 6, p. 401, 1889.
[18] *Math. and Phys. Papers*, vol. iii, p. 59.
[19] *Proc. Roy. Soc.*, 83 A, p. 357, 1910.

in which l is the free path of the gas molecules, A a constant, and the other symbols as above explained.

Obviously, l, other things being equal, is inversely proportional to the gas density, or pressure, if temperature is constant, and directly proportional to the absolute temperature if the pressure is constant. Hence,

$$V = \frac{2}{9} g r^2 \left(\frac{\sigma - \rho}{\mu}\right)\left(1 + \frac{B}{rp}\right) \text{ - - - - - - - - - - } (1)$$

in which B is a constant for any given temperature, p the gas pressure, or, if preferred, barometric height.

Now, a series of valuable experiments by McKeehan [20] has shown that for 21° C., and when p is the pressure in terms of millimetres of mercury,

$$B = .0075 \pm 3.$$

The value of μ, for dry air, is also closely known from the work of a number of experimenters, all of whom obtained substantially the same results. From a careful review of the whole subject, Millikan [21] finds that at 23° C.,

$$u = 1824 \times 10^{-7}, \left(\text{more recently } 18226 \times 10^{-8}\right),$$

and that, for the temperature, t, Centigrade,

$$\mu_t = \frac{150\,38\, T^{\frac{3}{2}}}{T + 124} \times 10^{-7}, \text{ approximately,}$$

where $T = 273.11 + t$.

It is easy, therefore, to compute, by the aid of equation (1), the velocity of fall of volcanic dust, assuming gravity to be the only driving force. There is, of course, radiation pressure, both toward and from the earth, as well as slight convective and other disturbances, but presumably gravitation exerts the controlling influence.

The following table of approximate velocities and times of fall for volcanic dust was computed by substituting in equation (1) the given numerical values, namely:

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[20] *Phys. Rev.*. 33, p. 153, 1911.

[21] *Ann. der Phys.*, 41, p. 759, 1913.

$g = 981 \frac{\text{cm.}}{\text{sec.}}$
$r = .000092$ cm.
$\sigma = 2.3$, approximate density of Krakatoa dust.
$\rho = 0$, being negligible relative to σ.
$\mu = 1416 \times 10^{-7}$, appropriate to $-55°$ C., roughly the temperature of the isothermal region in middle latitudes.
$B = .0056$, appropriate to $-55°$ C.
p = millimetres barometric pressure.

According to this table, it appears that spherical grains of sand of the size assumed, 1.85 microns in diameter, would require about one year to fall from only that elevation already reached by sounding balloons, 35.08 kilometres,[22] down to the under surface of the isothermal region, at the height of 11 kilometres.

Velocity and Time of Fall

Height in kilometres	Barometric pressure	Centimetres per second	Seconds per centimetre
40	1.84	1.0215	0.979
30	8.63	0.2414	4.143
20	40.99	0.0745	13.427
15	89.66	0.0503	19.874
11*	168.00	0.0408	24.492
0	760.00	0.0258†	38.760†

* Isothermal level of middle latitudes.
† Temperature 21° C.

As a matter of fact, volcanic dust, at least much of it, consists of thin-shelled bubbles or fine fragments of bubbles, and, therefore, must settle much slower than solid spheres, the kind above assumed. Indeed, the finest dust from Krakatoa, which reached a great altitude, probably not less than 40 nor more than 80 kilometres, was from two and a half to three years in reaching the earth, or, presumably, as above explained, the upper cloud levels.

At any rate, volcanic dust is so fine, and the upper atmosphere above 11 kilometres so free from moisture and vertical convection, that once such dust is thrown into this region, as it obviously was by the explosions of Skaptar Jökull and Asamayama in 1783, Babuyan in 1831, Krakatoa in 1883, Santâ Maria and Pelé in 1902, Katmai in 1912, and many others, it must require, as a rule, because of its slow descent, from one to three years to get

[22] *L'Astronomie*, 27, p. 329, 1913.

back to the earth. And this clearly has always been the case since the earth first assumed substantially its present condition, or had a cool crust and a gaseous envelope.

Obviously, then, it is only necessary to determine the present action of such dust on incoming solar and outgoing terrestrial radiation in order to reach a logical deduction as to what its effect on climate must have been in the past if, through extensive volcanic activity, it ever more or less continuously filled the upper atmosphere for a long or even considerable term of years, as may have happened several times during the geologic ages. And the same conclusion in regard to the possible effect of dust on the climates of the past clearly applies with equal force to the climates of the future.

Action of Dust on Solar Radiation.—Since solar radiation at the point of maximum intensity has a wave-length less than 5×10^{-5} cm.,[23] or half a micron, and since fully three-fourths of the total solar energy belongs to spectral regions whose wave-lengths are less than 10^{-4} cm., or one micron, it follows that the cubes of solar wave-lengths must, on the whole, be regarded as small in comparison with the volume of a volcanic dust particle, the diameter of which, as above explained, is nearly 2 microns. Hence, in discussing the action of volcanic dust on incoming solar radiation, we can, with more or less justification, assume the particles to be opaque through reflection or otherwise, and, therefore, use Rayleigh's[24] arguments as applied to a similar case.

Let r be the radius of the particle, n the number of particles per cubic centimetre, and a the projected joint area of these particles. Then, for random and sparsely scattered particles,

$$a = n\pi r^2$$

Hence, on dividing a plane parallel to the wave-front into Fresnel zones, it is seen that for each centimetre traversed the amplitude of the radiation is reduced in the ratio of 1 to $1 - n\pi r^2$. Therefore, if A is the initial amplitude, and A_x the amplitude after passing through x centimetres of the uniformly dusty region, assuming $n\pi r^2$ to be only a small fraction of a square centimetre,

$$A_x = A\,(1 - n\pi r^2)^x = Ae^{-n\pi r^2 x}$$

[23] Abbot and Fowle, *Annals Astrophys. Obsy.*, Smithsonian Inst., vol. ii. p. 104, 1908.

[24] *Phil. Mag.*, 47, p. 375, 1899.

Further, if I is the initial and I_x the final intensity, then

$$I_x = Ie^{-2n\pi r^2 x}$$

Hence, in the case of volcanic dust, where, as already explained, $r = 92 \times 10^{-6}$ centimetre,

$$A_x = Ae^{-n\pi x (92)^2 10^{-12}}$$

and

$$I_x = Ie^{-2n\pi x (92)^2 10^{-12}}$$

Presumably, the particles of dust are not absolutely opaque and, therefore, I_x probably is a little larger than the value here given, though even so this value is at least a first approximation.

Action of Dust on Terrestrial Radiation.—Terrestrial radiation, at the point of maximum intensity, has a wave-length of roughly, 12×10^{-4} centimetres, and, therefore, the wave-lengths of nearly all outgoing radiation are large in comparison with the diameters of those volcanic dust particles that remain long suspended in the atmosphere. Hence, while such particles abundantly *reflect* solar radiation, as is obvious from the whiteness of the sky when filled by them, they can only *scatter* radiation from the earth, according to the laws first formulated by Rayleigh,[25] whose papers must be consulted by those who would fully understand the equations which here will be assumed and not derived.

Let E be the intensity of terrestrial radiation as it enters the dusty shell, or as it enters the isothermal region, and E_y its intensity after it has penetrated this region, supposed uniformly dusty, a distance y centimetres; then, remembering that the dust particles are supposed to be spherical, according to Rayleigh,

$$E_y = Ee^{-hy}$$

where

$$h = 24\pi^3 n \frac{(K' - K)^2}{(K' + 2K)^2} \frac{T^2}{\lambda^4},$$

in which n is the number of particles per cubic centimetre, K the dielectric constant of the medium, K' the dielectric constant of the material of the particles, T the volume of a single particle, and λ the wave-length of the radiation concerned.

[25] *Loc. cit.*

But $K = 1$, and, since the dust seems generally to be a kind of glass, it may not be far wrong to assume that $K' = 7$. Hence, with these values,

$$h = 11\pi^3 n \frac{T^2}{\lambda^4}, \text{ nearly.}$$

Relative Action of Dust on Solar and Terrestrial Radiation.—To determine whether such a dust layer as the one under discussion will increase or decrease earth temperatures it is necessary to compare its action on short wave-length solar radiation with its action on long wave-length radiation from the earth.

In the case of solar radiation, as explained,

$$I_x = Ie^{-2n\pi x (92)^2 10^{-12}}$$

Clearly, then, the intensity of the solar radiation is reduced in the ratio of 1 to e, or

$$I_x : I = 1 : e$$

when $x = \frac{10^{12}}{2n\pi (92)^2}$ centimetres $= \frac{188}{n}$ kilometres, approximately.

On the other hand, in the case of terrestrial radiation, where

$$E_y = Ee^{-11\pi^3 n \frac{T^2}{\lambda^4} y},$$

the intensity is reduced in the ratio of $1 : e$, or

$$E_y : E = 1 : e,$$

when

$$y = \frac{\lambda^4}{11\pi^4 n T^2} \text{ centimetres,}$$

in which

$$T = \frac{4}{3}\pi (92)^3 10^{-18}$$

and

$$\lambda = 12 \times 10^{-4}, \text{ the region of maximum intensity.}$$

Hence,

$$y = \frac{5700}{n} \text{ kilometres, approximately.}$$

Therefore, finally,

$$y : x = 30 : 1, \text{ roughly,}$$

or the shell of volcanic dust, the particles all being the size given, is some thirty-fold more effective in shutting solar radiation out than it is in keeping terrestrial radiation in. In other words, the veil of dust produces an inverse greenhouse effect, and hence, if the dust veil were indefinitely maintained, the ultimate equilibrium temperature of the earth would be lower than it is when no such veil exists.

The ratio 30 to 1 in favor of terrestrial radiation in its ability to penetrate the dusty atmosphere may at first seem quite too large, but it should be remembered that the dust particles in question are to terrestrial radiation in general as air molecules are to solar radiation, in the sense that in both cases but little more than mere scattering takes place. Now it is obvious that the dust particles are many fold more effective in intercepting solar radiation, which they appear to do chiefly by reflection, than is an equal mass of air molecules which simply scatter it; and hence it may well be that the above theoretically determined ratio, 30 to 1, is no larger than the ratio that actually exists, or, at any rate, that it is of the correct order.

It must be distinctly understood that certain of the assumptions upon which the foregoing is based—uniformity of size, complete opacity and sphericity of the dust particles, for instance—are only approximately correct, but they are the best that at present can be made, and doubtless give at least the order of magnitude of the effects, which, indeed, for the present purpose, is quite sufficient.

It may be well, in this connection, to call attention to the fact that the excessively fine dust particles, or particles whose diameters are half, or less, the wave-length of solar radiation (region of maximum intensity), and which, therefore, remain longest in suspension, shut out solar radiation many fold more effectively than they hold back terrestrial radiation. This is because both radiations, solar and terrestrial, are simply scattered by such small particles, and scattered in proportion to the inverse fourth power of the wave-length. Indeed, since the ratio of solar wave-length to terrestrial wave-length (region of maximum intensity in both cases) is, roughly, 1 to 25, and the ratio of their fourth powers as 1 to 39×10^4, about, it follows that the interception of outgoing radiation by the very finest and therefore most persistent

dust is wholly negligible in comparison with its interception of incoming solar radiation.

Number of Dust Particles.—The intensity of the solar radiation, I_x, after it has passed through x centimetres of the dust layer of the atmosphere, is given, as previously explained by the equation,

$$I_x = Ie^{-2n\pi x (92)^2 \times 10^{-12}}$$

But, according to numerous observations made during the summer and fall of 1912, when the solar radiation had passed entirely through the dust layer at such an angle that it met, roughly, twice as many dust particles as it would have met had it come in normally, or from the zenith, it was reduced by about 20 per cent. That is to say, under these conditions

$$I_x = 0.8\, I.$$

Hence

$$10 = 8e^{2nx\pi \cdot (92)^2\, 10^{-12}}$$

Let $nx = 2N$. the total number of particles passed in a cylinder of one square centimetre cross section. Then

$$10 = 8e^{4N\pi\, (92)^2\, 10^{-12}}$$

Hence the number of particles in a vertical cylinder of one square centimetre cross section is given, roughly, by the equation

$$N = 34 \times 10^4.$$

Temperature Correction Due to Dust Radiation.—With the number and size of the dust particles known it is easy to determine at least an upper limit to the effect of the direct radiation of the particles themselves on the temperature of the earth.

The temperature of the dust particles, obviously, is very nearly that of the upper atmosphere in which they float, that is, approximately −55° C., or 218° C. absolute. Also, as previously explained, the quantity of radiation from the atmosphere below the isothermal region is substantially that which would be given off by a full radiator at 256° C. absolute.

Now assume the dust particles to be concentrated side by side on a common plane, and, further, assume them to be full radiators —conditions that would raise their effect to the theoretical upper limit. Let E be the intensity or quantity per square centimetre

of the outgoing planetary radiation, and D the intensity of the incoming dust radiation. Then

$$E : D = (256)^4 : a(218)^4,$$

in which a is the projected area of all the particles in a vertical cylinder of one square centimetre cross section.

But

$$a = 34\pi\ 10^4(92)^2 10^{-12} = 9 \times 10^{-3}.$$

Hence

$$E = 211\ D.$$

Now, when the radiation D is absorbed by the lower atmosphere, it follows that its temperature will be so increased that, when equilibrium is reached, the intensity of its new radiation will be to that of its old as 212 is to 211. Hence ΔT, the effective temperature increase of the lower atmosphere, is given by the equation

$$\frac{(256 + \Delta T)^4}{(256)^4} = \frac{212}{211},$$

from which

$$\Delta T = 0^\circ.3\ C.$$

But, as stated above, the dust particles, presumably, are not full radiators, and, therefore, probably one-fifth of a degree C. is as great an increase in temperature as may reasonably be expected from this source. But this *increase*, 0.2° C., is small in comparison with the *decrease*, 6° C. to 7° C., caused by the interception of solar radiation, already explained. *Hence it appears reasonably certain that the sum total of all the temperature effects produced by volcanic dust in the upper atmosphere, equal in amount to that put there by the explosion of Katmai, must be, if long continued, a lowering of the surface temperature by several degrees C.*

Total Quantity of Dust.—Let $nx = 2N$, the total number of particles passed in a cylinder of one square centimetre cross section. Then, as explained above,

$$10 = 8e^{4N\pi\,(92)^2 \times 10^{-12}}.$$

Hence

$$N = 34 \times 10^4$$

roughly = number of particles in a vertical cylinder of one square centimetre cross section.

If A is the entire area of the earth in square centimetres, then the total number of dust particles, assuming the dustiness everywhere as just found, is

$$NA = 1734 \times 10^{21}.$$

But the radius of each particle is 92×10^{-6} cm., and its volume, assuming it spherical, 33×10^{-13} cubic centimetre. Hence the total volume of the dust, assuming the particles spherical, is equal, roughly, to a cube 179 metres, or about 587 feet, on the side, an amount that certainly is not prohibitively large.

As just stated, the total quantity of dust sufficient, as explained, to cut down the intensity of the direct solar radiation by 20 per cent., and therefore, if indefinitely continued, capable, presumably, of producing an ice age, is astonishingly small—only the 174th part of a cubic kilometre, or the 727th part of a cubic mile, even assuming that the particles are spherical. Since, however, in large measure, the particles are more or less flat, it follows that the actual total mass of the dust necessary and sufficient to reduce the intensity of direct solar radiation by 20 per cent. probably is not more than the 1500th part of a cubic mile, or the 350th part of a cubic kilometre.

Hence, even this small amount of solid material distributed once a year, or even once in two years, through the upper atmosphere, would be more than sufficient to maintain continuously, or nearly so, the low temperature requisite to the production of an ice age; nor would it make any great difference where the volcanoes productive of the dust might be situated, provided only that it was driven high into the isothermal region or stratosphere, since, from whatever point of introduction, the winds of the upper atmosphere would soon spread it more or less evenly over the entire earth.

A little calculation shows, too, that this quantity of dust yearly, during a period of 100,000 years, would produce a layer over the earth only about half a millimetre, or one-fiftieth of an inch, thick, and therefore one could hardly expect to find any marked accumulation of it, even if it had filled the atmosphere for much longer periods.

Whether periods of explosive volcanic activity—and in this case, since the locality of the volcano is a matter of small importance, the whole earth must be considered—occurred at such

times as to synchronize with the ice ages and with other epochs of great climatic change is, of course, a problem for the geologist to solve. However, this much appears well-nigh certain: Since the beginning of reliable records, say 160 years ago, the average temperature of the earth has been perceptibly lower, possibly as much as 0.5° C., than it would have been if during all this time there had been no volcanic explosions violent enough to put dust into the isothermal region of the atmosphere. Similarly, on the other hand, if, during this period, violent volcanic explosions had been three or four times more numerous than they actually were, the average temperatures probably would have been 1° C. to 2° C. lower, or low enough, if long continued, to depress the snow line roughly 300 metres, and thus to begin a moderate ice age.

Effect of Dust on the Interzonal Gradient.—If I is the initial intensity of radiation of a given wave-length and aI its intensity after passing a unit distance through a homogeneous absorbing or scattering medium, then its remaining intensity, after traveling n units distance through this medium, will be $I\ a^n$. But n, in the case of solar radiation passing through the atmosphere, is proportional to the secant of the zenith distance of the sun; and from this in turn it is evident that, in general, variations of dust in the upper atmosphere must change the temperatures of the high latitude regions more than these within the Tropics. Hence, an increase of such dust would steepen the interzonal temperature gradients, strengthen the winds, and make heavier the rain and snowfall, a condition favorable to extensive glaciation. Of course, the increased circulation would, in turn, more or less reduce the new temperature difference, but, nevertheless, a portion, at least, of the increase clearly would remain, and with it the corresponding increases of wind and rain.

Chapter IV.

VULCANISM: OBSERVATIONAL.

It will be interesting and profitable now to consider the supplementary portion of the theory of the relation of vulcanism to climate. That is, to consider the observational evidence, pyrheliometric or other kind, bearing on the effect of volcanic dust on solar radiation, and thus obtain some idea of those absolute values essential to even a rough determination of the climatic consequence of volcanic dust in the high atmosphere.

Pyrheliometric Records.—Direct measurement of solar radiation by means of the pyrheliometer, an instrument that measures the total heat of sunshine, shows marked fluctuations from year to year in the intensity of this radiation as received at the surface of the earth. This subject has been carefully studied by Dr. H. H. Kimball,[26] of the United States Weather Bureau, who prepared the accompanying table, graphically represented by Fig. 190. Since the yearly values are given in terms of the average value for the entire period, it is obvious that percentages of this average do not represent the full effect of the disturbing causes, of which volcanic dust certainly is the chief.

The following table of intensities was computed from observational data obtained at the following stations:

Montpellier, France, monthly means (noon values)	1883–1900
Pavlovsk, Russia, monthly maxima	1892–1913
Lausanne, Switzerland, monthly means (noon values)	1896–1904
Warsaw, Russia, monthly maxima	1901–1913
Washington, D. C., and Mount Weather, Va., monthly means for air mass 2.0	1905–1913
Simla, India, monthly means (noon values)	1906–1913
Paris, France, monthly maxima	1907–1913

The marked decrease in the pyrheliometric readings for 1884, 1885, and 1886 doubtless were largely, if not almost wholly, due to the eruption of Krakatoa in the summer of 1883; the decreased values of 1888 to 1892, inclusive, occurred during a period of exceptional volcanic activity, but were probably due essentially to the violent eruptions of Bandaisan (1888), Bogoslof (1890), and

[26] *M. W. R.*, 46, p. 355, 1918.

Awoe, on Great Sangir (1892); the low values of 1903 to the eruptions of Santâ Maria (1902), Pelé (1902) and Colima (1903); and the low values of 1912–1913, to the explosion, June 6, 1912, of Katmai. The slight depression in the curve cor-

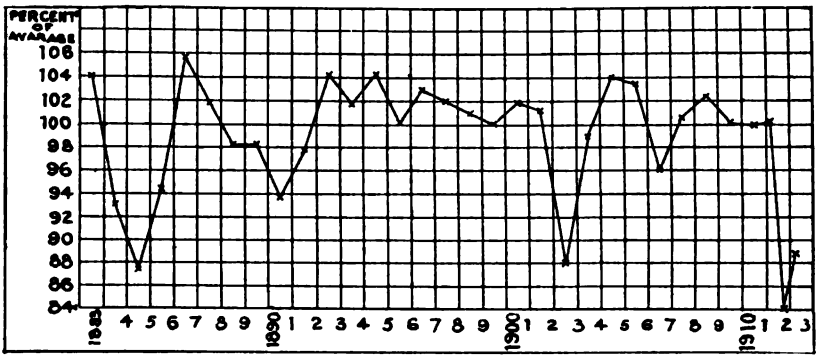
Year	Number of stations	Radiation
1883	1	103
1884	1	92
1885	1	89
1886	1	96
1887	1	105
1888	1	101
1889	1	100
1890	1	96
1891	1	95
1892	2	99
1893	2	104
1894	2	102
1895	2	103
1896	3	103
1897	3	103
1898	3	104
1899	3	103
1900	3	101
1901	3	102
1902	3	99
1903	3	88
1904	3	96
1905	3	100
1906	3	102
1907	5	98
1908	5	99
1909	5	102
1910	5	102
1911	5	103
1912	5	92
1913	5	93

responding to the year 1907, during which no violent eruptions were reported (this does not exclude the possibility of such occurrence in remote and unfrequented regions), according to Dr. Kimball, probably was caused by local haze at Washington, D. C., where his observations were made, and elsewhere, and this supposition is partially supported by the fact that his values for

the year were not uniformly low, and by the further fact, inferred from a publication by Gorczynski,[27] that during that year the solar radiation was but little below normal at Warsaw, Russia.

There is, then, abundant pyrheliometric evidence that volcanic dust in the upper atmosphere actually does produce that decrease in direct solar radiation that theory indicates it should, and, as the theory is well founded and the observations were carefully taken, this mutual confirmation may be regarded as conclusive both of the existence of volcanic dust in the upper atmosphere (isothermal region) and of its efficiency in intercepting direct radiation from the sun.

FIG. 190.



Annual average, pyrheliometric values.

It should be remembered, however, in this connection, that the intensity of the solar radiation at the surface of the earth depends not only upon the dustiness of the earth's atmosphere, but also upon the dustiness, and, of course, the temperature, of the solar atmosphere.

Obviously, dust in the sun's envelope must more or less shut in solar radiation just as, and in the same manner that, dust in the earth's envelope shuts it out. Hence it follows that when this dust is greatest, other things being equal, the output of solar energy will be least, and when the dust is least, other things being equal, the output of energy will be greatest. Not only may the intensity of the emitted radiation vary because of changes in the transparency of the solar atmosphere, but also because of any variations

[27] *C. R.*, 157, p. 84, 1913.

in the temperature of the effective solar surface, which, it would seem, might well be hottest when most agitated, or at the times of spot maxima, and coolest when most quiescent, or at the times of spot minima.

Now, the dustiness of the solar atmosphere, manifesting itself as a corona, certainly does vary through a considerable range from a maximum when the sun-spots are most numerous to a minimum when they are fewest, and, therefore, partly because of changes in the transparency of the solar envelope, and partly because of changes in the solar surface temperatures, if, as in all probability they do, such temperature changes take place, we should expect the solar constant also to vary from one value at the time of spot maximum to another at the time of spot minimum, and to vary as determined by the controlling factor, dust or temperature.

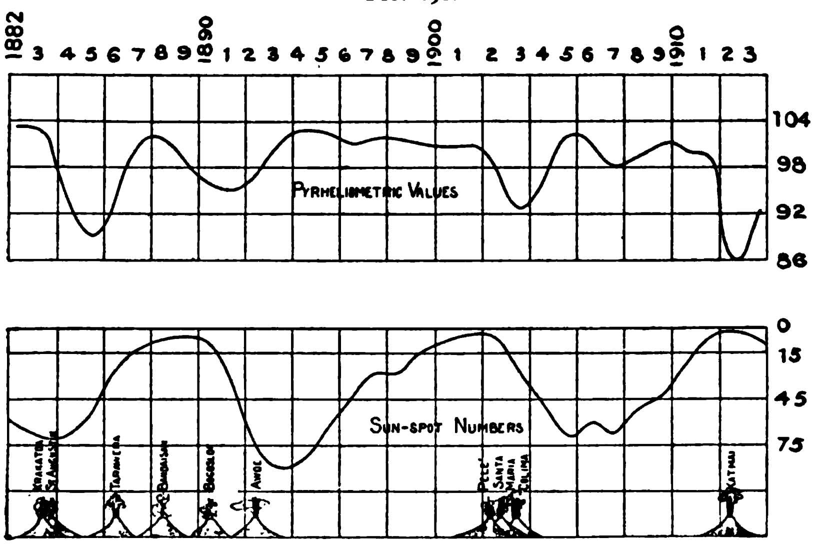
If the above reasoning is correct, it follows that pyrheliometric readings are functions of, among other things, both the solar atmosphere and our own terrestrial atmosphere; and as the former is altered chiefly by sun-spots or at least varies with their production and existence, and the latter by volcanic explosions, a means is at hand for comparing the relative importance of the two radiation screens.

Fig. 191 shows one such comparison. The upper curve gives smoothed annual average pyrheliometric readings (not solar constants, though closely proportional to them) and the lower curve sun-spot numbers. It will be noticed that in their most pronounced features the two curves have but little in common, and that the great drops in the pyrheliometric values occur simultaneously with violent volcanic explosions, as already explained, and not at the times of sun-spot changes. *Hence it appears that the dust in our own atmosphere, and not the condition of the sun, is a very important, if not the controlling, factor in determining the magnitudes and times of occurrence of great and abrupt changes of insolation intensity at the surface of the earth.*

Temperatures at the Surface of the Earth.—If a veil of dust actually should intercept as much as one-fifth of the direct solar radiation, as Fig. 190 indicates that at times it does, it would seem that in those years the temperature of the atmosphere at the surface of the earth should be somewhat below the normal. Of course, the great supply of heat in the ocean would produce a lag

in this effect, particularly over the oceans themselves, and, besides, there must be both an increase of sky light by scattering and some interception of earth radiation by the dust which, since it is at great altitudes, receives the full, or nearly the full, planetary radiation of the earth. This increase of sky radiation, together with the return terrestrial radiation, obviously compensates in some measure for the loss of direct insolation. However, measurements made by Abbot [28] at Bassour, Algeria, during the summer of 1912, show that at this time and place the direct radia-

FIG. 191.



Relation of pyrheliometric values to sun-spot numbers and volcanic eruptions.

tion and the sky radiation, which obviously included both the scattered solar radiation and some return terrestrial radiation, were together less by about 10 per cent. than their normal combined values; and there is no reason to think that in this respect Bassour was at all different from other places, certainly a large portion of the northern hemisphere, at least, covered by the veil of dust. Clearly, then, if this decrease in the radiation received were universal and should continue indefinitely, the ultimate radiation of the earth would also decrease to the same extent, or 10 per cent. Now, since the earth, or rather the water vapor of

[28] *Smithsonian Miscellaneous Collections,* vol. lx, No. 29, 1913.

the atmosphere, radiates substantially as a black body and, therefore, proportionally to the fourth power of its absolute temperature, it follows that a 10 per cent. change in its radiation would indicate about a 2.5 per cent. change in its temperature. But the effective temperature of the earth as a full radiator, which it closely approaches, is about 256° A.[29] Hence a change of 10 per cent. in the radiation emitted would imply 6.4° C. change in temperature, an amount which, if long enough continued, would be more than sufficient to produce glaciation equal probably to the most extensive of any known ice age.

As above implied, not much lowering of the temperature could be expected to take place immediately; however, some early cooling over land areas might well be anticipated. To test this point the temperature records of a number of high altitude (together with two or three very dry) inland stations have been examined. High altitudes were chosen because it was thought that the temperature effects of dust in the upper atmosphere probably are most clearly marked above the very and irregularly dusty layers of the lower atmosphere; and the condition that the stations should also be inland was imposed because these are freer, presumably, than many coast stations, from fortuitous season changes. Thus, stations in the eastern portion of the United States were rejected because of the great differences in the winters, for example, of this section depending upon the prevailing direction of the wind,[30] a condition wholly independent, so far as known, of variations in the intensity of direct radiation.

The number of stations was still further limited by the available recent data. Hence the records finally selected, and kindly put in shape by the Climatological Division of the United States Weather Bureau, Mr. P. C. Day in charge, were obtained at the following places:

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[29] Abbot and Fowle, *Annals Astrophys. Obsy.*, Smithsonian Institution, vol. ii. p. 175, 1908.

[30] Humphreys, *Monthly Weather Review*, vol. xlii, p. 672, 1914.

TABLE II.
Stations Whose Data Were Used.

AMERICA.

Name.	Latitude.	Longitude.	Elevation in feet.
Baker	44° 46′ N.	117° 50′ W.	3,466
Bismarck	46° 47′ N.	100° 38′ W.	1,674
Cheyenne	41° 08′ N.	104° 48′ W.	6,088
Denver	39° 45′ N.	105° 00′ W.	5,291
Dodge City	37° 45′ N.	100° 00′ W.	2,509
El Paso	31° 47′ N.	106° 30′ W.	3,762
Helena	46° 34′ N.	112° 04′ W.	4,110
Huron	44° 21′ N.	98° 14′ W.	1,306
North Platte	41° 08′ N.	100° 45′ W.	2,821
Red Bluff	40° 10′ N.	122° 15′ W.	332
Sacramento	38° 35′ N.	121° 30′ W.	69
Salt Lake City	40° 46′ N.	111° 54′ W.	4,360
San Antonio	29° 27′ N.	98° 28′ W.	701
Santa Fe	35° 41′ N.	105° 57′ W.	7,013
Spokane	47° 40′ N.	117° 25′ W.	1,929
Winnemucca	40° 58′ N.	117° 43′ W.	4,344
Yuma	32° 45′ N.	114° 36′ W.	141
EUROPE.			
Mont Ventoux	44° 10′ N.	5° 16′ E.	6,234
Obir	46° 30′ N.	14° 29′ E.	6,716
Pic du Midi	42° 56′ N.	0° 8′ E.	9,380
Puy de Dôme	45° 46′ N.	2° 57′ E.	4,813
Säntis	47° 15′ N.	9° 20′ E.	8,202
Schneekoppe	50° 44′ N.	15° 44′ E.	5,359
Sonnblick	47° 3′ N.	12° 57′ E.	10,190
INDIA.			
Simla	31° 6′ N.	77° 12′ E.	7,232

In Table III the first column gives the year in question. The second column gives the average departure in degrees F., for the seventeen American stations, of the annual average maximum, as determined from the monthly average maxima, from the normal annual maximum, or average of a great many annual average maxima. The third column gives smoothed values, determined from the actual values in the second column as follows:

$$S = \frac{a + 2b + c}{4},$$

in which S is the smoothed value, b the actual value pertaining to the particular year for which S is being computed, a and c the actual values for the next previous and the next succeeding years, respectively. The fourth and fifth columns give, respectively, the actual and the smoothed average departures of the annual average minima, while the sixth and seventh columns give the corresponding average departures of the annual average means.

TABLE III.
Average Temperature Departures from Temperature Normals.
AMERICA.

Year.	Maxima.		Minima.		Means.	
	Actual.	Smoothed.	Actual.	Smoothed.	Actual.	Smoothed
1880	−1.3	+0.03	−1.8	−0.68	−1.7	−0.50
1881	+0.2	−0.30	+0.6	−0.20	+0.1	−0.48
1882	−0.3	−0.50	−0.2	−0.20	−0.4	−0.50
1883	−1.6	−1.33	−1.0	−0.70	−1.3	−1.15
1884	−1.8	−1.20	−0.6	−0.28	−1.6	−1.05
1885	+0.4	−0.18	+1.1	+0.43	+0.3	−0.30
1886	+0.3	+0.35	+0.1	+0.10	−0.2	−0.03
1887	+0.4	+0.38	−0.9	−0.45	0.0	+0.07
1888	+0.4	+0.53	−0.1	−0.13	+0.5	+0.53
1889	+0.9	+0.63	+0.6	+0.23	+1.1	+0.85
1890	+0.3	+0.15	−0.2	−0.05	+0.7	+0.58
1891	−0.9	−0.58	−0.4	−0.38	−0.2	+0.05
1892	−0.8	−0.85	−0.1	−0.33	−0.1	−0.20
1893	−0.9	−0.73	−0.7	−0.38	−0.4	−0.08
1894	−0.3	−0.55	+0.4	−0.18	+0.6	+0.13
1895	−0.7	−0.35	−0.8	−0.08	−0.3	+0.25
1896	+0.3	−0.18	+0.9	+0.28	+1.0	+0.45
1897	−0.6	−0.30	+0.1	+0.13	+0.1	+0.28
1898	−0.3	−0.65	−0.6	−0.45	−0.1	−0.13
1899	−0.8	−0.13	−0.7	−0.10	−0.4	+0.25
1900	+1.4	+0.78	+1.6	+0.90	+1.9	+1.23
1901	+1.1	+0.83	+1.1	+1.08	+1.5	+1.35
1902	−0.3	−0.13	+0.5	+0.38	+0.5	+0.53
1903	−1.0	−0.43	−0.6	−0.05	−0.4	+0.18
1904	+0.6	−0.15	+0.5	+0.05	+1.0	+0.38
1905	−0.8	−0.30	−0.2	+0.08	−0.1	+0.33
1906	−0.2	−0.30	+0.2	+0.08	+0.5	+0.33
1907	0.0	+0.10	+0.1	+0.10	+0.4	+0.50
1908	+0.6	+0.15	0.0	−0.08	+0.7	+0.43
1909	−0.6	+0.38	−0.4	−0.05	−0.1	+0.55
1910	+2.1	+0.80	+0.6	+0.08	+1.7	+0.75
1911	−0.4	+0.03	−0.5	−0.35	−0.3	+0.05
1912	−1.2	−0.70	−1.0	−0.63	−0.9	−0.53

Fig. 192 shows the graphical equivalents of the smoothed portions of Table III.

It will be noticed that the three curves of Fig. 192, marked maximum, minimum, and mean, respectively, are, in general, quite similar to each other. Hence, because of this mutual check and general agreement, it seems reasonably certain that any one set of temperature data, the means, for instance, furnishes a fairly safe guide to the actual temperature and climatic fluctuations from year to year or period to period.

Table IV gives the weighted actual average departures and the smoothed departures in degrees F. of the annual mean temperatures of the selected seventeen American, seven European, and one Indian stations listed in Table I.

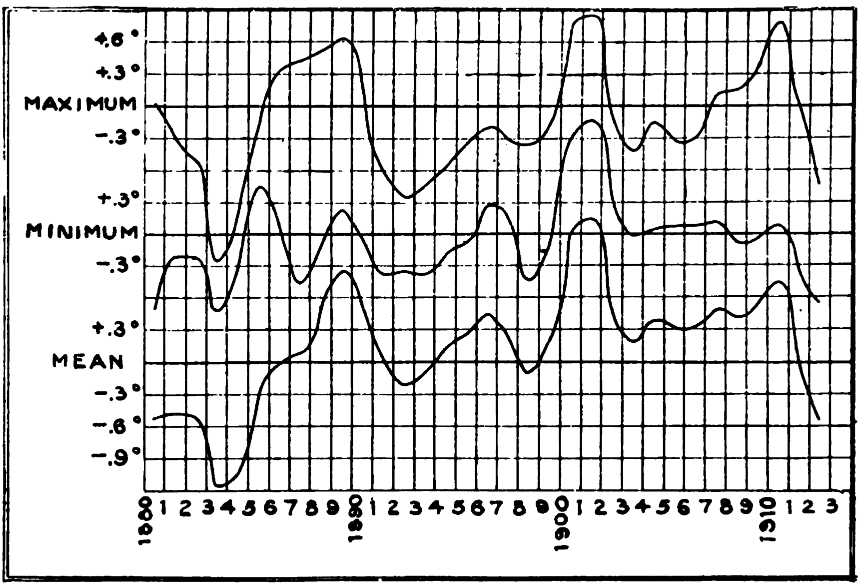
TABLE IV.

Weighted Departures of Mean Temperatures from Normal Temperatures.

WORLD.

Date.	Actual.	Smoothed.	Date.	Actual.	Smoothed.
1872	−0.78	−0.30	1893	−0.34	−0.06
1873	−0.65	−0.47	1894	+0.34	+0.03
1874	+0.20	−0.34	1895	−0.21	+0.10
1875	−1.12	−0.61	1896	+0.49	+0.28
1876	−0.40	−0.60	1897	+0.34	+0.45
1877	−0.48	−0.32	1898	+0.61	+0.46
1878	+0.07	0.00	1899	+0.27	+0.59
1879	+0.33	+0.04	1900	+1.19	+0.76
1880	−0.50	−0.13	1901	+0.40	+0.55
1881	+0.14	−0.02	1902	+0.20	+0.13
1882	+0.14	−0.16	1903	−0.30	+0.10
1883	−1.04	−0.68	1904	+0.81	+0.20
1884	−0.79	−0.61	1905	−0.51	+0.01
1885	+0.17	−0.09	1906	+0.23	+0.05
1886	+0.11	+0.03	1907	+0.23	+0.30
1887	−0.29	−0.05	1908	+0.51	+0.21
1888	+0.26	+0.24	1909	−0.43	+0.11
1889	+0.74	+0.57	1910	+0.69	+0.30
1890	+0.54	+0.40	1911	+0.23	+0.09
1891	−0.21	+0.06	1912	−0.80	−0.40
1892	+0.10	−0.09			

FIG. 192.



Smoothed averages of the annual average temperature departures of 17 American stations.

The average departures were calculated in accordance with the more or less correctly coefficiented equation,

$$D = \frac{4A + 2E + I}{7},$$

in which D is the weighted departure, A the smoothed average American, E the smoothed average European, and I the smoothed Indian, departure of the mean annual temperature from the normal annual temperature.

Table IV, extended, as well as the scanty early data, mainly from the given stations, will permit, back to 1872, is graphically represented by the continuous, light curve at the bottom of Fig. 181. In 1880 and again in 1901 the curve probably does not very closely represent world-wide temperature departures, being, presumably, at both places quite too low, owing, in each case, to an abnormally cold single month in America.

The dotted curve from 1907 to 1911 gives the average temperature departures for the American stations only, and presumably represents world temperature departures much more closely than does the continuous light line for the same time. This is because of two or three exceptionally cold summer months in Europe.

The dotted curve from 1872 to 1900 gives the smoothed averages of the annual temperature departures from the normal temperatures of the following stations as computed from the actual departures given by Nordmann[81]; Sierra Leone, Recife (or Pernambuco), Port au Prince, Trinité, Jamaica, Habana, Manila, Hong Kong, Zikawei, Batavia, Bombay, Island of Rodriguez, Island of Mauritius.

All these, or practically all, are low-level stations, and most of them either tropical or semi-tropical, and, therefore, should show in general, from altitude influence alone, a smaller, and from latitude influence alone, a greater, abnormality than do the stations whose temperature departures are given by the continuous fine-line curve. Hence, all things considered, the average temperature departures as calculated from the two sets of stations agree remarkably well, so that one can say with, presumably, a fair degree of confidence, that the heavy curve, T, approximately represents the average of the departures of the mean annual temperatures from the normal annual temperatures of equatorial and

[81] *Revue Générale des Sciences,* August, 1903, pp. 803–808. Annual Report, Smithsonian Institution, 1903, pp. 139–149.

high altitude regions of the earth, or that T, with the above restrictions, is the curve of world temperatures.

Much additional statistical evidence bearing on this point and supporting the conclusion just given, has been published by Mieike.[32] This consists of the average annual temperatures from 1870 to 1910 of 487 widely distributed stations, with, however, numerous and extensive breaks—in fact, the records of only a few stations cover the entire period. By grouping these stations according to zones, tropical, subtropical, warm temperate, cold temperate and frigid, and then averaging and smoothing the zonal annual temperature departures, giving all equal weight, values were found which run substantially parallel to those already found but of less (about half) amplitude, quite as anticipated from the fact that stations above the dust, fogs and many clouds of the lower atmosphere must be more sensitive to variations in the transparency of the outer atmosphere and to solar changes than are those (the great majority) located at, or not more than a few hundred metres above, sea level. Either curve might, therefore, be used in a discussion of the causes and periods of temperature changes, but in what follows the curve of larger amplitudes or the curve of high altitude stations will be used because: (*a*) data for it but not for the other are available through the period of the Katmai veil of dust, (*b*) it is freer from surface disturbances and therefore more representative of solar and high atmospheric conditions, (*c*) high altitude temperatures are more effective than those of sea level in modifying glacial conditions.

Relation of World Temperatures to Pyrheliometric Values.—Curve *P*, also of Fig. 193, gives the smoothed course of the annual average pyrheliometric readings, as computed from the actual values given in Fig. 190. The insolation intensity data, covering the whole of the depression that had its minimum in 1885, were obtained at a single place, Montpellier, France, by a single observer, L. J. Eon,[33] who confined himself to noon observations with a Crova actinometer. It may be, therefore, that merely local and temporary disturbances produced a local insolation curve that was not quite parallel to the curve for the entire world. At any rate, the drop in the solar radiation values obviously was due to dust put into the atmosphere by the explosion of Krakatoa in August, 1883, and it would seem that the effects of this dust both

[32] *Aus dem Archiv der Deutschen Seewarte.* 36, Nov. 3, 1913.

[33] *Bulletin météorologique du Département de l'Hérault*, 1900.

on the surface temperatures and on pyrheliometric values must have been greater during the latter part of 1883 and in 1884 than they were in 1885, when much of the dust certainly had already settled out of the atmosphere, and this supposition is well supported by the pyrheliometric and temperature drops that immediately followed the volcanic explosions of 1903 and 1912, and their partial recovery within a single year. Nevertheless, the pyrheliometric values must be accepted as obtained. Indeed, this exceptional lag is not quite unprecedented, since the coldest year following the similar, though more violent, explosion of Asamayama, just one hundred years earlier, was not the year of the explosion, 1783, nor the following year, but 1785.

It is probable that in the earlier, as certainly in the later, of these unusual cases the dust was thrown to such great altitudes that the finer portions were nearly, or quite, two years in reaching the lower level of the isothermal region. Clearly, too, much of this dust, while perfectly dry, probably was so fine as merely to scatter even solar radiation, and yet on reaching the more humid portions of the atmosphere the particles may have gathered sufficient moisture to assume reflecting size, and, therefore, seriously to interfere with insolation. This is merely suggested, but in no wise insisted upon, as a possible explanation of the unusual pyrheliometric lag after the explosion of Krakatoa.

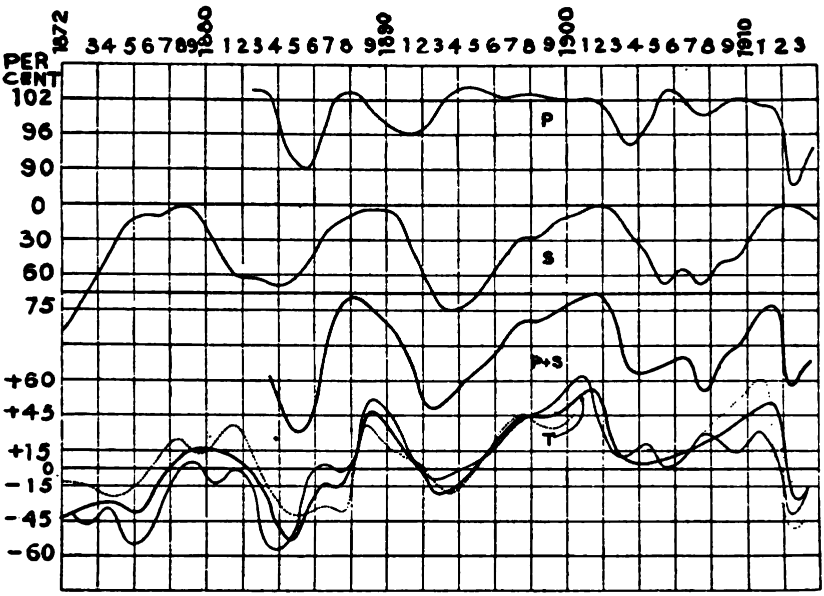
It is obvious, from a mere glance, that the pyrheliometric and the temperature curves, or curves P and T, have much in common. This is especially marked by the large and practically simultaneous drops in the two curves in 1912, following the eruption of Katmai. But while a relation between these curves thus appears certain, the agreement is so far from perfect as to force the conclusion that pyrheliometric values constitute only one factor in the determination of average world temperatures.

Sun-spots and Temperature.—It has been known for a long time that the curve of sun-spot numbers, curve S, Fig. 193, and the curve of earth temperatures, curve T, follow or parallel each other in a general way, in the sense that the fewer the spots the higher the temperature, with, however, puzzling discrepancies here and there. Both these facts, the general agreement between the phenomena in question and also their specific discrepancies, are well shown by the curves S and T of Fig. 193, and, while the discrepancies are marked, it is obvious that, on the whole, the agreement is quite too close to leave any doubt of the reality of some sort

of connection between sun-spots and atmospheric temperatures, Just how or by what process this relation conceivably may exist will be discussed below.

Combined Effect of Insolation Intensity and Sun-spot Influence on Atmospheric Temperatures.—Since it is obvious that the insolation intensity and the number of sun-spots each exerts an influence on the temperature of the earth, it is clear that some sort of a combination of the two curves *P* and *S* should more closely parallel the temperature curve, *T*, than does either alone. It is

FIG. 193.



Smoothed pyrheliometric, sun-spot, and temperature curves.

probable that the sun-spot effect is not directly proportional to the actual number of spots, but, however this may be, the direct combination of the curves *P* and *S* gives the resultant *P* + *S*, which, as a glance at the figures shows, actually parallels the curve of temperatures, *T*, with remarkable fidelity. Exactly this same combination, from 1880 to 1909, has been made by Abbot and Fowle,[34] whose lead in this important particular is here being followed, and the resultant curve found to run closely parallel to the curve of "smoothed annual mean departures" of the maximum temperatures of fifteen stations in the United States.

[34] *Smithsonian Miscellaneous Collections*, vol. lx, No. 29, 1913.

Probably the most striking point of agreement, one that must strongly be insisted upon, as shown by Fig. 193, between the combination curve and the temperature, occurs in 1912, when, in spite of the fact that the sun-spots were at a minimum, indicating that, according to rule, the temperature should be high, the temperature curve dropped greatly and abruptly; obviously, because of the simultaneous and corresponding decrease in the intensity of solar radiation produced by the extensive veil of Katmai's dust, precisely as happened at spot minima after the explosion of Asama in 1783. Both cases, since they occurred during spot minima, show distinctly the great influence volcanic dust has on terrestrial temperatures.

Temperature Variations Since 1750 as Influenced by Sun-spots and Volcanic Eruptions.—Sun-spot numbers [85] month by month are fairly well known since July, 1749, and so, too, are the annual temperature variations [86] from about the same time, and, therefore, the data at hand for comparing these two phenomena over a continuous period of a little more than 164 years, or from at least the beginning of the year 1750 to the present date. Fig. 194* makes this comparison easy. The bottom curves give the smoothed annual temperature departures, as computed from Köppen's actual annual departures, using all stations, while the top curve follows Wolfer's annual average sun-spot numbers. Of course, the earlier observations, both of sun-spots and of temperatures, were few in number and more or less unsatisfactory in comparison with those obtained during the past thirty, or even forty, years. Nevertheless, it is clear from Fig. 194 that at least since 1750, the date of our earliest records, and presumably, therefore, since an indefinitely distant time in the past, the two phenomena, atmospheric temperature and sun-spot numbers, have in general varied together, with, however, marked discrepancies from time to time. These we shall now consider, and shall show that they occurred, in every important case, immediately after violent volcanic eruptions.

Volcanic Disturbances of Atmospheric Temperature Since 1750.—It must be distinctly remembered that the earlier temperature records, because of their limited number, if for no other reason, give only the general trend of world temperatures. Again,

[85] Wolfer, *Astronomische Mitteilungen*, 93, 1902, and later numbers.

[86] Köppen, *Zeit. Oesterreich. Gesell. für Meteorologie*, vol. viii, pp. 241 and 257, 1873.

* Fig. 194 folded plate, attached to inside of back cover.

the record back to 1750 of even violent volcanic eruptions is necessarily incomplete; and, besides, *not all great eruptions decrease the surface temperature—only those that drive a lot of dust into the isothermal region, or very high atmosphere,* and even then decrease it perceptibly in only those regions, usually extensive and at times world-wide, over which the dust spreads. Pronounced and long-continued sky phenomena, therefore, of the type that followed the eruption of Krakatoa, furnish the best evidence of volcanic violence in the sense here used. Finally, there can be no particular test save where the temperature is low in comparison with that which the number of sun-spots would indicate. Obviously, then, no matter how close the actual relation between the phenomena may be, the errors and the incompleteness of the recorded data would prevent the discovery of more than a general relation.

Of course, it will naturally occur to one to ask about special cases, such as the cold years of 1783–4–5, and, in particular, 1816, the famous "year without a summer," "poverty year," or "eighteen hundred and froze-to-death." The first of these, 1783–5, followed, as already explained, the great explosion of Asama in 1783, while the second, the "year without a summer," that was cold the world over, followed the eruption of Tomboro, which killed 56,000 people [37] and blew up so much dust that "for three days there was darkness at a distance of 300 miles."[38]

There is a detail in the temperature curve for the years 1886–7 that needs special attention. The temporary depression where, seemingly, the temperature should be steadily rising, obviously was due to the great eruption of Tarawera (June 10, 1886), in New Zealand. This volcano is a little more than 38° south of the equator, and, therefore, furnishes a good example of an eruption on one side of the equator affecting the temperature far to the other side. Doubtless, however, when the dust gets but a little way into the isothermal region the effect is greatest on the volcano's side of the equator.

But if the temperature was decreased by Tarawera, why, one might ask, was not the pyrheliometric curve similarly affected? It was, for several months after the eruption, as the individual monthly values show,[39] but the annual means, plotted in the figure,

[37] Schneider, "Die Vulcanischen Erscheinungen der Erde," p. 1, 1911.

[38] *Rept. Krakatoa Committee Royal Society*, 1888, p. 393.

[39] *Bulletin Météorologique du Département de l'Hérault*, p. 136, 1900.

have the effect of making the pyrheliometric disturbance from Tarawera appear only as a retardation in the recovery from the effects of Krakatoa.

Neglecting the smaller irregularities which may or may not have been of world-wide occurrence, and remembering that, other things being equal, temperature maxima are to be expected at the times of spot minima and temperature minima at the times of spot maxima, the marked discrepancies and their probable explanations may be tabulated as follows:

Temperature and Sun-spot Discrepancies.

Date.	Nature of discrepancy.	Probable cause.
1755–6	Cold	Kötlugia, Iceland, 1755, Oct. 17.
1766–7	Cold	Hecla, Iceland, 1766. Apr. 15 to Sept. 7. Mayon, Luzon, 1766.
1778–9	Warm	Maximum number (annual) of sun-spots ever recorded and unusually short spot period. Can it be that the solar constant actually was notably greater than usual at this time?
1784–5–6	Cold	Asama, Japan, 1783, most frightful eruption on record. Skaptar Jökull, Iceland, 1783, June 8 and 18. Vesuvius, Italy, 1785.
1799	Cold	Fuego (?), Guatemala. (Uncertain.)
1809	Cold	St. George (?), Azores, 1808. (Uncertain.) Etna (?), Sicily, 1809. (Uncertain.)
1812–13–14–15–16	Cold	Soufrière, St. Vincent, 1812, Apr. 30. Mayon, Luzon, 1814. Tomboro, Sumbawa, 1815, Apr. 7 to 12, very great.
1831–2	Cold	Graham's Island, 1831, July 10 to early in Aug. Babujan Islands, 1831. Pichincha, Ecuador, 1831.
1836–7–8	Cold	Coseguina Nicaragua, 1835, Jan. 20. Awatska, Kamchatka, 1837.
1856–7	Cold	Cotopaxi (?), and others, 1855–6. (Uncertain.)
1872–3	Cold	Vesuvius, Italy, 1872, Apr. 23 to May 3. Merapi, Java, 1872, April.
1875–6	Cold	Vatna Jökull, Iceland, 1875, March 29 and during April.
1884–5–6	Cold	Krakatoa, Straits of Sunda, 1883, Aug. 27, greatest since 1783. Saint Augustin, Alaska, 1883, Oct. 6. Tarawera, New Zealand, 1886, June 10.
1890–1–2	Cold	Bogoslof, Aleutian Islands, 1890, Feb. Awoe, Great Sangir, 1892, June 7.
1902–3–4	Cold	Pelé, Martinique, 1902, May 8. Santa Maria, Guatemala, 1902, Oct. 24. Colima, Mexico, 1903, Feb. and Mar.
1912–13	Cold	Katmai, Alaska, 1912, June 6.

For the sake of completeness as well as for such little value and interest it may have the following list is added of still earlier great volcanic eruptions and the kinds of seasons that history[40] reports to have followed. Taken by themselves these ancient or preinstrumental records help but little to connect effect with cause. Nevertheless, it is at least pleasing to know that they report precisely those general weather or seasonal conditions which from later and reliable instrumental observations we infer must have happened, provided only that the explosions were sufficiently great.

Date of eruption.	Volcano.	Type of season, etc.
79, Aug. 24	Vesuvius. (Destruction of Pompeii.)	About 80, severe drought for several years in middle Asia. (Accords with low temperature but signifies very little.)
1631, Dec. 16	Vesuvius. (Most violent since 79. Height of ash cloud, measured by Braccini, 48 kilometres.)	1632, Apr. 27, destructive snow in Transylvania (Siebenbürgen); May 17, frost in Saxony; hot dry summer in Italy; Oct. 4, very cold in France, 37 soldiers frozen between Montpellier and Baziers. 1633, severe winter; May 22, snow and severe cold in Transylvania.
1636, May 18 to winter	Hecla	1637, long, severe winter.
1680	Celebes	1681, severe drought and cold spring (Evelyn's Diary).
1693, Feb. 13 till Aug.	Hecla	1694, hard, snowy winter in both Italy and Spain; May 17, all vineyards of Troyes destroyed by frost.
1694	Celebes	
1694	Amboyna	1695, long, severe and dry winter; cool summer.
1694, Nov. 20 till 1695, April	Gunong Api	
1707, May 20 to Aug.	Vesuvius	1708, very mild winter; cold summer; Dec. 10 to 20, very heavy snow in France.
1707	Japanese volcano	1709, from Jan. 6, for a month and a half extraordinary severe cold in nearly all Europe; Adriatic Sea and Thames frozen; snow 10 feet deep in Spain and Portugal; 50 days frost in England from Dec. 25, 1708 till March 12, 1709; mild winter in Constantinople, but very severe in eastern North America; May 17, snow in Oedenburg; May 17 and 18, frost in Alsace; cool, rainy summer.
1707, May 25 and July 25	Santorin	
1721, May 11 till autumn	Kotlugia	1722, cool, wet year.

[40] Hennig, *Abhand, des K. Preuss. Meteor. Inst.*, Bd. II, No. 4, 1904.

Confining attention to the first of the above lists, since the value of the second in this connection is doubtful, it will be seen that excepting some ill-defined cases, all of the seeming irregularities in the temperature curve, and all of the known volcanic eruptions, are satisfactorily accounted for.

It may be concluded, therefore, that the variations in the average temperature of the atmosphere of the kind and magnitude shown by actual records depend jointly upon volcanic eruptions, through the action of dust on radiation, as already explained, and upon sun-spot numbers, through, presumably, some intermediate action they have upon the atmosphere—possibly of the nature explained in the next chapter.

Magnitude and Importance of Actual Temperature Changes.—The actual temperature range from sun-spot maximum to sun-spot minimum varies, roughly, from 0.5° C. to 1° C., or possibly more, while the effect of volcanic dust appears to be fully as great—on rare occasions even much greater. In some ways, and in respect to many things, a range of average temperatures of even 1° C. is well-nigh negligible, and, therefore, however important the results may seem to the scientist, the ultra-utilitarian would be justified in asking, "What of it?"

Much of it, in a distinctly practical as well as in a purely scientific sense, as is true of every fact of nature. For instance, during the summer, or growing season, a change of 0.5° C. produces a latitude shift of the isotherms by fully 80 miles. Hence, if there is but little or no volcanic dust to interfere, during sun-spot minima cereals and other crops may be successfully grown 50 to 150 miles farther north (or south in the southern hemisphere) than at the times of sun-spot maxima. This alone is of great practical importance, especially to those who live near the thermal limits of crop production.

In addition to changing the area over which crop production is possible, a change of average temperature also affects, in some cases greatly, the time of plant development. Thus Walter [41] has shown that a change of only 0.7° C. may alter, and in Mauritius has been observed actually to alter by as much as an entire year, the time required for the maturing of sugar cane. Hence the temperature changes that normally accompany sun-spot varia-

[41] "On the Influence of Forests on Rainfall and the Probable Effect of 'Déboisement' on Agriculture in Mauritius" (1908).

tions, though small in absolute magnitude, are of great importance, and, by availing ourselves of the reasonable foreknowledge we have of these changes, may easily be made of still greater importance.

In forecasting these small but important climatic changes it must be distinctly remembered that to the fairly periodic, and therefore predictable, sun-spot influence must be added the irregular and unpredictable volcanic effects. But even here the case is not bad for the forecaster, because the volcanic dust always produces, qualitatively, the same effect—a cooling—and because both the amount of this cooling and its duration (generally only one or two years, as already explained) may approximately be estimated from the nature of the volcanic explosion itself.

Chapter V.

OTHER FACTORS OF CLIMATIC CONTROL.

(9, 10, 11, 12, 13, 14.)

9. SUN-SPOTS.

As already stated, the average temperature of the earth as a whole varies inversely with the frequency of sun-spots.

How Sun-spots May Change Earth Temperatures.—If the solar constant should remain the same from spot maximum to spot minimum it clearly would not be easy to see at a glance why the surface temperature of the earth should vary as it does with spot numbers; and the situation is still more difficult if, as observations appear to indicate, the lowest temperatures occur when the solar constant is greatest and the highest temperatures occur when this constant is least. There is, however, a possible explanation of the paradox, and, while it may not contain the whole truth, it nevertheless is sufficient to show *a priori* that in all probability our temperatures do change from spot maxima to spot minima without a corresponding change in the solar constant, and also to show that a decrease in our surface temperatures may accompany even a slight increase in the solar constant.

The explanation in question has already been given elsewhere,[42] and the original paper must be consulted by those who wish to weigh all the details of the argument. Briefly, however, the argument is as follows:

1. At the times of spot maxima the solar corona is much more extensive than it is at the times of spot minima—a well-known observation.

2. This corona consists, in part at least, of reflecting particles, as many eclipse observations have shown, and so may be regarded as dust in the solar atmosphere.

3. The brightness of the sun, as every solar observer knows, drops off from centre to limb.

4. This drop, as reported by various observers, is greater the shorter the wave-length, and due, almost certainly, to diffuse scattering.

From the observational facts it follows that during spot

[42] Humphreys, *Astrophys. Jr.*, 32, p. 97, 1910.

minima, other things being equal, the solar spectrum must necessarily be richer in violet and ultra-violet radiation than it is during spot maxima.

But, as experiment has shown,[43] ultra-violet radiation of shorter wave-length than λ 1850 is strongly absorbed by oxygen, with the result that some of the oxygen is converted into ozone. Hence, since the atmosphere of the isothermal region is cold and dry (conditions favorable to the stability of ozone) and since of the gases of the upper atmosphere only oxygen is appreciably absorptive of radiations between λ 1250 and λ 1900 [44] the stratosphere was believed to contain more or less ozone, a belief now fully confirmed by several observers,[45] and long ago virtually confirmed by Ångström.[46] In so far, then, as this ozone is produced by the action of ultra-violet solar radiation, it is also logical to expect it to be greater in quantity when the short wave-length radiation, to which it is due, is most intense, or, presumably, therefore, at the times of spot minima. Now, according to the experiments of Ladenburg and Lehmann,[47] while ozone is somewhat absorptive of solar radiation it is several-fold more absorptive, in fact, highly absorptive, of terrestrial radiation. Hence, in this case, as in the case of the absorption of radiation by dust, already considered, equation *A*, page 571, is applicable.

In this equation let a be the coefficient of absorption of the ozone in the isothermal region for solar radiation, and b its coefficient of absorption for earth radiation. To be definite, let $a = 0.02$ and $b = 0.10$ at the time of spot maximum, and for a spot minimum let $a = 0.03$ and $b = 0.15$, quantities that would require really very little ozone. Then, since the earth radiates practically as a full radiator, or black body, at the absolute temperature 256° C., if T *max.* and T *min.* are the equilibrium temperatures at the time of spot maximum and spot minimum, respectively,

$$\left(\frac{T\text{max.}}{256}\right)^4 = \frac{521}{500}; \ T\text{max.} = 258^\circ.65$$

[43] Lyman, *Astrophys. Jr.*, 27, p. 87, 1908.

[44] Lyman (*l. c.*).

[45] Fabry and Buisson. *C. R.*. 156. p. 782. 1913: *Journal de Physique*, 3. p. 196, 1913; Fowler and Strutt, *Proc. Roy. Soc. A.*, 93, p. 577, 1917; Fowle, *Smithsonian Miscellaneous Collections V.*. 68. No. 8. 1917.

[46] *Arkiv för Matematik. Astronomi och Fysik*. 1. p. 395, 1904.

[47] *Annalen der Physik*, 21, p. 305, 1906.

and

$$\left(\frac{T\text{min.}}{256}\right)^4 = \frac{2129}{2000};\ T\text{min.} = 260^\circ.05$$

That is, under these conditions, and if the solar constant should remain exactly the same, the temperature at the time of spot minimum would be 1°.4 C. warmer than at the time of spot maximum. Hence, even a slight increase in the solar constant at the time of spot maximum might still leave the temperature a trifle cooler than at the time of spot minimum.

Obviously, too, a greater extent of cirrus and cirrus haze during spot maxima than during spot minima (a condition some observers have thought to exist; and due, perhaps, if it does exist, to auroral effects) would also tend to produce the concurrent temperature difference.

Of course, it is not asserted that either the ozone, or the cirrus haze, actually does vary in the manner assumed—observations are inadequate to decide either question; but as such changes are in complete accord with laboratory experiments, it seems that the possibility of their occurrence in nature, and their effects, should be mentioned.

INFLUENCE OF CARBON DIOXIDE ON TEMPERATURES.

It was stated in the early part of this discussion, under the carbon dioxide theory of ice ages, that the question of the possible effect a change in the amount of carbon dioxide in the atmosphere might have on temperatures would be taken up later. The way to this is now open through the above discussion of ozone. Like ozone, carbon dioxide also is more absorptive of terrestrial radiation than of solar energy. Hence, increasing the carbon dioxide in the atmosphere, and thereby increasing its amount in the isothermal region where it can be treated as a shell external to the radiating earth, obviously must have the same general effect on the temperature of the earth as increasing the ozone of this region would have. That is, other things being equal, a greater or less temperature increase would follow the introduction into the atmosphere of a larger amount of carbon dioxide.

Because of the constant mixing caused by vertical convection, it is probable that the percentage of carbon dioxide is very nearly as great at the under surface of the isothermal region as it is at the surface of the earth. If so, then the carbon dioxide of the

isothermal region is equivalent, roughly, to a layer 40 centimetres thick at normal atmospheric pressure. In high latitudes, where the isothermal level is low, the equivalent layer probably is thicker than this, and in equatorial regions probably thinner. Now, according to the experiments of Schaefer,[48] a layer of carbon dioxide 40 centimetres thick is sufficient to produce very nearly full absorption, and, therefore, no increase in the amount of carbon dioxide in the atmosphere could very much increase its temperature.

An approximate idea of the possible temperature change of the lower atmosphere as a result of the presence of carbon dioxide in the isothermal region can be obtained from known data. Thus, Abbot and Fowle[49] have computed that carbon dioxide may absorb 14 per cent. of the radiation from a black body at the temperature of 282.2° C. absolute. But as this is not many degrees, 25 or so, above the effective temperature of the earth as a radiator, it follows that 14 per cent. is, roughly, the upper limit to which terrestrial radiation can be absorbed by carbon dioxide in the isothermal region, while its absorption of solar radiation is very nearly negligible.

Assuming that the present amount of carbon dioxide in the isothermal region absorbs 1 per cent. of the solar radiation and 10 per cent. of the outgoing earth radiation (values that seem to be, roughly, of the correct order), and using equation *A*, page 571, it will be seen, if the experiments here referred to and the assumptions are substantially correct, that doubling or even multiplying by several-fold the present amount of carbon dioxide, which would leave the absorption of solar radiation practically unchanged, and increase the absorption of terrestrial radiation at most to only 14 per cent., could increase the intensity of the radiation received at the surface of the earth about half of 1 per cent., and, therefore, the average temperature by no more than about 1.3° C. Similarly, reducing the carbon dioxide by one-half could decrease the temperature by no more than approximately the same amount, 1.3° C.

It is not certain to what extent the percentage of carbon dioxide in the atmosphere has actually varied during the geologic past, but, if the above reasoning is correct, it seems that surface

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[48] *Ann. der Physik*, 16, p. 93, 1905.

[49] *Annals Astrophys. Obsy., Smithsonian Inst.*, vol. ii, p. 172, 1908.

temperatures could never have been much increased above their present values through the action of this particular agent alone. Furthermore, the fact, so far as known, that within the tropics, at least, plant growth was quite as vigorous during the ice ages as it is now, shows that for a very long time, even in the geological sense, carbon dioxide has been abundant in the atmosphere—probably never much less abundant than at present. Hence, it seems likely that a decrease in temperature of a fraction of one degree is all that can reasonably be accounted for in this way.

Finally, if the above reasoning is correct, it seems that changes in the amount of carbon dioxide in the atmosphere might have been a factor in the production of certain climatic changes of the past, but that it could not, of itself, have produced the ice ages.

10. LAND ELEVATION.

Since many changes in land elevation are known to have taken place during the different geologic ages it is necessary, in considering the climates of the past, to inquire what climatic effects such variations in level would of themselves produce. Changes in area will be considered later.

The substantial answer to this question obviously is found in the present effects of elevation on climate. That is to say, the effects of elevation must then have been distant, local, and universal, just as they now are. The distant effects obviously often extended, as they now extend, many hundreds of miles to the leeward of favorably situated high mountain ranges and consisted, as they now consist, essentially in a decrease of precipitation, owing to the extraction of moisture from the atmosphere by forced convection and the consequent tendency towards, or even culmination in, desert conditions beyond. The second or local effects clearly were both an increase in the local precipitation, especially on the windward side, and an average decrease in the temperature, approximately the same as at present, of about 1° C. for each 180 metres, 200 metres, and 250 metres difference in elevation on mountains, hills and plateaus, respectively.[50]

In one important case, namely, when the surface is extensive and snow-covered (probably to some extent also when bare), this relation of temperature decrease between mountain, hill and plateau does not hold, as is obvious from the following considera-

[50] Hann, "Lehrbuch der Meteorologie," 2nd ed., p. 101.

tion. During long, clear winter nights, such as obtain in high latitudes, the surface becomes greatly chilled through comparatively free radiation to the still colder air far above and even to empty space beyond. Hence, the surface air also is chilled and its density made correspondingly greater. It, therefore, flows away to lower levels and at the same time its temperature is increased through increase of pressure, or at least prevented from falling so low as it otherwise would. When the slope is steep, as it usually is on the sides of hills and mountains, this flow clearly must be more or less rapid, especially along narrow valleys or ravines, and, therefore, an approach established, within this portion of the air current, to the adiabatic temperature gradient of about 1° C. per each 100 metres' change in elevation. On the other hand, when the slope is very gentle, as it is over the interior of Greenland and over much of the explored portion of the Antarctic Continent, air drainage necessarily is sluggish and unable to keep pace with the surface cooling. Hence, in such cases the change of temperature with change in elevation (counting from sea level) can be, and usually is, far greater than adiabatic, or 1° C., about, per 100 metres. Hence, such regions, when there are no higher surrounding mountains, can and often do establish: (1) a circulation of the upper air from the ocean to the higher portions of the plateau; (2) a well-defined surface temperature inversion, or, for the first few hundred metres, an increase of temperature with increase of elevation; (3) a slow settling of this air onto the cold surface below; (4) the precipitation, without cloud, of fine snow crystals—"frost snow"; (5) drainage of this chilled and relatively dense air to lower levels; (6) drifting of the snow with the winds and the consequent extension, so far as temperature and other conditions will permit, of the ice-covered or glaciated area.

All these conditions obtain to-day over the two great glaciers that still remain, that of Greenland and that of Antarctica, and, presumably, therefore, must also have obtained to a greater or less extent over all great glacial fields wherever and whenever found. Mere changes in level, therefore, might cause, and doubtless often have caused, somewhat corresponding fluctuations in the climates of both local and leeward regions, and especially in the extent and depth of local glaciations; but this obvious fact does not in the least justify the assumption that either the universal

low temperatures and extensive glaciations of ice ages or the world-wide genial temperatures of the intervening periods had, on the whole, any such origin.

It may be worth while in this connection to call attention to the fact that the thickness or depth of a glacial sheet cannot continue indefinitely to increase with increase of elevation, but, on the contrary, for each given locality must have a level of maximum development, above which it somewhat rapidly decreases.

An important if not the chief contributing factor to this result is the relation of saturation to temperature, illustrated by Fig. 195, which shows by its shaded areas the relative amounts of precipitation for each 5-degree decrease in the temperature of saturated air, assuming the volume to remain constant. Now as the wind blows up and over a mountain range its temperature decreases somewhat regularly with increase of elevation, and, therefore, at whatever temperature precipitation begins it must, as Fig. 195 shows, continue to decrease in amount as the cloud reaches higher and higher elevations—the effect of the accompanying volume increase on vapor capacity being much less than the effect of the temperature decrease.

If the precipitation always began at the same level, it is obvious that this would be the level not only of initial but also of maximum precipitation. But as the level of initial precipitation actually varies through a considerable range, it follows that the level of maximum catch lies somewhere within this range, probably, too, well within its lower half. Obviously, then, the level of maximum snowfall is not at, nor even close to, the tops of very high mountains, but far down near the lower snow limit.

In addition to this, the generally increasing steepness of the higher reaches causes more frequent avalanches and a greater speed of flow. In short, the higher mountain levels not only catch less snow than do the somewhat lower, but also more rapidly shed what they do catch.

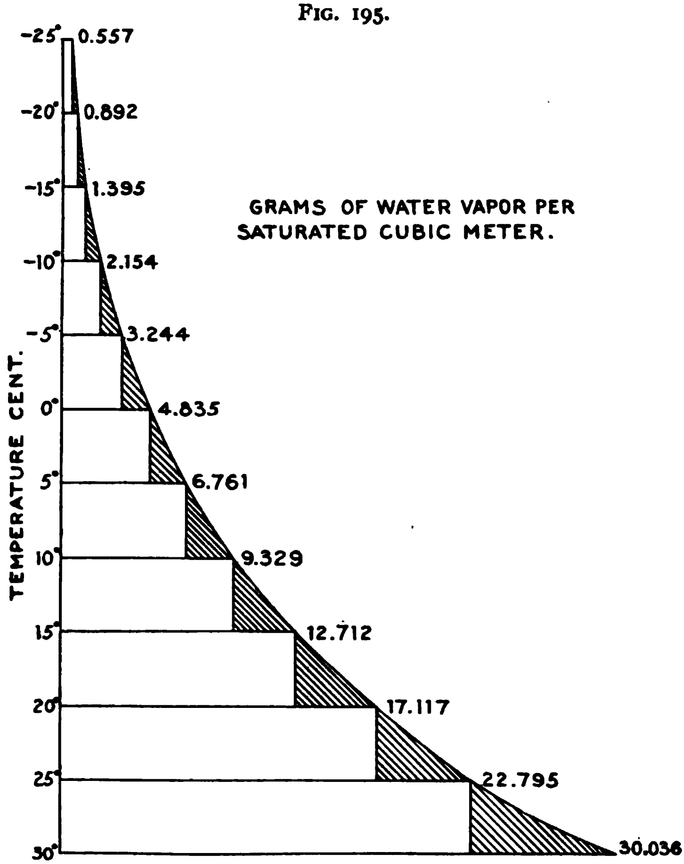
A full discussion of this subject would, of course, require an account of the rate of melting, evaporation, drift, glacial flow, and probably other phenomena; but the above, presumably, is sufficient to make it clear why maximum glaciation is not at the greatest elevations, but, on the contrary, at distinctly lower levels.

The universal climatic effect of land elevation, mentioned above, a greater or less lowering of the average temperature over,

perhaps, the entire earth, is a logical consequence of the other two. As is well known, water vapor is by far the world's greatest conservator of heat. Hence, anything that diminishes the average amount of this substance in the atmosphere obviously must somewhat lower surface temperatures.

Now precipitation through which the average humidity is

FIG. 195.



lowered is induced chiefly by vertical convection, a condition which mountain ranges vigorously and extensively produce, (*a*) by mechanical deflection of the winds, (*b*) by updrafts induced on their sides during summer insolation, and (*c*) by the air drainage down their valleys of clear nights. The first two types of convection are very efficient in removing moisture from the

atmosphere, while the third spreads the cold of the exposed higher levels—exposed in the sense that they are above much of the protecting vapor blanket, and, therefore, subjected to more rapid cooling.

Clearly, then, assuming the same total extent of land area, the average temperature of the world is lower when there are a number of high mountain ranges than when there are but few or none—lower because the exposed heights themselves are cold, because they drain their cold air over adjacent regions, and because they dry the atmosphere and thus prevent other portions of the earth from having as efficient protection from heat loss as they otherwise would.

This lowering of the average temperature necessarily decreases the rate of evaporation and the saturation quantity, and, thereby, presumably, further accentuates the cooling. Hence, high mountain ranges, especially when along the coasts of extensive continents, lead to low temperatures, local and windward glaciation, and reduced leeward precipitation. On the other hand, the absence of mountain ranges permits of a relatively humid atmosphere and comparatively high average temperatures.

Conceivably, therefore, the great world-wide climatic changes of the past originated in corresponding changes of level, gradual or cataclysmic. Nevertheless, it seems most improbable (apparently it has not yet been definitely proved one way or the other) that all the several glaciated continents rose and sank together—that there neither is, nor ever was, a simultaneous swing, up and down, of all, or nearly all, continental areas on the one hand and ocean beds on the other; at least not to any such extent as this hypothesis would demand. To be sure, there is much and increasing evidence [51] that in geologically very recent times there has been an increase of sea level relative to that of the land of, roughly, 50 metres in many tropical and subtropical regions where, so far as known, glaciers have never existed. But the simplest interpretation of this appears to be, not that so extensive and such different ocean beds have everywhere sunk to substantially the same extent and at about the same time, but rather that the phenomenon is only the inevitable result of the melting of the extensive glacial sheets of the Quaternary ice age.

[51] Vaughan, *Bull. Amer. Geographic Soc.*, vol. xlvi, p. 426, 1914; Daly, *Amer. Jr. Sci.*, 41, 153, 1916.

It will be assumed, therefore, that all effects, of whatever kind, changes in land elevation may have had on climate, local and leeward, were essentially as above explained, and hence that such changes, however important as contributory factors, constituted neither the sufficient initiating nor the whole sustaining cause either of the glacial or of the interglacial climates if, as is generally believed, these were simultaneously world-wide.

II. CHANGES IN LAND AREA.

In addition to changes in land elevation, and largely because of such changes, there have also been many variations in land extent and, consequently, in the ratio, both local and world-wide, of water surface to land surface. The importance of this phenomenon may be inferred from the fact that, according to Schuchert,[52] North America alone has been submerged at least 17 times over areas that range from 154,000 to 4,000,000 square miles, or fully half its present extent. Hence, in a discussion of how the climates of the past have varied, and why, it is necessary to take this additional factor into consideration. An exact or quantitative evaluation of this factor is impossible, but its qualitative effect, and even a rough approximation to its numerical value, may be inferred from the present climates, along the same latitudes, of continents and of islands, and also from the climatic contrast between the northern and the southern hemispheres, as shown in Fig. 197, by the mean annual isotherms. That is, any appreciable increase in the ratio of land to ocean surface presumably accentuated the seasonal contrasts, or made the summers warmer and the winters colder than they otherwise would have been. It must also have accentuated the latitude contrast or made warm regions warmer and cold ones still colder, as is also illustrated by Fig. 197. In short, any increase of land area must, in general, have rendered the local climate more continental and less marine, and all this for the obvious reason that while the solid portions of the earth, rock, sand, and soil, have no power of avoiding great temperature ranges through either evaporation, convection, or flow to other latitudes, all three methods belong abundantly to the ocean.

From this, in turn, since the range of animal species is partly restricted and delimited by the extremes of temperature, and the

[52] *Bull Geol. Soc. America*, vol. xx, p. 601, 1910.

range of vegetable species limited both by these extremes and by the length of the growing season, it follows that an age of great land extent must, through its fossils, give evidence of an excessive zonal climatic contrast, or appear in middle and higher latitudes to have been one of harsh climates, while an age of small land extent, even though the world as a whole had the same average temperature as before, must similarly record a much less zonal contrast and a spread of genial climates to far higher latitudes.

In the above attention has been confined strictly to land surfaces, their elevation and their areas, but continental mountain ranges and plateaus are not the only portions of the earth subject to changes in elevation and extent. Ocean beds and submarine ridges must also vary in both particulars, and, in turn, profoundly modify both local temperatures and zonal contrasts. Some of these ridges doubtless have alternately risen quite to the surface, perhaps at times much higher, and again sunk to considerable depths, and in each case inevitably have produced greater or less climatic changes through the resulting alterations in the oceanic and atmospheric circulations, and, perhaps, in other ways.

12. 13. ATMOSPHERIC AND OCEANIC CIRCULATION.

As everyone knows, practically the entire amount of heat that maintains the surface temperature of the earth is the result of the absorption of solar energy, chiefly by the lower atmosphere and by the various superficial coverings of the surface—vegetation, soil, rock, and water. On the average, the daily supply of this heat per unit area is much greater within the Tropics than it is at higher latitudes, a condition that maintains two distinct, though largely interdependent, systems of perpetual circulation, the atmospheric and the oceanic, that tend to equalize the temperatures of the earth—that keep the tropical regions from becoming unbearably hot and the frigid zones from being unendurably cold.

Clearly, then, any modification of either of these circulations would produce a corresponding change of climates, and the cause of this modification might properly be regarded as the cause of the given climatic changes themselves. But preliminary to a consideration of the probable climatic effects of any such modification it will be necessary, first, briefly to examine both circulations as they now exist.

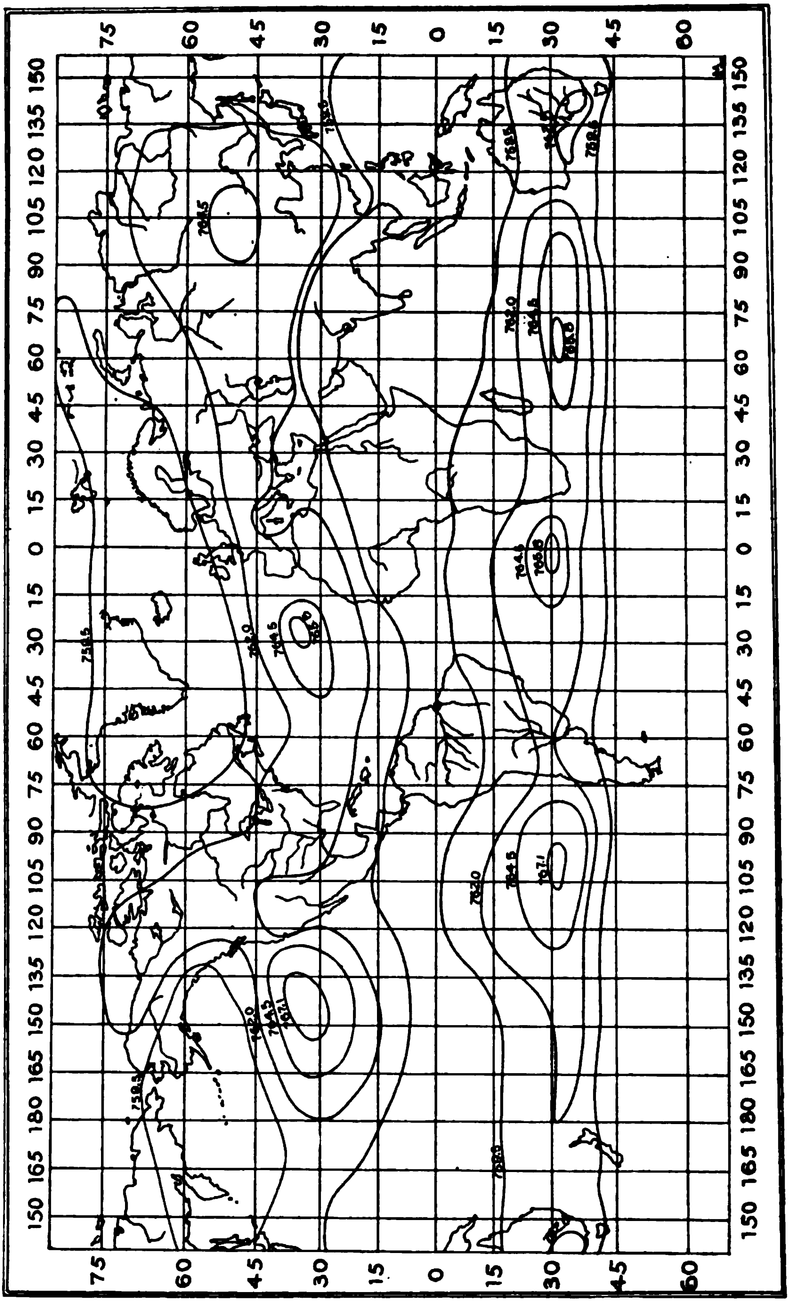
Present Atmospheric and Oceanic Circulations.—Despite innumerable disturbances, great and small, the fundamental circulation of the atmosphere is from equatorial toward polar regions and return, over a surface that moves from west to east with a linear velocity that gradually decreases with increase of latitude. As a result of these conditions the prevailing winds of latitudes higher than 32°, roughly, N. and S., blow from the west towards the east, or, to be exact, have a west-to-east component, while the winds between these latitudes, in the neighborhood, therefore, of the equator and the tropics, commonly blow, in the same general sense as above, from the east towards the west.

Now, the rotation of the earth has forced it to take the shape of an oblate spheroid—has bulged it at the equator and flattened it at the poles—and this distortion from the perfect sphere is such that an object on the surface is in equilibrium, so far as moving north or south is concerned, only when it has the same angular velocity about the axis of the earth that the earth itself has, or, in short, when it has no motion over the surface east or west. Obviously, then, an object moving from west to east, or with an angular velocity exceeding that of the earth, would be in equilibrium on a bulge greater than that which actually exists, and hence all winds with an eastward component, and, therefore, as stated, the prevailing winds outside latitudes 32° N. and S., must tend to climb up towards the equator. On the other hand, an object moving from east to west, or with an angular velocity less than that of the earth, as do the prevailing winds between latitudes 32° N. and S., must tend to slide down towards the adjacent or nearest pole. These opposite drifts of the winds, therefore, the drift of the winds of high latitudes toward the equator and the drift of the tropical winds toward the adjacent pole, obviously have produced the two belts of relatively high barometric pressure that roughly parallel the equator, the one at about latitude 32° N.; the other at approximately latitude 32° S.

Each belt of high pressure is greatly disturbed where it crosses a continental area, owing chiefly to temperature inequalities, but it also happens that even on the oceans the belts are of unequal intensity, and, what is of especial importance, have relatively fixed absolute pressure maxima or anticyclonic centres.

These permanent or nearly permanent centres of high pressure, or centres of action as they have been called, are five in

FIG. 196.



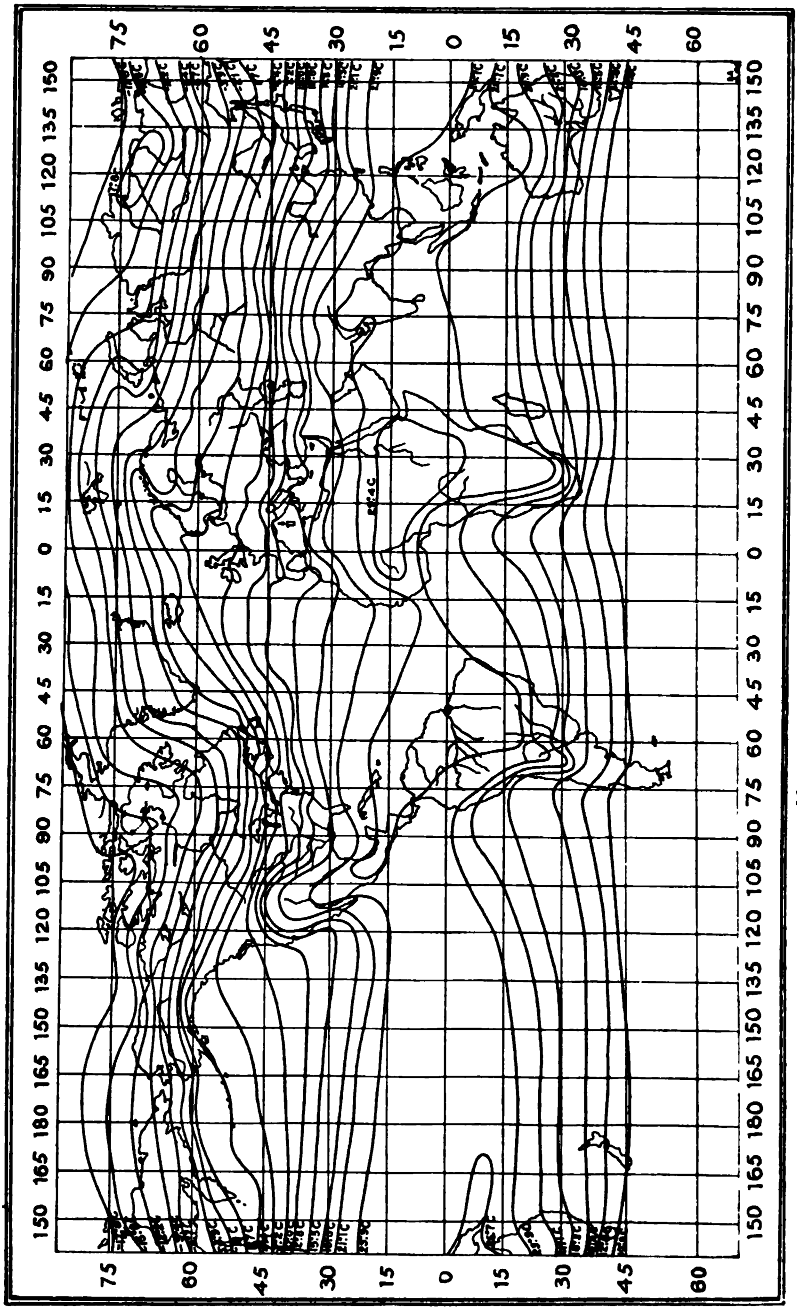
Mean annual isobars. (Buchan.)

number, as a reference to Fig. 196 will show; two in the northern hemisphere, one off the coast of southern California, the other near the Azores; and three in the southern hemisphere, one off the coast of Chile, another just west of South Africa, and a third between South Africa and Australia. Further, as is shown by Fig. 197, there is, in the region of each of these permanent anticyclonic centres, a marked equatorial deflection of the annual average isotherms, showing clearly that while each high pressure belt as a whole is caused by the mechanical squeeze of the opposite air drifts, above explained, the additional pressure that produces each maximum is a result of the local relatively low temperature, which causes a corresponding contraction and, therefore, increased density of the local atmosphere. And, finally, these particular low air temperatures in turn are caused by cold ocean currents, as is obvious from Fig. 198, and again from Fig. 199, which show the interrelations here considered between barometric pressure, air isotherms, and ocean currents.

That both the mere existence of these anticyclonic centres and their several geographic locations are of great importance to the climates of neighboring regions, through their directing influence on the prevailing winds, is obvious from Figs. 200 and 201, that give the directions and indicate the average force and steadiness of ocean winds at different seasons. But all this influence on the winds, however vital to many a local climate, can properly be said to be only one of the important climatic effects of the existing ocean currents.

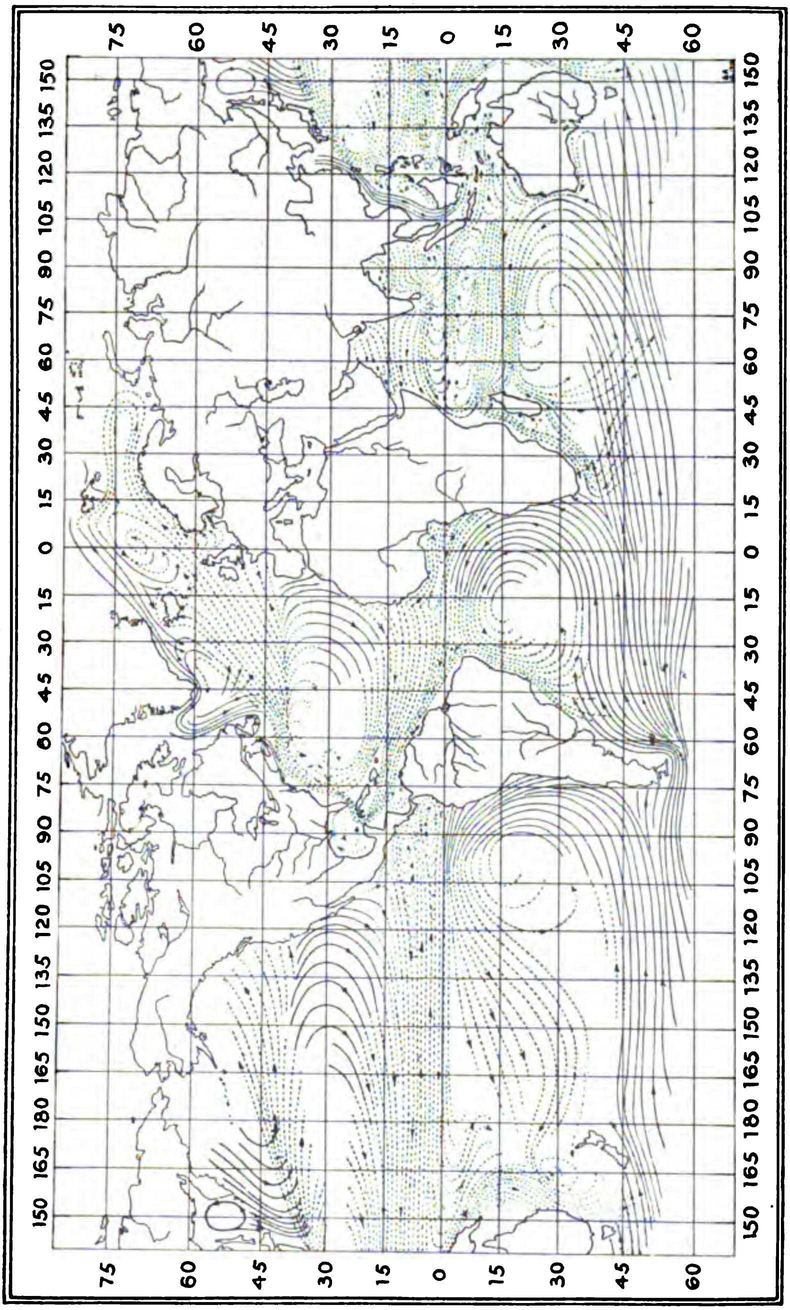
In addition to the several permanent anticyclonic centres there are, also (see Figs. 200 and 201), one permanent cyclonic centre, the Icelandic "low," and one semi-permanent or winter cyclonic centre, the Aleutian "low," both of which are strongly influenced by ocean currents, the first by the Gulf Stream and the second by the Japan Current. The Icelandic "low" results from the temperature contrast between the air over the relatively warm water and that over the glacial fields of Greenland and Iceland, in consequence of which there is established a circulation in the sense of an overflow from the warm region and counter underflows from both the adjacent cold regions. This thermally induced circulation in conjunction with the rotation of the earth produces cyclonic conditions, and as the temperature contrasts are permanent (the Icelandic and the Greenland glaciers remain

FIG. 197.



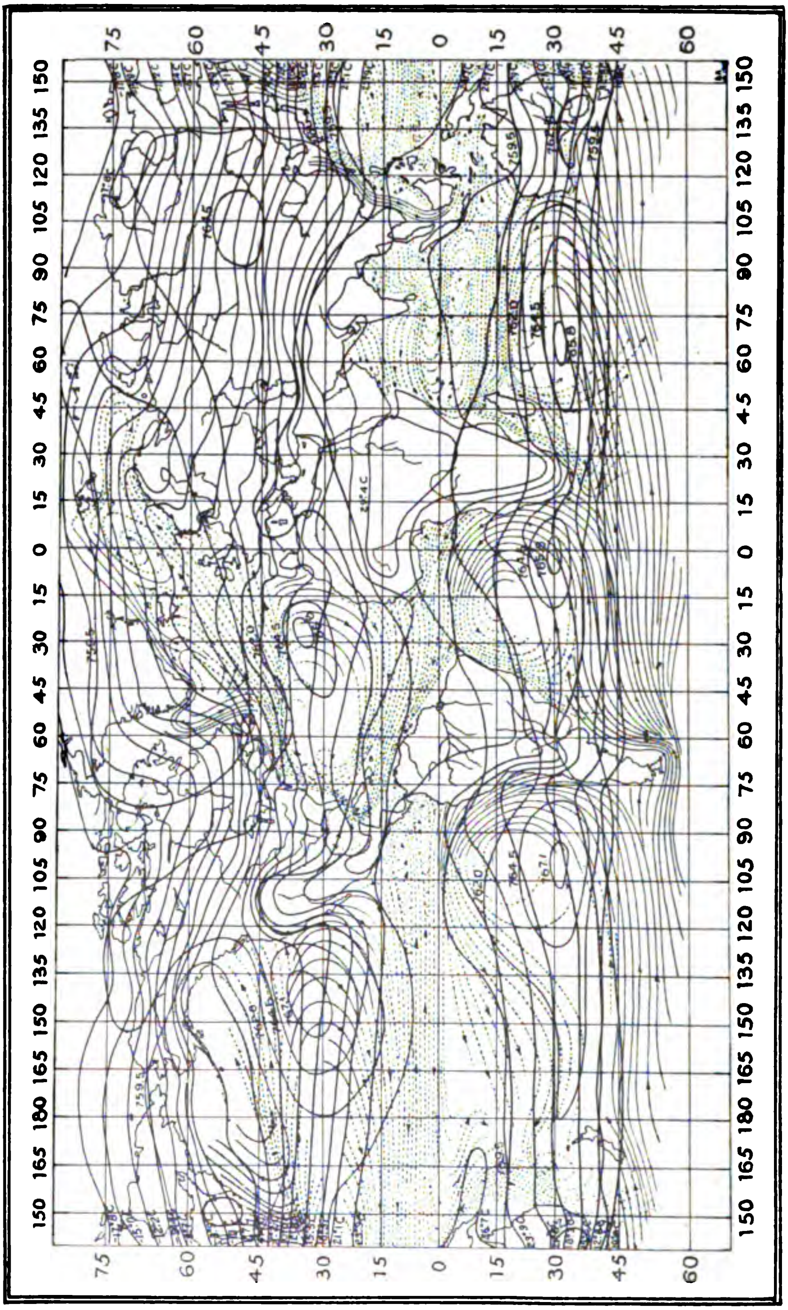
Mean annual isotherms. (Buchan.)

Fig. 198.



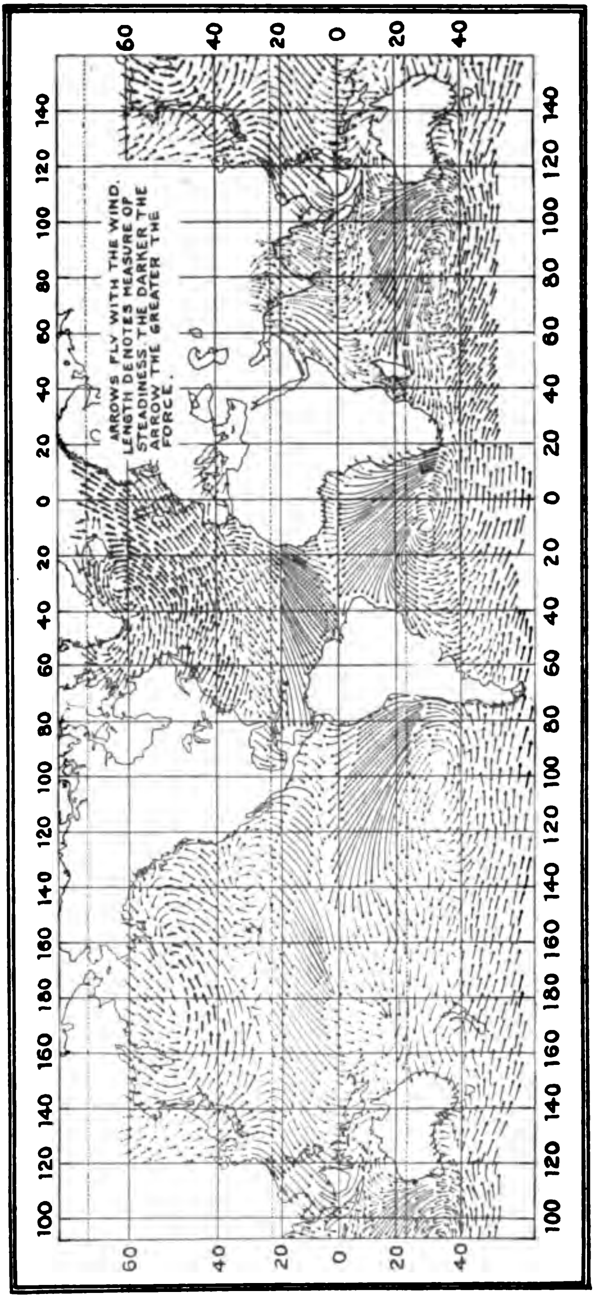
Ocean currents; full lines, cold; dotted lines, warm.

FIG. 199.



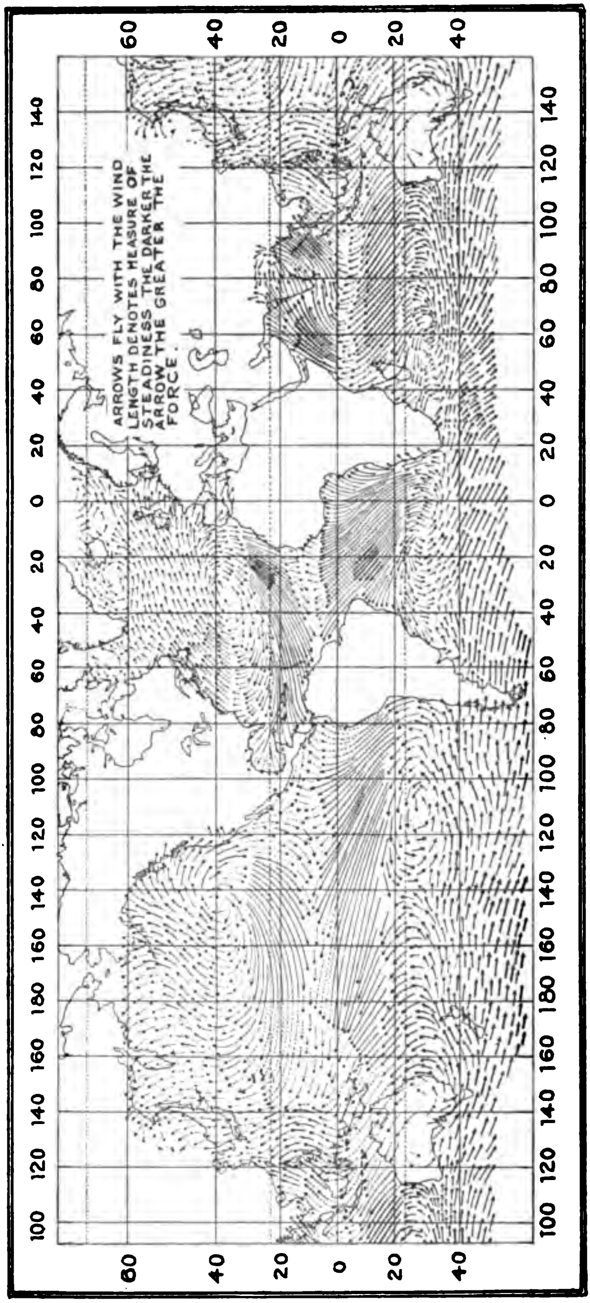
Superposition of ocean currents on mean annual isotherms and isobars.

FIG. 200.



Normal wind direction and velocity, January and February. (Köppen.)

Fig. 201.



Normal wind direction and velocity, July and August. (Köppen.)

throughout the year and indefinitely), the cyclonic conditions themselves of this region are also permanent.

The Aleutian "low," on the contrary, that is flanked by the Alaskan peninsula and the peninsulas of northeastern Siberia, has a circulation similar to that of the Icelandic "low," and for similar reasons, only while the adjacent land areas are covered with ice and snow and, therefore, are relatively cold. In the spring, when the snow melts away, the cyclonic conditions also disappear, there then being nothing to sustain them.

But the formation and geographic location of the above cyclonic and anticyclonic conditions—direction and force of winds, nature and frequency of storms, and the like—do not exhaust the more important climatic effects of existing ocean currents, as a glance at the annual average isotherms and ocean currents of the North Atlantic (Fig. 199) will show. In fact, the January isotherm of Chicago, Buffalo, and Boston (very approximately on the latitude of Rome), though not particularly cold, passes, under the influence of the Gulf Stream, through Iceland and on well beyond the Arctic Circle to the north of Norway and Sweden; while, under the same influence, frost is practically unknown and semitropical vegetation flourishes on the Scilly Isles in the latitude of northern Newfoundland. Similarly, it is said that because of the Japan Current frost rarely, if ever, occurs on one or two of the Aleutian islands.

Obviously, then, as the above several examples show, the ocean currents, driven by the winds, deflected by the rotation of the earth and by continental and island barriers, and otherwise modified by various minor causes, are both directly and indirectly of the greatest importance to the climates of many parts of the world; directly through the immense thermal interchange they establish between the torrid and the frigid zones, and indirectly through the centres of action, the permanent and semi-permanent "highs" and "lows," they create and maintain.

Possible Changes in the Oceanic Circulation and the Obvious Climatic Results.—Clearly, if the immense system of hot-water heating by which the temperature of the whole surface of the earth tends to become equalized should be greatly modified, say by the opening of a valve here or the closing of another there, correspondingly great climatic modifications surely would have to follow. And there are several, perhaps many, such valves that

have been opened or closed, irregularly, and from time to time, since the beginning of geological records. One such valve now partially open, perhaps at one time closed and at another still wider open than at present, lies between South America and the Antarctic continent. Another, now but a little way open, is Bering Strait, which doubtless has greatly changed from one to another geologic age. Still another, now wholly closed, but at one time probably wide open, is the Central American region between the Caribbean Sea and the Pacific Ocean. This particular valve, if now widely opened, would, on the one hand, obliterate the Gulf Stream proper and probably diminish the Antilles current, and, on the other, greatly increase the Japan Current; and, of course, in each case induce widespread and marked climatic changes. Yet another valve, now rather wide open, that merits special mention, a valve that may have suffered many changes and have undergone its latest opening only in recent times, geologically speaking, is found in that ridge which by way of Iceland and the Faroe Islands connects Greenland with north Scotland.

With this Greenland-Scotland valve closed and even with all the other valves, channels of flow and deflecting obstructions, substantially as they now are, it is well-nigh certain that the Icelandic "low" would shift to some point between Greenland and Newfoundland; that Labrador and the Hudson Bay region would receive a greatly increased precipitation; that the Norwegian Sea would become largely, if not wholly, ice-covered; and, finally, that Norway and Sweden, since they have the same latitude as Greenland, would be swept by winds of practically Arctic temperature and, therefore, eventually would become, like Greenland itself, almost wholly ice-capped. Indeed, any decided change in either the average intensity or average position of the Icelandic "low," if continued for even a few weeks, seems to produce a marked influence on the weather of west and north Europe. In general, whenever the average position of this "low" during a winter month is considerably to the west of its normal place, as occasionally happens, the average temperature of north Europe is likely to be several degrees below normal.[53] That is, the above conclusion that a permanent or age-long shift of the Icelandic "low" far to the west of its present position would

[53] Hann, "Handbuch der Meteorologie," 3rd ed., pp. 632–3, 637, 639.

lead to, or, at least, permit, the reglaciation of portions of north Europe appears to be abundantly supported by direct observation. Nor would these be all the profound climatic changes that probably, indeed well-nigh certainly, would follow the closing of the Greenland-Scotland valve, but they are sufficient, if granted, to indicate how vitally important the direction and magnitude of the ocean currents are to our climates and to local glaciation.

Doubtless many other valves have contributed their part in the control of the earth's great water circulation and the regulation of its climatic details, but it would be tedious to take all of them up individually, and for the present purpose unnecessary, since it is desired here only to make clear the fact that oceanic circulation is a vital factor in the production and control of many a local climate.

14. SURFACE COVERING.

The contrast between land and water in respect to their climatic effects obviously is largely due to the inequality of their surfaces both as radiators and as absorbers. The values of these properties are approximately known for the ocean, but not for the continents. In fact, there are no fixed values for land areas; nor can there be, since their surfaces undergo many and great changes. Bare soil, luxuriant vegetation, and snow, for instance, are among the surface coverings that have very unequal powers of radiation and absorption; and, therefore, the changes from one to the other over any extensive area necessarily is a matter of climatic importance.

To illustrate an extreme, but seemingly possible, climatic effect due to change of surface, assume:

(1) That continental areas are unusually extensive; (2) that mountains are abundant and high; (3) that during a period of a few years volcanic explosions are frequent (one a year, say) and of such nature as to put large quantities of dust in the upper atmosphere, as did the Krakatoa and other explosions of historic times.

The low temperatures due to the volcanic dust would lead to an abnormal extent of continental snow covering, and this, in turn, by its power of reflection, would render the insolation correspondingly less effective—would virtually decrease the solar constant, for all the reflected radiation that passes directly to

space had as well never have reached the earth at all, so far as producing any heating is concerned. In this way an ice covering of the land areas, far more extensive than previously existed, might be initiated by the volcanic explosions, and then perpetuated over long periods, if not even added to, by the highly reflecting snow cover.

CHRONOLOGICAL RELATION OF GEOLOGICAL EVENTS.

It is rarely possible to state in terms of years even approximately when any prehistoric geological event occurred or, similarly, to measure its duration, but it is possible in large measure accurately to decipher the order of their occurrence and to learn their chronological relations to each other.

Our present knowledge of geological chronology has been summarized by several geologists—Schuchert in his " Climates of Geologic Time,"[54] for instance—all of whom, though differing in details, have much in common. In the course of his summary Schuchert[55] says:

> The data at hand show that the earth since the beginning of geologic history has periodically undergone more or less widespread glaciation, and that the cold climates have been of short geologic duration. So far as known, there were seven periods of decided temperature changes, and of these at least four were glacial climates. The greatest intensity of these reduced temperatures varied between the hemispheres, for in earliest Proterozoic and Pleistocene time it lay in the northern, while in late Proterozoic and Permic time it was more equatorial than boreal. The three other probable periods of cooled climates are as yet too little known to make out their centres of greatest intensity.
>
> Of the four more or less well-determined glacial periods, at least three (the earliest Proterozoic, Permic, and Pleistocene) occurred during or directly after times of intensive mountain-making, while the fourth (late Proterozoic) apparently also followed a period of elevation. The Table Mountain tillites of South Africa, if correctly correlated, fall in with the time of the making of the great Caledonian Mountains in the northern hemisphere. On the other hand, the very marked and world-wide mountain-making period, with decided volcanic activity, during late Mesozoic and earliest Eocene times, was not accompanied by a glacial climate, but only a cooled one. The cooled period of the Liassic also followed a mountain-making period, that of late Triassic time. We may therefore state that cooled and cold climates, as a rule, occur during or immediately follow periods of marked mountain-making—a conclusion also arrived at independently by Ramsay.[56]

[54] Carnegie Institution of Washington, Publication No. 192, p. 285, 1914.
[55] *l. c.*, p. 286.
[56] Oversigt af Finska Vet.-Soc. Forhandl., vol. lii, pp. 1–48, 1910.

From the above, with which most geologists are in accord, it appears that neither cold climates nor even cool climates ever occurred except when mountains were either building or at least geologically young, and, also, so far as may be judged from the Strand line of North America, when the ratio of land area to ocean was relatively large. This coincidence appears to have occurred too frequently to admit of the idea that it was mere chance or due to anything less than some sort of a casual relation—surely not that the cold climates produced the mountains, nor necessarily that mountain building was wholly responsible for glaciation, but rather that crustal uplift and the other geological phenomena, whatever they were, that went with it so combined as to produce at one time a small and at another a great climatic change. If, also, as supported by geological evidence, vulcanism (only violently explosive volcanoes are of especial climatological importance), was, in general, most active during the epochs of mountain building, but unequally so for different mountain systems, then it follows that at each of these particular times there obtained to a greater or less degree precisely those formations and conditions—high mountains, extensive land area, restricted oceanic circulation and much volcanic dust—which are known to be effective to-day in reducing temperature, and which, presumably, are entirely sufficient, when properly coöperating, to produce not only a cool climate, such as, relatively speaking, now prevails, but even climates that are severely glacial. Which of these several causes was most effective in producing the low temperatures of any given age in the past (they must have been unequally active at different epochs) it may be impossible to determine, but the brief duration of distinctly glacial ages in comparison to the much longer periods when mild climates prevailed seems strongly to favor the assumption that vigorous vulcanism was often at least an important contributing factor.

But whatever else geologic chronology, as it is now understood, may teach in regard to the climatic changes of the past, the practical coincidence of cold ages with mountain building epochs appears at once and irretrievably to negative the entire group of cosmical ice-age theories—those that assume all great climatic changes to depend upon the sun, a condition in space, or anything else wholly outside the earth. Such theories, any solar theory, for instance, must also assume that those external changes, solar

changes, say, which caused a marked lowering of terrestrial temperatures, occurred only at or about the times of mountain building. That is, they must assume that either (*a*) the solar changes caused the mountain building, or (*b*) that mountain building caused the solar changes, or, finally (*c*) that both had some unknown but simultaneously acting common cause. But each of these assumptions is wholly untenable—it has no support whatever in the logic of cause and effect—and, therefore, it seems that any theory that implicitly or otherwise is definitely hung on the horns of this dilemma, as is every cosmical ice-age theory, nebular, or what not, must itself be abandoned.

This is not intended in the least to deny or even to question the existence of small solar changes of comparatively short duration, but only to emphasize the fact that forces within the earth itself suffice to modify its own climate, and that there is much and accumulating evidence that these and these alone have actually caused great changes time and again in the geologic past; done so by building mountains, and by tearing them down; by emerging continents, and by submerging them; by restricting ocean currents, and by making wide their paths; by filling the atmosphere with volcanic dust, and by clearing it of foreign substances; and by every possible combination of these and other such phenomena.

CONCLUSION.

It appears from various considerations that, with a constant or nearly constant output of solar energy, the earth itself possesses the inherent ability to profoundly modify its own climates, whether only local or world-wide. Thus, a mere change in land elevation, whether of plateau or of mountain range, a thing that appears often to have happened, must alter both the local and the leeward climates, and, by reducing the general humidity, somewhat lower the average temperature. Besides, a change in land elevation of any considerable extent is pretty certain to be accompanied by a somewhat corresponding variation in continental area, and such modification of shore lines and ocean beds that greater or less changes must follow in the directions, temperatures, and magnitudes of ocean currents, in the location and intensity of permanent "highs" and permanent "lows," in the direction, force and temperature of local winds, in the amount

and kind of local precipitation, and in a host of other meteorological phenomena.

Again, as the laws of radiation indicate must be true, and as observations, at least back to 1750, the date of the earliest reliable records, show, the temperature of the lower atmosphere depends in part upon the amount of dust in the upper air, in the sense that when this amount is great the average temperature at the surface of the earth is below normal, and when the dust is absent this temperature is comparatively high. Hence, as there appear to have been several periods of great volcanic activity in the past with intervening periods of volcanic quiescence, it is inferred that volcanic dust in the upper atmosphere was at least one important factor in some, if not all, of the great and universal climatic changes that have left their records in abandoned beaches and forsaken moraines.

How these various causes of climatic changes were related to each other during the geologic past is not yet entirely clear. This the geologist, most interested and most competent to judge, must determine. May it be that extensive upheavals and great volcanic activity often were synchronous? If so, the climatic effects of each obviously were added to those of the other, and hence it may be that the greatest of our past climatic changes were caused by the roughly synchronous variations in continental level and volcanic activity; universal cold periods coming with increase in vulcanism, increase in elevation and the obstruction of interzonal oceanic circulation; universal mild periods when volcanic dust seldom veiled the skies, when the continents had sunk or been eroded to low levels and when there was great freedom of oceanic circulation from equatorial to polar regions; mild universal climatic oscillations with temporary changes in vulcanism; and mere local climatic changes with variations in such local climatic controls as near-by elevations and neighboring ocean currents. Finally, as the past is the pledge of the future, it is but reasonable to suppose that the world is yet to know many another climatic change, in an irregular but well-nigh endless series, often local and usually slight, though always important, but occasionally, it may be, as in the ages gone—whether towards the auspiciously genial or into the fatefully disastrous—universal, profound and momentous.

APPENDIX I.

GRADIENT WIND VELOCITY TABLES.

To be used only in the absence of local disturbances—thunderstorms, line squalls, and the like—or strong horizontal temperature gradients, and when the isobars, as drawn, are free from any considerable reduction or other errors. Also to be used with discretion in the case of east winds in the middle latitudes, since at an altitude of 1 kilometre or more their actual velocities are likely to be less than the computed, as explained on page 133.

To find from the following tables the probable wind velocity at 1 to 2 kilometres elevation over any given place, one notes, *a*, the current system of winds, cyclonic or anticyclonic, at that place (this determines which table to use); *b*, the latitude of the place in question (this determines the latitude division of the table in which the desired value is to be found); *c*, the pressure gradient shown on the concurrent weather map in terms of the difference of the barometer reading in millimetres per 100 kilometres at right angles to the nearest isobar (this, through the closest tabulated gradient, locates the gradient division of the value sought); and, finally, *d*, the radius of curvature, in kilometres, of this isobar (a sufficiently close practical value of r) at the place in question, on line with which the desired velocity is given in metres per second, kilometres per hour, and miles per hour. The wind at the given levels is roughly parallel to the corresponding surface isobar, and so directed that on following it one will have the lower pressure to his left.

In using these tables in conjunction with weather maps whose barometric interval is 0.1 inch, the interval used in the United States, it is only necessary in taking step *c* to note the number of miles between the 0.1 inch isobars and then select from the second expressions in the first columns the nearest to an equal gradient.

The actual gradient, radius of curvature, density, and latitude usually will all differ somewhat from the tabulated values, but as the latter, except the density, which may be computed approxi-

mately, are given for small intervals it would be easy to add, with their proper signs, interpolated corrections. In practice, however, this will hardly be necessary, partly because of the great number of intervals directly supplied by the tables themselves, and partly because actual velocities and computed gradient velocities are likely to differ too much to justify minute corrections. They differ because the atmosphere never attains to a fixed or steady state of motion; because the actual density is likely to differ from that assumed; and because the gradient at the level for which computation is made is not, as a rule, exactly the same as that given on the maps.

In regions of great elevation, 1 kilometre or more, the isobaric lines, if drawn, as they commonly are, in accordance with values obtained by reduction of the barometer to sea level, may be seriously in error during both unusually warm and exceptionally cold weather. Obviously, therefore, it is not safe, at such times and places, to use the reduced distribution of isobars for the calculation of gradient winds—nor, indeed, for any other purpose.

The first line in each section of the anticyclonic table gives the maximum velocity for the given density and pressure gradient and the corresponding radius of curvature of the path. It will be noticed that this limiting radius grows smaller as the gradient is decreased, in accordance with the fact that steep gradients and strong winds can not occur near the centre of an anticyclonic region.

TABLE I.

Gradient Wind Velocity for Cyclonic Movement.

(Computed for $\rho = .0011$, a density that obtains at an elevation of about 1 kilometre above sea level.)

$\frac{\Delta B\ (mm.)}{100\ km.}$ = barometric gradient, or difference of barometric reading in millimetres per 100 kilometres at right angles to isobars $= \frac{\Delta B\ (\text{tenths } in.)}{158\ mi.}$

r = radius of curvature of isobars in kilometres.

$V\frac{m}{s}$ = velocity in metres per second.

$V\frac{km}{hr}$ = velocity in kilometres per hour.

$V\frac{mi}{hr}$ = velocity in miles per hour.

Latitude 25°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.2\ mm.}{100\ km.} = \frac{0.1\ in.}{789\ mi.}$	100	2.73	9.83	6.11	$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	100	5.99	21.56	13.40
	200	3.13	11.27	7.00		200	7.38	26.57	16.51
	300	3.33	11.99	7.45		300	8.18	29.45	18.30
	400	3.45	12.42	7.72		400	8.71	31.36	19.49
	500	3.53	12.71	7.90		500	9.10	32.76	20.36
	600	3.58	12.89	8.01		600	9.40	33.84	21.03
	700	3.63	13.07	8.12		700	9.64	34.70	21.56
	800	3.66	13.18	8.19		800	9.83	35.39	21.99
	900	3.69	13.28	8.25		900	9.99	35.96	22.34
	1000	3.71	13.36	8.30		1000	10.13	36.47	22.66
	1200	3.74	13.46	8.36		1200	10.35	37.26	23.15
	1500	3.78	13.61	8.46		1500	10.58	38.09	23.67
	2000	3.81	13.72	8.53		2000	10.84	39.02	24.25
	3000	3.85	13.86	8.61		3000	11.12	40.03	24.87
	∞	3.93	14.15	8.79		∞	11.79	42.44	26.37
$\frac{0.4\ mm.}{100\ km.} = \frac{0.1\ in.}{395\ mi.}$	100	4.53	16.31	10.13	$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	100	7.24	26.06	16.19
	200	5.45	19.62	12.19		200	9.06	32.62	20.27
	300	5.95	21.42	13.31		300	10.15	36.54	22.70
	400	6.27	22.57	14.02		400	10.90	39.24	24.38
	500	6.49	23.36	14.52		500	11.46	41.26	25.64
	600	6.66	23.98	14.90		600	11.90	42.84	26.62
	700	6.79	24.44	15.19		700	12.25	44.10	27.40
	800	6.90	24.84	15.43		800	12.54	45.14	28.05
	900	6.98	25.13	15.62		900	12.78	46.01	28.59
	1000	7.05	25.38	15.77		1000	12.99	46.76	29.05
	1200	7.17	25.81	16.04		1200	13.32	47.95	29.79
	1500	7.29	26.24	16.30		1500	13.69	49.28	30.62
	2000	7.42	26.71	16.60		2000	14.11	50.80	31.57
	3000	7.55	27.18	16.89		3000	14.58	52.49	32.62
	∞	7.86	28.30	17.58		∞	15.72	56.59	35.16

Latitude 25° (Continued.)

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{1.0\ mm.}{100\ km.} = \frac{0.1\ in.}{158\ mi.}$	100	8.35	30.06	18.68	$\frac{2.5\ mm.}{100\ km.} = \frac{0.1\ in.}{63\ mi.}$	100	14.59	52.52	32.63
	200	10.58	38.09	23.67		200	19.21	69.16	42.97
	300	11.94	42.98	26.71		300	22.28	80.21	49.84
	400	12.90	46.44	28.86		400	24.60	88.56	55.03
	500	13.63	49.07	30.49		500	26.44	95.18	59.14
	600	14.20	51.12	31.76		600	27.97	100.69	62.57
	700	14.67	52.81	32.81		700	29.27	105.37	65.47
	800	15.06	54.22	33.69		800	30.40	109.44	68.00
	900	15.39	55.40	34.42		900	31.38	112.97	70.20
	1000	15.67	56.41	35.05		1000	32.26	116.14	72.17
	1200	16.13	58.07	36.08		1200	33.74	121.46	75.47
	1500	16.65	59.94	37.25		1500	35.50	127.80	79.41
	2000	17.24	62.06	38.56		2000	37.64	135.50	84.20
	3000	17.92	64.51	40.08		3000	40.34	145.22	90.24
	∞	19.65	70.74	43.96		∞	49.13	176.87	109.90
$\frac{1.5\ mm.}{100\ km.} = \frac{0.1\ in.}{105\ mi.}$	100	10.75	38.70	24.05	$\frac{3.0\ mm.}{100\ km.} = \frac{0.1\ in}{53\ mi.}$	100	16.23	58.43	36.31
	200	13.87	49.93	31.03		200	21.49	77.36	48.07
	300	15.87	57.13	35.50		300	25.04	90.14	56.01
	400	17.32	62.35	38.74		400	27.74	99.86	62.19
	500	18.44	66.38	41.23		500	29.92	107.71	66.93
	600	19.35	69.66	43.28		600	31.73	114.23	70.98
	700	20.11	72.40	44.99		700	33.28	119.81	74.45
	800	20.75	74.70	46.42		800	34.64	124.70	77.49
	900	21.30	76.68	47.65		900	35.83	128.99	80.15
	1000	21.78	78.41	48.72		1000	36.89	132.80	82.52
	1200	22.59	81.32	50.53		1200	38.71	139.36	86.59
	1500	23.50	84.60	52.57		1500	40.88	147.17	91.45
	2000	24.58	88.49	54.99		2000	43.56	156.82	97.44
	3000	25.87	93.13	57.87		3000	47.01	169.24	105.16
	∞	29.48	106.13	65.95		∞	58.96	212.26	131.90
$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	100	12.79	46.04	28.61	$\frac{4.0\ mm.}{100\ km.} = \frac{0.1\ in.}{39\ mi.}$	100	19.15	68.94	42.84
	200	16.70	60.12	37.36		200	25.57	92.05	57.20
	300	19.26	69.34	43.09		300	29.99	107.96	67.08
	400	21.16	76.18	47.34		400	33.39	120.20	74.69
	500	22.65	81.54	50.67		500	36.17	130.21	80.91
	600	23.88	85.97	53.42		600	38.51	138.64	86.15
	700	24.92	89.71	55.74		700	40.53	145.91	90.67
	800	25.80	92.88	57.71		800	42.31	152.32	94.65
	900	26.58	95.69	59.46		900	43.89	158.00	98.18
	1000	27.26	98.14	60.98		1000	45.31	163.12	101.36
	1200	28.40	102.24	63.53		1200	47.77	171.97	106.86
	1500	29.74	107.06	66.52		1500	50.75	182.70	113.52
	2000	31.34	112.82	70.10		2000	54.51	196.24	121.94
	3000	33.31	119.92	74.52		3000	59.48	214.13	133.05
	∞	39.31	141.52	87.94		∞	78.61	283.00	175.85

Latitude 30°.

$\frac{\Delta B \text{ (mm.)}}{100 \text{ km.}}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$	$\frac{\Delta B \text{ (mm.)}}{100 \text{ km.}}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$
$\frac{0.2 \text{ mm.}}{100 \text{ km.}} = \frac{0.1 \text{ in.}}{789 \text{ mi.}}$	100	2.48	8.93	5.55	$\frac{0.8 \text{ mm.}}{100 \text{ km.}} = \frac{0.1 \text{ in.}}{197 \text{ mi.}}$	100	6.86	24.70	15.35
	200	2.79	10.04	6.24		200	8.43	30.35	18.86
	300	2.93	10.55	6.56		300	9.32	33.55	20.85
	400	3.01	10.84	6.74		400	9.92	35.71	22.19
	500	3.07	11.05	6.87		500	10.35	37.26	23.15
	600	3.10	11.16	6.93		600	10.68	38.45	23.89
	700	3.13	11.27	7.00		700	10.94	39.38	24.47
	800	3.15	11.34	7.05		800	11.16	40.18	24.97
	900	3.17	11.41	7.09		900	11.33	40.79	25.35
	1000	3.19	11.48	7.13		1000	11.48	41.33	25.68
	1200	3.21	11.56	7.18		1200	11.72	42.19	26.22
	1500	3.23	11.63	7.23		1500	11.98	43.13	26.80
	2000	3.25	11.70	7.27		2000	12.26	44.14	27.43
	3000	3.28	11.81	7.34		3000	12.57	45.25	28.11
	∞	3.32	11.95	7.43		∞	13.29	47.84	29.73
$\frac{0.4 \text{ mm.}}{100 \text{ km.}} = \frac{0.1 \text{ in.}}{395 \text{ mi.}}$	100	4.22	15.19	9.44	$\frac{1.0 \text{ mm.}}{100 \text{ km.}} = \frac{0.1 \text{ in.}}{158 \text{ mi.}}$	100	7.95	28.62	17.78
	200	4.96	17.86	11.10		200	9.90	35.64	22.15
	300	5.34	19.22	11.94		300	11.04	39.74	24.69
	400	5.58	20.09	12.48		400	11.82	42.55	26.44
	500	5.74	20.66	12.84		500	12.40	44.64	27.74
	600	5.86	21.10	13.11		600	12.84	46.22	28.72
	700	5.95	21.42	13.31		700	13.20	47.52	29.53
	800	6.02	21.67	13.47		800	13.49	48.56	30.17
	900	6.08	21.89	13.60		900	13.74	49.46	30.73
	1000	6.13	22.07	13.71		1000	13.95	50.22	31.21
	1200	6.21	22.36	13.89		1200	14.28	51.41	31.95
	1500	6.28	22.61	14.05		1500	14.65	52.74	32.77
	2000	6.37	22.93	14.25		2000	15.06	54.22	33.69
	3000	6.56	23.62	14.68		3000	15.51	55.84	34.70
	∞	6.64	23.90	14.85		∞	16.61	59.80	37.16
$\frac{0.6 \text{ mm.}}{100 \text{ km.}} = \frac{0.1 \text{ in.}}{263 \text{ mi.}}$	100	5.63	20.27	12.60	$\frac{1.5 \text{ mm.}}{100 \text{ km.}} = \frac{0.1 \text{ in.}}{105 \text{ mi.}}$	100	10.32	37.15	23.08
	200	6.80	24.48	15.21		200	13.12	47.23	29.35
	300	7.44	26.78	16.64		300	14.85	53.46	33.22
	400	7.85	28.26	17.56		400	16.07	57.85	35.95
	500	8.15	29.34	18.23		500	17.00	61.20	38.03
	600	8.37	30.13	18.72		600	17.73	63.83	39.66
	700	8.54	30.74	19.10		700	18.33	65.99	41.00
	800	8.68	31.25	19.42		800	18.84	67.82	42.14
	900	8.79	31.64	19.66		900	19.27	69.37	43.10
	1000	8.89	32.00	19.88		1000	19.63	70.67	43.91
	1200	9.04	32.54	20.22		1200	20.24	72.86	45.27
	1500	9.20	33.12	20.58		1500	20.92	75.31	46.80
	2000	9.37	33.73	20.96		2000	21.69	78.08	48.52
	3000	9.55	34.38	21.36		3000	22.58	81.29	50.51
	∞	9.97	35.89	22.30		∞	24.92	89.71	55.74

Latitude 30° (*Continued*).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
2.0 *mm.* / 100 *km.* =	100	12.34	44.42	27.60	3.0 *mm.* / 100 *km.* =	100	15.77	56.77	35.27
0.1 *in.*	200	15.90	57.24	35.57	0.1 *in.*	200	20.64	74.30	46.17
79 *mi.*	300	18.16	65.38	40.62	53 *mi.*	300	23.85	85.86	53.35
	400	19.79	71.24	44.27		400	26.24	94.46	58.70
	500	21.06	75.82	47.11		500	28.13	101.27	62.93
	600	22.08	79.49	49.39		600	29.85	107.46	66.77
	700	22.93	82.55	51.29		700	31.00	111.60	69.34
	800	23.64	85.10	52.88		800	32.14	115.70	71.89
	900	24.26	87.34	54.27		900	33.12	119.23	74.09
	1000	24.79	89.24	55.45		1000	33.99	122.36	76.03
	1200	25.68	92.45	57.45		1200	35.46	127.66	79.32
	1500	26.71	96.16	59.75		1500	37.19	133.88	83.19
	2000	27.89	100.40	62.39		2000	39.27	141.37	87.84
	3000	29.30	105.48	65.54		3000	41.84	150.62	93.59
	∞	33.22	119.59	74.31		∞	49.84	179.42	111.49
2.5 *mm.* / 100 *km.* =	100	14.14	50.90	31.63	4.0 *mm.* / 100 *km.* =	100	18.67	67.21	41.76
0.1 *in.*	200	18.38	66.17	41.12	0.1 *in.*	200	24.68	88.85	55.21
63 *mi.*	300	21.13	76.97	47.27	39 *mi.*	300	28.73	103.43	64.27
	400	23.15	83.34	51.78		400	31.88	114.77	71.32
	500	24.74	89.06	55.34		500	34.26	123.34	76.64
	600	26.05	93.78	58.27		600	36.31	130.72	81.23
	700	27.12	97.63	60.66		700	38.06	137.02	85.14
	800	28.05	100.98	62.75		800	39.59	142.52	88.56
	900	28.85	103.86	64.54		900	40.93	147.35	91.56
	1000	29.55	106.38	66.10		1000	42.12	151.63	94.22
	1200	30.74	110.44	68.63		1200	44.16	158.98	98.79
	1500	32.11	115.60	71.83		1500	46.48	167.33	103.98
	2000	33.73	121.43	75.45		2000	49.59	178.52	110.93
	3000	35.70	128.52	79.86		3000	53.41	192.28	119.48
	∞	41.53	149.51	92.90		∞	66.45	239.22	148.64

Latitude 35°.

0.2 *mm.* / 100 *km.* =	100	2.28	8.21	5.10	0.4 *mm.* / 100 *km.* =	100	3.94	14.18	8.81
0.1 *in.*	200	2.52	9.07	5.64	0.1 *in.*	200	4.55	16.38	10.18
789 *mi.*	300	2.62	9.43	5.86	395 *mi.*	300	4.85	17.46	10.85
	400	2.68	9.65	6.00		400	5.04	18.14	11.27
	500	2.72	9.79	6.08		500	5.16	18.58	11.55
	600	2.75	9.90	6.15		600	5.24	18.86	11.72
	700	2.77	9.97	6.20		700	5.31	19.12	11.88
	800	2.78	10.01	6.22		800	5.36	19.30	11.99
	900	2.79	10.04	6.24		900	5.41	19.48	12.10
	1000	2.80	10.08	6.26		1000	5.44	19.58	12.17
	1200	2.82	10.15	6.31		1200	5.49	19.76	12.28
	1500	2.83	10.19	6.33		1500	5.55	19.98	12.42
	2000	2.85	10.26	6.38		2000	5.61	20.20	12.55
	3000	2.86	10.30	6.40		3000	5.67	20.41	12.68
	∞	2.90	10.44	6.49		∞	5.79	20.84	12.95

Latitude 35° (*Continued*).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	100	5.32	19.15	11.90	$\frac{1.5\ mm.}{100\ km.} = \frac{0.1\ in.}{105\ mi.}$	100	9.93	35.75	22.21
	200	6.31	22.72	14.12		200	12.45	44.82	27.85
	300	6.83	24.59	15.28		300	13.95	50.22	31.21
	400	7.16	25.78	16.02		400	15.00	54.00	33.55
	500	7.39	26.60	16.53		500	15.77	56.77	35.28
	600	7.55	27.18	16.89		600	16.38	58.97	36.64
	700	7.68	27.65	17.18		700	16.87	60.73	37.74
	800	7.78	28.01	17.40		800	17.28	62.21	38.66
	900	7.87	28.33	17.60		900	17.61	63.40	39.39
	1000	7.94	28.58	17.76		1000	17.90	64.44	40.04
	1200	8.04	28.94	17.98		1200	18.36	66.10	41.07
	1500	8.16	29.38	18.26		1500	18.88	67.97	42.24
	2000	8.28	29.81	18.52		2000	19.46	70.06	43.53
	3000	8.41	30.28	18.82		3000	20.11	72.40	44.99
	∞	8.69	31.28	19.48		∞	21.72	78.19	48.58
$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	100	6.51	23.44	14.57	$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	100	11.93	42.95	26.69
	200	7.88	28.37	17.63		200	15.18	54.65	33.96
	300	8.62	31.03	19.28		300	17.19	61.88	38.45
	400	9.11	32.80	20.38		400	18.61	67.00	41.63
	500	9.45	34.02	21.14		500	19.69	70.88	44.04
	600	9.71	34.96	21.72		600	20.55	73.98	45.97
	700	9.91	35.68	22.17		700	21.25	76.50	47.53
	800	10.07	36.25	22.52		800	21.84	78.62	48.85
	900	10.20	36.72	22.82		900	22.34	80.42	49.97
	1000	10.31	37.12	23.07		1000	22.77	81.97	50.93
	1200	10.49	37.76	23.46		1200	23.48	84.53	52.52
	1500	10.68	38.45	23.89		1500	24.27	87.37	54.29
	2000	10.88	39.17	24.34		2000	25.18	90.65	56.33
	3000	11.10	39.96	24.83		3000	26.22	94.33	58.61
	∞	11.58	41.69	25.90		∞	28.96	104.26	64.78
$\frac{1.0\ mm.}{100\ km.} = \frac{0.1\ in.}{158\ mi.}$	100	7.59	27.32	16.98	$\frac{2.5\ mm.}{100\ km.} = \frac{0.1\ in.}{63\ mi.}$	100	13.72	49.39	30.69
	200	9.31	33.52	20.83		200	17.63	63.47	39.44
	300	10.27	36.97	22.97		300	20.10	72.36	44.96
	400	10.92	39.31	24.43		400	21.89	78.80	48.96
	500	11.38	40.97	25.46		500	23.26	83.74	52.03
	600	11.74	42.26	26.26		600	24.37	87.73	54.51
	700	12.02	43.27	26.89		700	25.29	91.04	56.57
	800	12.24	44.06	27.38		800	26.06	93.82	58.30
	900	12.43	44.75	27.81		900	26.72	96.19	59.77
	1000	12.59	45.32	28.16		1000	27.30	98.28	61.07
	1200	12.84	46.22	28.72		1200	28.25	101.70	63.19
	1500	13.11	47.20	29.33		1500	29.34	105.62	65.63
	2000	13.41	48.28	30.00		2000	30.41	109.48	68.03
	3000	13.73	49.43	30.71		3000	32.10	115.56	71.81
	∞	14.48	52.13	32.39		∞	36.20	130.32	80.98

Latitude 35° (Continued).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{3.0\ mm.}{100\ km.}$ =	100	15.34	55.22	34.31	$\frac{4.0\ mm.}{100\ km.}$ =	100	18.22	65.59	40.76
$\frac{0.1\ in.}{53\ mi.}$	200	19.86	71.50	44.43	$\frac{0.1\ in.}{39\ mi.}$	200	23.87	85.93	53.39
	300	22.77	81.97	50.93		300	27.59	99.32	61.71
	400	24.91	89.68	55.73		400	30.37	109.33	67.93
	500	26.57	95.65	59.43		500	32.57	117.25	72.86
	600	27.91	100.48	62.44		600	34.38	123.77	76.91
	700	29.04	104.54	64.96		700	35.91	129.28	80.33
	800	30.00	108.00	67.11		800	37.22	133.99	83.26
	900	30.82	110.95	68.94		900	38.37	138.13	85.83
	1000	31.55	113.58	70.58		1000	39.38	141.77	88.09
	1200	32.76	117.94	73.28		1200	41.10	147.96	91.94
	1500	34.15	122.94	76.39		1500	43.11	155.20	96.44
	2000	35.79	128.84	80.06		2000	45.54	163.94	101.87
	3000	37.76	135.94	84.47		3000	48.54	174.74	108.58
	∞	43.44	156.38	97.17		∞	57.92	208.51	129.56

Latitude 40°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.2\ mm.}{100\ km.}$ =	100	2.11	7.60	4.72	$\frac{0.6\ mm.}{100\ km.}$ =	100	5.04	18.14	11.27
$\frac{0.1\ in.}{789\ mi.}$	200	2.30	8.28	5.14	$\frac{0.1\ in.}{263\ mi.}$	200	5.90	21.24	13.20
	300	2.38	8.57	5.33		300	6.33	22.79	14.16
	400	2.43	8.75	5.44		400	6.60	23.76	14.76
	500	2.46	8.86	5.51		500	6.78	24.41	15.17
	600	2.48	8.93	5.55		600	6.91	24.88	15.46
	700	2.49	8.96	5.57		700	7.01	25.24	15.68
	800	2.50	9.00	5.59		800	7.08	25.49	15.84
	900	2.51	9.04	5.62		900	7.15	25.74	15.99
	1000	2.52	9.07	5.63		1000	7.20	25.92	16.11
	1200	2.53	9.11	5.66		1200	7.28	26.21	16.29
	1500	2.54	9.14	5.68		1500	7.37	26.53	16.49
	2000	2.55	9.18	5.70		2000	7.46	26.86	16.69
	3000	2.56	9.22	5.73		3000	7.55	27.18	16.89
	∞	2.58	9.29	5.77		∞	7.75	27.90	17.34
$\frac{0.4\ mm.}{100\ km.}$ =	100	3.71	13.36	8.30	$\frac{0.8\ mm.}{100\ km.}$ =	100	6.23	22.43	13.94
$\frac{0.1\ in.}{395\ mi.}$	200	4.22	15.19	9.44	$\frac{0.1\ in.}{197\ mi.}$	200	7.41	26.68	16.58
	300	4.46	16.06	9.98		300	8.04	28.94	17.98
	400	4.61	16.60	10.31		400	8.44	30.38	18.88
	500	4.70	16.92	10.51		500	8.72	31.39	19.50
	600	4.77	17.17	10.67		600	8.92	32.11	19.95
	700	4.82	17.35	10.75		700	9.08	32.69	20.31
	800	4.85	17.46	10.85		800	9.21	33.16	20.60
	900	4.89	17.60	10.94		900	9.31	33.52	20.83
	1000	4.91	17.68	10.99		1000	9.40	33.84	21.03
	1200	4.95	17.82	11.07		1200	9.53	34.31	21.32
	1500	4.99	17.96	11.16		1500	9.67	34.81	21.63
	2000	5.03	18.11	11.25		2000	9.82	35.35	21.97
	3000	5.08	18.29	11.36		3000	10.09	36.32	22.57
	∞	5.17	18.61	11.56		∞	10.34	37.22	23.13

Latitude 40° (Continued).

ΔB (mm.) / 100 km.	r	V_{s}^{m}	V_{hr}^{km}	V_{hr}^{mi}	ΔB (mm.) / 100 km.	r	V_{s}^{m}	V_{hr}^{km}	V_{hr}^{mi}
1.0 *mm.* =	100	7.28	26.21	16.29	2.5 *mm.* =	100	13.34	48.02	29.84
100 *km.*	200	8.80	31.68	19.69	100 *km.*	200	16.96	61.06	37.94
0.1 *in.*	300	9.63	34.67	21.54	0.1 *in.*	300	19.20	69.12	42.95
158 *mi.*	400	10.17	36.61	22.75	63 *mi.*	400	20.79	74.84	46.50
	500	10.55	37.98	23.60		500	21.99	79.16	49.19
	600	10.83	38.99	24.23		600	22.95	82.62	51.34
	700	11.06	39.82	24.74		700	23.73	85.43	53.08
	800	11.24	40.46	25.14		800	24.38	87.77	54.54
	900	11.39	41.00	25.48		900	24.94	89.78	55.79
	1000	11.51	41.44	25.75		1000	25.42	91.51	56.86
	1200	11.71	42.16	26.20		1200	26.20	94.32	58.66
	1500	11.91	42.88	26.64		1500	27.09	97.52	60.60
	2000	12.14	43.70	27.15		2000	28.09	101.12	62.83
	3000	12.38	45.57	28.32		3000	29.26	105.34	65.45
	∞	12.92	46.51	28.90		∞	32.30	116.28	72.25
1.5 *mm.* =	100	9.60	34.56	21.47	3.0 *mm.* =	100	14.95	53.82	33.44
100 *km.*	200	11.87	42.73	26.55	100 *km.*	200	19.17	69.01	42.88
0.1 *in.*	300	13.19	47.48	29.50	0.1 *in.*	300	21.83	78.59	48.83
105 *mi.*	400	14.09	50.72	31.52	53 *mi.*	400	23.74	85.46	53.10
	500	14.74	53.06	32.97		500	25.21	90.76	56.40
	600	15.25	54.90	34.11		600	26.39	95.00	59.03
	700	15.65	56.34	35.01		700	27.36	98.50	61.21
	800	15.98	57.53	35.75		800	28.18	101.45	63.04
	900	16.25	58.50	36.35		900	28.88	103.97	64.60
	1000	16.48	59.33	36.87		1000	29.49	106.16	65.96
	1200	16.86	60.70	37.72		1200	30.50	109.80	68.23
	1500	17.26	62.14	38.61		1500	31.65	113.94	70.80
	2000	17.71	63.76	39.62		2000	32.97	118.69	73.75
	3000	18.21	65.56	40.74		3000	34.50	124.20	77.17
	∞	19.38	69.77	43.35		∞	38.77	139.57	86.72
2.0 *mm.* =	100	11.57	41.65	25.88	4.0 *mm.* =	100	17.82	64.15	39.86
100 *km.*	200	14.55	52.38	32.55	100 *km.*	200	23.14	83.30	51.76
0.1 *in.*	300	16.35	58.86	36.57	0.1 *in.*	300	26.58	95.69	59.46
79 *mi.*	400	17.60	63.36	39.37	39 *mi.*	400	29.11	104.80	65.12
	500	18.52	66.67	41.43		500	31.08	111.89	69.53
	600	19.26	69.34	43.09		600	32.69	117.68	73.12
	700	19.84	71.42	44.38		700	34.04	122.54	76.14
	800	20.33	73.19	45.48		800	35.18	126.65	78.70
	900	20.74	74.66	46.39		900	36.18	130.25	80.93
	1000	21.10	75.96	47.20		1000	37.05	133.38	82.88
	1200	21.67	78.01	48.47		1200	38.50	138.60	86.12
	1500	22.31	80.32	49.91		1500	40.20	144.72	89.93
	2000	23.02	82.87	51.49		2000	42.19	151.88	94.37
	3000	23.83	85.79	53.31		3000	44.60	160.56	99.86
	∞	25.84	93.02	57.80		∞	51.69	186.08	115.62

Latitude 45°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$
$\frac{0.2\ mm.}{100\ km.} = \frac{0.1\ in.}{789\ mi.}$	100	1.97	7.09	4.41	$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	100	5.96	21.46	12.87
	200	2.13	7.67	4.77		200	7.01	25.24	15.68
	300	2.19	7.88	4.90		300	7.55	27.18	16.89
	400	2.23	8.03	4.99		400	7.89	28.40	17.42
	500	2.25	8.10	5.03		500	8.12	29.23	18.16
	600	2.27	8.17	5.08		600	8.29	29.84	18.54
	700	2.28	8.21	5.10		700	8.42	30.31	18.83
	800	2.29	8.24	5.12		800	8.52	30.67	19.06
	900	2.29	8.24	5.12		900	8.60	30.96	19.24
	1000	2.30	8.28	5.14		1000	8.67	31.21	19.39
	1200	2.31	8.32	5.17		1200	8.78	31.61	19.64
	1500	2.32	8.35	5.19		1500	8.89	32.00	19.84
	2000	2.33	8.39	5.21		2000	9.01	32.44	20.16
	3000	2.33	8.39	5.21		3000	9.14	32.90	20.44
	∞	2.35	8.46	5.26		∞	9.40	33.84	21.03
$\frac{0.4\ mm.}{100\ km.} = \frac{0.1\ in.}{395\ mi.}$	100	3.52	12.67	7.87	$\frac{1.0\ mm.}{100\ km.} = \frac{0.1\ in.}{158\ mi.}$	100	7.40	26.64	16.55
	200	3.94	14.18	8.81		200	8.36	30.10	18.70
	300	4.14	14.90	9.26		300	9.08	32.69	20.31
	400	4.26	15.34	9.53		400	9.54	34.34	21.34
	500	4.33	15.59	9.69		500	9.86	35.50	22.06
	600	4.39	15.80	9.82		600	10.10	36.36	22.59
	700	4.43	15.95	9.91		700	10.28	37.01	23.00
	800	4.46	16.06	9.98		800	10.43	37.55	23.33
	900	4.49	16.16	10.04		900	10.55	37.98	23.60
	1000	4.50	16.20	10.07		1000	10.65	38.34	23.82
	1200	4.53	16.31	10.13		1200	10.80	38.88	24.16
	1500	4.56	16.42	10.20		1500	10.97	39.49	24.54
	2000	4.60	16.56	10.29		2000	11.15	40.14	24.94
	3000	4.63	16.57	10.36		3000	11.33	40.79	25.35
	∞	4.70	16.92	10.51		∞	11.75	42.30	26.28
$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	100	4.81	17.32	10.76	$\frac{1.5\ mm.}{100\ km.} = \frac{0.1\ in.}{105\ mi.}$	100	9.28	33.41	20.72
	200	5.55	19.98	12.42		200	11.36	40.90	25.41
	300	5.92	21.31	13.24		300	12.54	45.14	28.05
	400	6.13	22.07	13.71		400	13.32	47.95	29.79
	500	6.28	22.61	14.05		500	13.88	49.97	31.05
	600	6.39	23.00	14.29		600	14.31	51.52	32.01
	700	6.47	23.29	14.47		700	14.65	52.74	32.77
	800	6.53	23.51	14.61		800	14.92	53.71	33.37
	900	6.58	23.69	14.72		900	15.15	54.54	33.89
	1000	6.62	23.83	14.81		1000	15.34	55.22	34.31
	1200	6.69	24.08	14.96		1200	15.64	56.30	34.98
	1500	6.75	24.30	15.10		1500	15.97	57.49	35.72
	2000	6.82	24.55	15.25		2000	16.33	58.79	36.53
	3000	6.90	24.84	15.43		3000	16.72	60.19	37.40
	∞	7.05	25.38	15.77		∞	17.62	63.43	39.41

Latitude 45° (Continued).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
2.0 *mm.* =	100	11.24	40.46	25.14	3.0 *mm.* =	100	14.59	52.52	32.63
100 *km.*	200	14.00	50.40	31.32	100 *km.*	200	18.55	66.78	41.50
0.1 *in.*	300	15.61	56.20	34.92	0.1 *in.*	300	21.00	75.60	46.98
79 *mi.*	400	16.72	60.19	37.40	53 *mi.*	400	22.72	81.79	50.82
	500	17.53	63.11	39.21		500	24.04	86.54	53.77
	600	18.16	65.38	40.62		600	25.08	90.29	56.10
	700	18.67	67.21	41.76		700	25.93	93.35	58.00
	800	19.08	68.69	42.68		800	26.64	95.90	59.59
	900	19.43	69.95	43.47		900	27.24	98.06	60.93
	1000	19.72	70.99	44.11		1000	27.77	99.97	62.12
	1200	20.20	72.72	45.23		1200	28.62	103.03	64.02
	1500	20.72	74.59	46.35		1500	29.58	106.49	66.17
	2000	21.30	76.68	46.78		2000	30.68	110.45	68.63
	3000	21.94	78.98	49.08		3000	31.94	114.98	71.45
	∞	23.49	84.56	52.54		∞	35.24	126.86	78.83

2.5 *mm.* =	100	12.99	46.76	29.05	4.0 *mm.* =	100	17.45	62.82	39.03
100 *km.*	200	16.37	58.93	36.62	100 *km.*	200	22.48	80.93	50.29
0.1 *in.*	300	18.41	66.28	41.18	0.1 *in.*	300	25.68	92.45	57.45
63 *mi.*	400	19.83	71.37	44.35	39 *mi.*	400	27.99	100.76	62.61
	500	20.90	75.24	46.75		500	29.78	107.21	66.62
	600	21.74	78.26	48.63		600	31.23	112.43	69.86
	700	22.41	80.68	50.13		700	32.42	116.71	72.52
	800	22.97	82.69	51.38		800	33.44	120.38	74.80
	900	23.44	84.38	52.43		900	34.31	123.52	76.75
	1000	23.85	85.86	53.35		1000	35.06	126.22	78.43
	1200	24.51	88.24	54.83		1200	36.32	130.75	81.25
	1500	25.25	90.90	56.48		1500	37.77	135.97	84.49
	2000	26.07	93.85	58.32		2000	39.45	142.02	88.25
	3000	27.01	97.24	60.42		3000	41.43	149.15	92.68
	∞	29.37	105.73	65.70		∞	46.99	169.16	105.11

Latitude 50°.

0.2 *mm.* =	100	1.86	6.70	4.16	0.4 *mm.* =	100	3.34	12.02	7.47
100 *km.*	200	1.99	7.16	4.45	100 *km.*	200	3.72	13.39	8.32
0.1 *in.*	300	2.04	7.34	4.56	0.1 *in.*	300	3.89	14.00	8.70
789 *mi.*	400	2.07	7.45	4.63	395 *mi.*	400	3.98	14.33	8.90
	500	2.09	7.52	4.67		500	4.04	14.54	9.04
	600	2.10	7.56	4.70		600	4.09	14.72	9.15
	700	2.11	7.60	4.72		700	4.12	14.83	9.22
	800	2.12	7.63	4.74		800	4.15	14.94	9.28
	900	2.13	7.67	4.77		900	4.17	15.01	9.33
	1000	2.13	7.67	4.77		1000	4.18	15.05	9.35
	1200	2.14	7.70	4.79		1200	4.21	15.16	9.42
	1500	2.14	7.70	4.79		1500	4.23	15.23	9.46
	2000	2.15	7.74	4.81		2000	4.26	15.34	9.53
	3000	2.15	7.74	4.81		3000	4.28	15.41	9.59
	∞	2.17	7.81	4.85		∞	4.34	15.62	9.71

Latitude 50° (Continued).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.6\ mm}{100\ km.}$ =	100	4.61	16.60	10.31	$\frac{1.5\ mm.}{100\ km.}$ =	100	9.01	32.44	20.16
$\frac{0.1\ in.}{263\ mi.}$	200	5.27	18.97	11.79	$\frac{0.1\ in.}{105\ mi.}$	200	10.92	39.31	24.43
	300	5.58	20.09	12.48		300	11.98	43.13	26.80
	400	5.76	20.74	12.89		400	12.67	45.61	28.34
	500	5.89	21.20	13.17		500	13.16	47.38	29.44
	600	5.97	21.49	13.35		600	13.53	48.71	30.27
	700	6.04	21.74	13.51		700	13.82	49.75	30.91
	800	6.09	21.92	13.62		800	14.06	50.62	31.45
	900	6.13	22.07	13.71		900	14.25	51.30	31.88
	1000	6.17	22.21	13.80		1000	14.41	51.88	32.24
	1200	6.22	22.39	13.91		1200	14.66	52.78	32.80
	1500	6.27	22.57	14.02		1500	14.93	53.75	33.40
	2000	6.33	22.79	14.16		2000	15.23	54.83	34.07
	3000	6.38	22.97	14.27		3000	15.54	55.94	34.76
	∞	6.51	23.44	14.57		∞	16.26	58.54	36.38
$\frac{0.8\ mm.}{100\ km.}$ =	100	5.73	20.63	12.82	$\frac{2.0\ mm.}{100\ km.}$ =	100	10.95	39.42	24.49
$\frac{0.1\ in.}{197\ mi.}$	200	6.68	24.05	14.94	$\frac{0.1\ in.}{79\ mi.}$	200	13.51	48.64	30.22
	300	7.15	25.74	15.99		300	14.99	53.96	33.53
	400	7.44	26.78	16.64		400	15.98	57.53	35.75
	500	7.63	27.47	17.07		500	16.70	60.12	37.36
	600	7.77	27.97	17.38		600	17.25	62.10	38.59
	700	7.88	28.37	17.63		700	17.69	63.68	39.57
	800	7.97	28.69	17.83		800	18.04	64.94	40.35
	900	8.03	28.91	17.96		900	18.34	66.02	41.02
	1000	8.09	29.12	18.09		1000	18.59	66.92	41.58
	1200	8.18	29.45	18.30		1200	18.99	68.36	42.48
	1500	8.27	29.77	18.50		1500	19.43	69.95	43.47
	2000	8.36	30.10	18.70		2000	19.91	71.68	44.54
	3000	8.46	30.46	18.93		3000	20.44	73.58	45.72
	∞	8.67	31.21	19.39		∞	21.69	78.08	48.52
$\frac{1.0\ mm.}{100\ km.}$ =	100	6.76	24.34	15.12	$\frac{2.5\ mm.}{100\ km.}$ =	100	12.69	45.68	28.38
$\frac{0.1\ in.}{158\ mi.}$	200	7.99	28.76	17.87	$\frac{0.1\ in.}{63\ mi.}$	200	15.86	57.10	35.48
	300	8.63	31.07	19.31		300	17.73	63.83	39.66
	400	9.02	32.47	20.18		400	19.02	68.47	42.55
	500	9.29	33.44	20.78		500	19.97	71.89	44.67
	600	9.50	34.20	21.25		600	20.71	74.56	46.33
	700	9.65	34.74	21.59		700	21.30	76.68	47.65
	800	9.77	35.17	21.85		800	21.79	78.44	48.74
	900	9.87	35.53	22.08		900	22.21	79.96	49.68
	1000	9.96	35.86	22.28		1000	22.55	81.18	50.44
	1200	10.09	36.32	22.57		1200	23.12	83.23	51.72
	1500	10.22	36.79	22.86		1500	23.74	85.46	53.10
	2000	10.36	37.30	23.18		2000	24.47	88.09	54.74
	3000	10.52	37.87	23.53		3000	25.21	90.76	56.40
	∞	10.84	39.02	24.25		∞	27.11	97.60	60.65

Latitude 50° (*Continued*).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{3.0\ mm.}{100\ km.} = \frac{0.1\ in.}{53\ mi.}$	100	14.28	51.41	31.95	$\frac{4.0\ mm.}{100\ km.} = \frac{0.1\ in.}{39\ mi.}$	100	17.12	61.63	38.29
	200	18.01	64.84	40.29		200	21.90	78.84	48.99
	300	20.27	72.97	45.34		300	24.89	89.60	55.67
	400	21.85	78.66	48.88		400	27.03	97.31	60.47
	500	23.03	82.91	51.52		500	28.66	103.18	64.11
	600	23.96	86.26	53.60		600	29.97	107.89	67.04
	700	24.72	88.99	55.30		700	31.04	111.74	69.43
	800	25.34	91.22	56.68		800	31.95	115.02	71.47
	900	25.87	93.13	57.87		900	32.72	117.79	73.19
	1000	26.33	94.79	58.90		1000	33.39	120.20	74.69
	1200	27.07	97.45	60.55		1200	34.50	124.20	77.17
	1500	27.89	100.40	62.39		1500	35.75	128.70	79.97
	2000	28.81	103.72	64.45		2000	37.19	133.88	83.19
	3000	29.86	107.50	66.80		3000	38.87	139.93	86.95
	∞	32.53	117.11	72.77		∞	43.37	156.13	97.01

Latitude 55°.

$\frac{0.2\ mm.}{100\ km.} = \frac{0.1\ in.}{789\ mi.}$	100	1.77	6.37	3.96	$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	100	4.44	15.98	9.93
	200	1.88	6.77	4.21		200	5.03	18.11	11.25
	300	1.93	6.95	4.32		300	5.30	19.08	11.86
	400	1.95	7.02	4.36		400	5.46	19.66	12.22
	500	1.96	7.06	4.39		500	5.57	20.05	12.46
	600	1.97	7.09	4.41		600	5.64	20.30	12.61
	700	1.98	7.13	4.43		700	5.70	20.52	12.75
	800	1.99	7.16	4.45		800	5.74	20.66	12.84
	900	1.99	7.16	4.45		900	5.77	20.77	12.91
	1000	1.99	7.16	4.45		1000	5.80	20.88	12.97
	1200	2.00	7.20	4.47		1200	5.85	21.06	13.09
	1500	2.00	7.20	4.47		1500	5.89	21.20	13.17
	2000	2.01	7.24	4.50		2000	5.94	21.38	13.29
	3000	2.02	7.27	4.52		3000	5.99	21.56	13.40
	∞	2.03	7.31	4.54		∞	6.08	21.89	13.60
$\frac{0.4\ mm.}{100\ km.} = \frac{0.1\ in.}{395\ mi.}$	100	3.20	11.52	7.16	$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	100	5.54	19.94	12.39
	200	3.53	12.71	7.90		200	6.40	23.04	14.32
	300	3.68	13.25	8.23		300	6.82	24.55	15.25
	400	3.76	13.54	8.41		400	7.07	25.45	15.81
	500	3.81	13.72	8.53		500	7.24	26.06	16.19
	600	3.85	13.86	8.61		600	7.36	26.50	16.47
	700	3.88	13.97	8.68		700	7.45	26.82	16.67
	800	3.90	14.04	8.72		800	7.52	27.07	16.82
	900	3.91	14.08	8.75		900	7.58	27.29	16.96
	1000	3.93	14.15	8.79		1000	7.63	27.47	17.07
	1200	3.95	14.22	8.84		1200	7.70	27.72	17.22
	1500	3.97	14.29	8.88		1500	7.77	27.97	17.38
	2000	3.99	14.36	8.92		2000	7.85	38.26	17.56
	3000	4.01	14.44	8.97		3000	7.94	28.58	17.76
	∞	4.06	14.62	9.08		∞	8.11	29.20	18.14

Latitude 55° (Continued).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{1.0\ mm.}{100\ km.}=$	100	6.55	23.58	14.65	$\frac{2.5\ mm.}{100\ km.}=$	100	12.43	44.75	27.81
$\frac{0.1\ in.}{158\ mi.}$	200	7.08	27.65	17.18	$\frac{0.1\ in.}{63\ mi.}$	200	15.41	55.48	34.47
	300	8.24	29.66	18.43		300	17.15	61.74	38.36
	400	8.59	30.92	19.21		400	18.22	65.59	40.76
	500	8.83	31.79	19.75		500	19.19	69.08	42.92
	600	9.01	32.44	20.16		600	19.85	71.46	44.40
	700	9.14	32.90	20.44		700	20.38	73.37	45.59
	800	9.25	33.30	20.69		800	20.82	74.95	46.57
	900	9.33	33.59	20.87		900	21.18	76.25	47.38
	1000	9.40	33.84	21.03		1000	21.49	77.36	48.07
	1200	9.51	34.24	21.28		1200	21.98	79.13	49.17
	1500	9.63	34.67	21.54		1500	22.52	81.07	50.37
	2000	9.74	35.06	21.79		2000	23.12	83.23	51.72
	3000	9.87	35.53	22.08		3000	23.77	85.57	53.17
	∞	10.14	36.50	22.68		∞	25.35	91.26	56.71
$\frac{1.5\ mm.}{100\ km.}=$	100	8.77	31.57	19.62	$\frac{3.0\ mm.}{100\ km.}=$	100	14.00	50.40	31.32
$\frac{0.1\ in.}{105\ mi.}$	200	10.55	37.98	23.60	$\frac{0.1\ in.}{53\ mi.}$	200	17.54	63.14	39.23
	300	11.51	41.44	25.75		300	19.65	70.74	43.96
	400	12.13	43.67	27.14		400	21.00	75.60	46.98
	500	12.56	45.22	28.10		500	22.18	79.85	49.62
	600	12.89	46.40	28.83		600	23.03	82.91	51.52
	700	13.14	47.30	29.39		700	23.70	85.32	53.02
	800	13.35	48.06	29.86		800	24.26	87.34	54.27
	900	13.51	48.64	30.22		900	24.73	89.03	55.32
	1000	13.65	49.14	30.53		1000	25.13	90.47	56.22
	1200	13.87	49.93	31.03		1200	25.79	92.84	57.69
	1500	14.10	50.76	31.56		1500	26.50	95.40	59.28
	2000	14.35	51.66	32.10		2000	27.30	98.28	61.07
	3000	14.62	52.63	32.70		3000	28.20	101.52	63.08
	∞	15.21	54.76	34.03		∞	30.42	109.51	68.05
$\frac{2.0\ mm.}{100\ km.}=$	100	10.70	38.52	23.94	$\frac{4.0\ mm.}{100\ km.}=$	100	16.84	60.62	37.67
$\frac{0.1\ in.}{79\ mi.}$	200	13.10	47.16	29.30	$\frac{0.1\ in.}{39\ mi.}$	200	21.40	77.04	47.87
	300	14.45	52.02	32.32		300	24.21	87.16	54.16
	400	15.35	55.26	34.34		400	26.10	93.96	58.38
	500	16.00	57.60	35.79		500	27.70	99.72	61.96
	600	16.48	59.33	36.87		600	28.91	104.08	64.67
	700	16.88	60.77	37.76		700	29.88	107.57	66.84
	800	17.19	61.88	38.45		800	30.70	110.52	68.67
	900	17.45	62.82	39.03		900	31.39	113.00	70.22
	1000	17.67	63.61	39.53		1000	31.99	115.16	71.56
	1200	18.02	64.87	40.31		1200	32.95	118.62	73.70
	1500	18.39	66.20	41.14		1500	34.08	122.69	76.24
	2000	18.80	67.68	42.05		2000	35.34	127.22	79.05
	3000	19.25	69.30	43.06		3000	36.79	132.44	82.29
	∞	20.28	73.01	45.37		∞	40.56	146.02	90.73

Latitude 60°.

$\frac{\Delta B \text{ (mm.)}}{100 \text{ km.}}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B \text{ (mm.)}}{100 \text{ km.}}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.2 \text{ mm.}}{100 \text{ km.}} =$	100	1.69	6.08	3.78	$\frac{0.8 \text{ mm.}}{100 \text{ km.}} =$	100	5.38	19.37	12.04
$\frac{0.1 \text{ in.}}{789 \text{ mi.}}$	200	1.79	6.44	4.00	$\frac{0.1 \text{ in.}}{197 \text{ mi.}}$	200	6.17	22.21	13.80
	300	1.83	6.59	4.10		300	6.54	23.54	14.63
	400	1.85	6.66	4.14		400	6.77	24.37	15.14
	500	1.86	6.70	4.16		500	6.92	24.91	15.48
	600	1.87	6.73	4.18		600	7.02	25.27	15.70
	700	1.88	6.77	4.21		700	7.10	25.56	15.88
	800	1.88	6.77	4.21		800	7.17	25.81	16.04
	900	1.89	6.80	4.23		900	7.22	25.99	16.15
	1000	1.89	6.80	4.23		1000	7.26	26.14	16.24
	1200	1.90	6.84	4.25		1200	7.32	26.35	16.37
	1500	1.90	6.84	4.25		1500	7.38	26.57	16.51
	2000	1.90	6.84	4.25		2000	7.45	26.82	16.67
	3000	1.91	6.88	4.28		3000	7.53	27.11	16.85
	∞	1.91	6.88	4.28		∞	7.60	27.36	17.00
$\frac{0.4 \text{ mm.}}{100 \text{ km.}} =$	100	3.08	11.09	6.89	$\frac{1.0 \text{ mm.}}{100 \text{ km.}} =$	100	6.37	22.93	14.25
$\frac{0.1 \text{ in.}}{395 \text{ mi.}}$	200	3.38	12.17	7.56	$\frac{0.1 \text{ in.}}{158 \text{ mi.}}$	200	7.42	26.71	16.60
	300	3.51	12.64	7.85		300	7.93	28.55	17.74
	400	3.58	12.89	8.01		400	8.25	29.70	18.45
	500	3.63	13.07	8.12		500	8.46	30.46	18.92
	600	3.66	13.18	8.19		600	8.61	31.00	19.26
	700	3.68	13.25	8.23		700	8.73	31.43	19.53
	800	3.70	13.32	8.28		800	8.82	31.75	19.73
	900	3.72	13.39	8.32		900	8.90	32.04	19.91
	1000	3.73	13.43	8.34		1000	8.96	32.26	20.05
	1200	3.74	13.46	8.36		1200	9.05	32.58	20.24
	1500	3.76	13.54	8.41		1500	9.15	32.94	20.47
	2000	3.78	13.61	8.46		2000	9.25	33.30	20.69
	3000	3.80	13.68	8.50		3000	9.37	33.73	20.96
	∞	3.80	13.68	8.50		∞	9.50	34.20	21.25
$\frac{0.6 \text{ mm.}}{100 \text{ km.}} =$	100	4.29	15.44	9.59	$\frac{1.5 \text{ mm.}}{100 \text{ km.}} =$	100	8.57	30.85	19.17
$\frac{0.1 \text{ in.}}{263 \text{ mi.}}$	200	4.83	17.39	10.81	$\frac{0.1 \text{ in.}}{105 \text{ mi.}}$	200	10.24	36.86	22.90
	300	5.07	18.25	11.34		300	11.12	40.03	24.87
	400	5.22	18.79	11.68		400	11.68	42.05	26.13
	500	5.31	19.12	11.88		500	12.08	43.49	27.02
	600	5.37	19.33	12.04		600	12.37	44.53	27.67
	700	5.42	19.51	12.12		700	12.59	45.32	28.16
	800	5.46	19.66	12.22		800	12.77	45.97	28.56
	900	5.49	19.76	12.28		900	12.02	46.51	28.90
	1000	5.51	19.84	12.33		1000	13.05	46.98	29.19
	1200	5.55	19.98	12.42		1200	13.23	47.63	29.60
	1500	5.59	20.12	12.50		1500	13.44	48.38	30.06
	2000	5.63	20.27	12.59		2000	13.65	49.14	30.53
	3000	5.67	20.41	12.68		3000	13.88	49.97	31.05
	∞	5.70	20.52	12.85		∞	14.25	51.30	31.88

Latitude 60° (*Continued*).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$
$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	100	10.48	37.73	23.44	$\frac{3.0\ mm.}{100\ km.} = \frac{0.1\ in.}{53\ mi.}$	100	14.77	53.17	33.04
	200	12.75	45.90	28.52		200	17.14	61.70	38.34
	300	14.01	50.44	31.34		300	19.12	68.83	42.77
	400	14.83	53.39	33.17		400	20.48	73.73	45.81
	500	15.42	55.51	34.49		500	21.47	77.29	48.03
	600	15.86	57.10	35.48		600	22.24	80.06	49.75
	700	16.21	58.36	36.26		700	22.86	82.30	51.14
	800	16.49	59.36	36.88		800	23.36	84.10	52.26
	900	16.72	60.19	37.40		900	23.79	85.64	53.22
	1000	16.92	60.91	37.85		1000	24.15	86.94	54.02
	1200	17.22	61.99	38.52		1200	24.73	89.03	55.32
	1500	17.56	63.22	39.28		1500	25.38	91.37	56.77
	2000	17.92	64.51	40.08		2000	26.08	93.89	58.34
	3000	18.30	65.88	40.94		3000	26.87	96.73	60.10
	∞	18.99	68.36	42.48		∞	28.49	102.56	63.73
$\frac{2.5\ mm.}{100\ km.} = \frac{0.1\ in.}{63\ mi.}$	100	12.20	43.92	27.29	$\frac{4.0\ mm.}{100\ km.} = \frac{0.1\ in.}{39\ mi.}$	100	16.59	59.72	37.11
	200	15.03	54.11	33.62		200	20.96	75.46	46.89
	300	16.66	59.98	37.27		300	23.63	85.07	52.86
	400	17.75	63.90	39.71		400	25.00	91.80	57.04
	500	18.54	66.74	41.47		500	26.90	96.84	60.17
	600	19.14	68.90	42.81		600	28.01	100.84	62.66
	700	19.62	70.63	43.89		700	28.91	104.08	64.67
	800	20.01	72.04	44.76		800	29.66	106.78	66.35
	900	20.34	73.22	45.50		900	30.29	109.04	67.76
	1000	20.61	74.20	46.10		1000	30.84	111.02	68.98
	1200	21.05	75.78	47.09		1200	31.73	114.23	70.98
	1500	21.53	77.51	48.16		1500	32.72	117.79	73.19
	2000	22.05	79.38	49.32		2000	33.83	121.79	75.68
	3000	22.63	81.47	50.62		3000	35.11	126.40	78.54
	∞	23.74	85.46	53.10		∞	37.98	136.73	84.96

Table II.

Gradient Wind Velocity for Anticyclonic Movement.

Latitude 25°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.2\ mm.}{100\ km.} = \frac{0.1\ in.}{789\ mi.}$	255.1	7.86	28.30	17.59
	300	5.67	20.41	12.68
	400	4.91	17.68	10.99
	500	4.63	16.67	10.36
	600	4.47	16.09	10.00
	700	4.38	15.77	9.80
	800	4.31	15.52	9.64
	900	4.26	15.34	9.53
	1000	4.22	15.19	9.44
	1200	4.17	15.01	9.33
	1500	4.11	14.78	9.18
	2000	4.07	14.65	9.10
	3000	4.02	14.47	8.99
	∞	3.93	14.15	8.79
$\frac{0.4\ mm.}{100\ km.} = \frac{0.1\ in.}{395\ mi.}$	510.2	15.72	56.59	35.16
	600	11.34	40.82	25.36
	700	10.34	37.22	23.13
	800	9.82	35.35	21.97
	900	9.48	34.13	21.21
	1000	9.25	33.30	20.69
	1200	8.94	32.18	20.00
	1500	8.68	31.25	19.42
	2000	8.44	30.38	18.88
	3000	8.23	29.63	18.41
	∞	7.86	28.30	17.58
$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	765.3	23.58	84.89	52.75
	800	19.52	70.27	43.66
	900	17.01	61.24	38.05
	1000	15.89	57.20	35.54
	1200	14.72	52.99	32.93
	1500	13.87	49.93	31.03
	2000	13.21	47.56	29.55
	3000	12.66	45.58	28.32
	∞	11.79	42.44	26.37
$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	1020.4	31.44	113.18	70.33
	1200	22.67	81.61	50.71
	1500	20.09	72.32	44.94
	2000	18.50	66.60	41.38
	3000	17.35	62.46	38.81
	∞	15.72	56.59	35.16
$\frac{1.0\ mm.}{100\ km.} = \frac{0.1\ in.}{158\ mi.}$	1275.5	39.30	141.48	87.91
	1500	28.34	102.02	63.39
	2000	24.54	88.34	54.89
	3000	22.36	80.50	50.02
	∞	19.65	70.74	43.96
$\frac{1.5\ mm.}{100\ km.} = \frac{0.1\ in.}{105\ mi.}$	1913.3	58.96	212.26	131.89
	2000	48.80	175.68	109.16
	3000	36.81	132.52	82.34
	∞	29.48	106.13	65.95
$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	2551	78.62	283.03	175.87
	3000	56.68	204.05	126.79
	∞	39.31	141.52	87.94

Latitude 30°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
0.2 mm. =	182.25	6.64	23.90	14.85
100 km.	200	5.12	18.43	11.45
0.1 in.	300	4.08	14.69	9.13
789 mi.	400	3.82	13.75	8.54
	500	3.70	13.32	8.28
	600	3.62	13.03	8.10
	700	3.57	12.85	7.98
	800	3.54	12.74	7.92
	900	3.51	12.64	7.85
	1000	3.49	12.56	7.80
	1200	3.46	12.46	7.74
	1500	3.43	12.35	7.67
	2000	3.40	12.24	7.61
	3000	3.38	12.17	7.56
	∞	3.32	11.95	7.43
0.4 mm. =	364.5	13.28	47.81	29.71
100 km.	400	10.24	36.86	22.90
0.1 in.	500	8.74	31.46	19.55
395 mi.	600	8.17	29.41	18.27
	700	7.85	28 26	17.56
	800	7.65	27.54	17.11
	900	7.50	27.00	16.78
	1000	7.40	26.64	16.55
	1200	7.24	26.06	16.19
	1500	7.11	25.60	15.91
	2000	6.98	25.13	15.62
	3000	6.86	24.70	15.35
	∞	6.64	23.90	14.85
0.6 mm. =	546.75	19.94	71.78	44.60
100 km.	600	15.36	55.30	34.36
0.1 in.	700	13.58	48.89	30.38
263 mi.	800	12.76	45.94	28.55
	900	12.26	44.14	27.43
	1000	11.91	42.88	26.64
	1200	11.49	41.36	25.70
	1500	11.09	39.92	24.81
	2000	10.76	38.74	24.07
	3000	10.47	37.69	23.42
	∞	9.97	35.89	22.30
0.8 mm. =	729	26.58	95.69	59.46
100 km.	800	20.48	73.73	45.81
0.1 in.	900	18.51	66.64	41.41
197 mi.	1000	17.48	62.93	39.10
	1200	16.34	58.82	36.55
	1500	15.48	55.73	34.63
	2000	14.79	53.24	33.08
	3000	14.21	51.16	31.79
	∞	13.29	47.84	29.73
1.0 mm. =	911.25	33.22	119.59	74.31
100 km.	1000	25.60	92.16	57.26
0.1 in.	1200	22.29	80.24	49.86
158 mi.	1500	20.43	73.55	45.70
	2000	19.12	68.83	42.77
	3000	18.11	65.20	40.51
	∞	16.61	59.80	37.16
1.5 mm. =	1367.9	48.84	179.42	111.49
100 km.	1500	38.40	138.24	85.90
0.1 in.	2000	31.89	114.80	71.33
105 mi.	3000	28.68	103.25	64.16
	∞	24.92	89.71	55.74
2.0 mm. =	1822.5	66.44	239.18	148.62
100 km.	2000	51.20	184.32	114.53
0.1 in.	3000	40.85	147.06	95.76
79 mi.	∞	33.22	119.59	74.31
2.5 mm. =	2278.1	83.06	299.02	185.80
100 km.	3000	55.72	200.59	124.64
0.1 in.	∞	41.53	149.51	92.90
63 mi.				
3.0 mm. =	2733.8	99.68	358.85	222.98
100 km.	3000	76.79	276.44	171.77
0.1 in.	∞	49.84	179.42	111.49
53 mi.				

Latitude 35°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
0.2 *mm.* =	138.5	5.80	20.88	12.88
100 *km.*	200	3.73	13.43	8.35
0.1 *in.*	300	3.34	12.02	7.47
789 *mi.*	400	3.20	11.52	7.16
	500	3.13	11.27	7.00
	600	3.09	11.12	6.91
	700	3.06	11.02	6.85
	800	3.03	10.91	6.78
	900	3.02	10.87	6.75
	1000	3.00	10.80	6.71
	1200	2.98	10.73	6.67
	1500	2.97	10.69	6.64
	2000	2.95	10.62	6.60
	3000	2.93	10.55	6.56
	∞	2.90	10.44	6.49
0.4 *mm.* =	277.0	11.58	41.69	25.90
100 *km.*	300	9.07	32.65	20.30
0.1 *in.*	400	7.46	26.86	16.61
395 *mi.*	500	6.95	25.02	15.55
	600	6.68	24.05	14.94
	700	6.52	23.47	14.58
	800	6.41	23.08	14.34
	900	6.32	22.75	14.14
	1000	6.26	22.54	14.01
	1200	6.17	22.21	13.80
	1500	6.09	21.92	13.62
	2000	6.01	21.64	13.45
	3000	5.93	21.35	13.27
	∞	5.79	20.84	12.95
0.6 *mm.* =	415.5	17.38	62.57	38.88
100 *km.*	500	12.31	44.32	27.54
0.1 *in.*	600	11.18	40.25	25.01
263 *mi.*	700	10.61	38.20	23.74
	800	10.26	36.94	22.95
	900	10.02	36.07	22.41
	1000	9.85	35.46	22.03
	1200	9.61	34.60	21.50
	1500	9.39	33.80	21.00
	2000	9.19	33.08	20.56
	3000	9.01	32.44	20.16
	∞	8.69	31.28	19.48
0.8 *mm.* =	554.0	23.16	83.38	51.81
100 *km.*	600	18.14	65.30	40.58
0.1 *in.*	700	15.91	57.28	35.59
197 *mi.*	800	14.90	53.64	33.33
	900	14.30	51.48	31.99
	1000	13.89	50.00	31.07
	1200	13.36	48.10	29.89

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
0.8 *mm.* =	1500	12.92	46.51	28.90
100 *km.*	2000	12.52	45.07	28.01
0.1 *in.*	3000	12.17	43.81	27.22
197 *mi.*	∞	11.58	41.69	25.90
1.0 *mm.* =	692.5	28.96	104.26	64.78
100 *km.*	700	26.24	94.46	58.70
0.1 *in.*	800	21.19	76.28	47.40
158 *mi.*	900	19.57	70.45	43.78
	1000	18.63	67.07	41.68
	1200	17.55	63.18	39.26
	1500	16.71	60.16	37.38
	2000	16.01	57.64	35.82
	3000	15.43	55.55	34.52
	∞	14.48	52.13	32.39
1.5 *mm.* =	1038.7	43.44	156.38	97.15
100 *km.*	1200	31.79	114.44	71.11
0.1 *in.*	1500	27.95	100.62	62.52
105 *mi.*	2000	25.66	92.38	57.40
	3000	24.02	86.47	53.73
	∞	21.72	78.19	48.58
2.0 *mm.* =	1385.0	57.92	208.51	129.56
100 *km.*	1500	45.36	163.30	101.47
0.1 *in.*	2000	37.26	134.14	83.33
79 *mi.*	3000	33.41	120.28	74.70
	∞	28.96	104.26	64.78
2.5 *mm.* =	1731.2	72.40	260.64	161.95
100 *km.*	2000	52.98	190.73	118.51
0.1 *in.*	3000	43.87	157.93	98.13
63 *mi.*	∞	36.20	130.32	80.98
3.0 *mm.* =	2077.5	86.88	312.77	194.35
100 *km.*	3000	55.89	201.20	125.02
0.1 *in.*	∞	43.44	156.38	97.17
53 *mi.*				
4.0 *mm.* =	2770.0	115.84	417.02	259.12
100 *km.*	3000	90.72	326.59	202.93
0.1 *in.*	∞	57.92	208.51	129.56
39 *mi.*				

Latitude 40°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.2\ mm.}{100\ km.} = \frac{0.1\ in.}{789\ mi.}$	110.25	5.16	18.58	11.55
	200	3.10	11.16	6.93
	300	2.88	10.37	6.44
	400	2.79	10.04	6.24
	500	2.74	9.86	6.13
	600	2.72	9.79	6.08
	700	2.70	9.72	6.04
	800	2.68	9.65	6.00
	900	2.67	9.61	5.97
	1000	2.66	9.58	5.95
	1200	2.65	9.54	5.93
	1500	2.63	9.47	5.88
	2000	2.62	9.43	5.86
	3000	2.61	9.40	5.84
	∞	2.58	9.29	5.77
$\frac{0.4\ mm.}{100\ km.} = \frac{0.1\ in.}{395\ mi.}$	220.5	10.34	37.22	23.13
	300	6.83	24.59	15.28
	400	6.19	22.28	13.84
	500	5.92	21.31	13.24
	600	5.76	20.74	12.89
	700	5.66	20.38	12.66
	800	5.59	20.12	12.50
	900	5.53	19.91	12.37
	1000	5.49	19.76	12.28
	1200	5.43	19.55	12.15
	1500	5.37	19.33	12.01
	2000	5.32	19.15	11.90
	3000	5.27	18.97	11.79
	∞	5.17	18.61	11.56
$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	330.75	15.50	55.80	34.67
	400	10.95	39.42	24.50
	500	9.80	35.28	21.92
	600	9.29	33.44	20.78
	700	8.98	32.33	20.09
	800	8.78	31.61	19.64
	900	8.64	31.10	19.32
	1000	8.53	30.71	19.08
	1200	8.38	30.17	18.79
	1500	8.23	29.63	18.41
	2000	8.10	29.16	18.12
	3000	7.98	28.73	17.85
	∞	7.75	27.90	17.34
$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	441.0	20.68	74.45	46.26
	500	15.39	55.40	34.42
	600	13.65	49.14	30.53
	700	12.86	46.30	28.77
	800	12.38	44.57	27.69
	900	12.06	43.42	26.98
	1000	11.83	42.59	26.46
$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	1200	11.52	41.47	25.77
	1500	11.24	40.46	25.14
	2000	10.98	39.53	24.56
	3000	10.75	58.70	24.05
	∞	10.34	37.22	23.13
$\frac{1.0\ mm.}{100\ km.} = \frac{1.0\ in.}{158\ mi.}$	551.25	25.84	93.02	57.80
	600	20.12	72.43	45.01
	700	17.69	63.68	39.57
	800	16.59	59.72	37.11
	900	15.93	57.35	35.64
	1000	15.48	55.73	34.63
	1200	14.89	53.60	33.31
	1500	14.39	51.80	32.19
	2000	13.96	50.26	31.23
	3000	13.58	48.89	30.38
	∞	12.92	46.51	28.90
$\frac{1.5\ mm.}{100\ km.} = \frac{0.1\ in.}{105\ mi.}$	826.9	38.76	139.54	86.71
	900	30.18	108.65	67.51
	1000	27.38	98.57	61.25
	1200	24.89	89.60	55.68
	1500	23.22	83.59	51.94
	2000	21.95	79.02	49.10
	3000	20.94	75.38	46.84
	∞	19.38	69.77	43.35
$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	1102.5	51.68	186.05	115.61
	1200	40.23	144.83	89.99
	1500	34.13	122.87	76.35
	2000	30.96	111.46	69.26
	3000	29.77	107.17	66.59
	∞	25.84	93.02	57.80
$\frac{2.5\ mm.}{100\ km.} = \frac{0.1\ in.}{63\ mi.}$	1378.1	64.60	232.56	144.50
	1500	50.29	181.04	112.50
	2000	41.48	149.33	92.79
	3000	37.23	134.03	83.28
	∞	32.30	116.28	72.25
$\frac{3.0\ mm.}{100\ km.} = \frac{0.1\ in.}{53\ mi.}$	1653.75	77.54	279.14	173.45
	2000	54.76	197.14	122.50
	3000	46.43	167.15	103.86
	∞	38.77	139.57	86.72
$\frac{4.0\ mm.}{100\ km.} = \frac{0.1\ in.}{39\ mi.}$	2205.0	103.38	372.17	231.25
	3000	68.25	245.70	152.67
	∞	51.69	186.08	115.62

Latitude 45°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V \frac{m}{s}$	$V \frac{km}{hr}$	$V \frac{mi}{hr}$
0.2 *mm.* =	91.33	4.70	16.92	10.51
100 *km.*	100	3.62	13.03	8.10
0.1 *in.*	200	2.70	9.72	6.04
789 *mi.*	300	2.56	9.22	5.73
	400	2.50	9.00	5.59
	500	2.47	8.89	5.52
	600	2.45	8.82	5.48
	700	2.43	8.75	5.44
	800	2.42	8.71	5.41
	900	2.41	8.68	5.39
	1000	2.41	8.68	5.39
	1200	2.40	8.64	5.37
	1500	2.39	8.60	5.34
	2000	2.38	8.57	5.33
	3000	2.36	8.50	5.28
	∞	2.35	8.46	5.26
0.4 *mm.* =	182.66	9.40	33.84	21.03
100 *km.*	200	7.24	26.06	16.19
0.1 *in.*	300	5.78	20.81	12.93
395 *mi.*	400	5.41	19.48	12.10
	500	5.23	18.83	11.70
	600	5.12	18.43	11.45
	700	5.05	18.18	11.30
	800	5.00	18.00	11.18
	900	4.97	17.89	11.12
	1000	4.94	17.78	11.05
	1200	4.89	17.60	10.94
	1500	4.85	17.46	10.85
	2000	4.81	17.32	10.76
	3000	4.77	17.17	10.67
	∞	4.70	16.92	10.51
0.6 *mm.* =	274.0	14.10	50.76	31.54
100 *km.*	300	10.86	39.10	24.30
0.1 *in.*	400	9.02	32.47	20.18
263 *mi.*	500	8.43	30.35	18.86
	600	8.11	29.20	18.14
	700	7.92	28.51	17.72
	800	7.78	28.01	17.40
	900	7.69	27.68	17.20
	1000	7.61	27.40	17.03
	1200	7.50	27.00	16.78
	1500	7.40	26.64	16.55
	2000	7.31	26.32	16.35
	3000	7.21	25.96	16.13
	∞	7.05	25.38	15.77
0.8 *mm.* =	365.33	18.80	67.68	42.05
100 *km.*	400	14.48	52.13	32.39
0.1 *in.*	500	12.36	44.50	27.65
197 *mi.*	600	11.55	41.58	25.84
	700	11.11	40.00	24.85
	800	10.82	38.95	24.20
	900	10.61	38.20	23.74
	1000	10.46	37.66	23.40
	1200	10.25	36.90	22.93
	1500	10.05	36.18	22.48
	2000	9.87	35.53	22.08
	3000	9.70	34.92	21.70
	∞	9.40	33.84	21.03
1.0 *mm.* =	456.66	23.50	84.60	52.57
100 *km.*	500	18.10	65.16	40.49
0.1 *in.*	600	15.76	56.74	35.26
158 *mi.*	700	14.77	53.17	33.04
	800	14.19	51.08	31.74
	900	13.80	49.68	30.87
	1000	13.52	48.67	30.24
	1200	13.14	47.30	29.39
	1500	12.81	46.12	28.66
	2000	12.50	45.00	27.96
	3000	12.23	44.03	27.36
	∞	11.75	42.30	26.28
1.5 *mm.* =	685.0	35.24	126.86	78.83
100 *km.*	700	30.54	109.94	68.31
0.1 *in.*	800	25.50	91.80	57.04
105 *mi.*	900	23.64	85.10	52.88
	1000	22.55	81.18	50.44
	1200	21.28	76.61	47.60
	1500	20.28	73.01	45.37
	2000	19.45	70.02	43.51
	3000	18.76	67.54	41.97
	∞	17.62	63.43	39.41
2.0 *mm.* =	913.33	46.98	169.13	105.09
100 *km.*	1000	36.20	130.32	80.98
0.1 *in.*	1200	31.52	113.47	70.51
79 *mi.*	1500	28.89	104.00	64.62
	2000	27.04	97.34	60.48
	3000	25.61	92.20	57.29
	∞	23.49	84.56	52.54
2.5 *mm.* =	1141.66	58.74	211.46	131.39
100 *km.*	1200	47.93	172.55	107.22
0.1 *in.*	1500	39.40	141.84	88.14
63 *mi.*	2000	35.46	127.66	79.32
	3000	32.86	118.30	73.51
	∞	29.37	105.73	65.70

Latitude 45° (Continued).

Δ (mm.) / 100 km.	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	ΔB (mm.) / 100 km.	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
3.0 mm. / 100 km. = 0.1 in. / 53 mi	1370.0	70.48	253.73	157.66	4.0 mm. / 100 km. = 0.1 in. / 39. mi.	1826.66	93.98	338.33	210.23
	1500	54.30	195.48	121.46		2000	72.40	260.64	165.53
	2000	45.10	162.36	100.89		3000	57.77	207.97	129.23
	3000	40.56	146.02	90.73		∞	46.99	169.16	105.11
	∞	35.24	126.86	78.83					

Latitude 50°.

0.2 mm. / 100 km. = 0.1 in. / 789 mi.	77.66	4.34	15.62	9.71
	100	2.94	10.58	6.57
	200	2.43	8.75	5.44
	300	2.33	8.39	5.21
	400	2.29	8.24	5.12
	500	2.26	8.14	5.06
	600	2.24	8.06	5.01
	700	2.23	8.03	4.99
	800	2.23	8.03	4.99
	900	2.22	7.99	4.96
	1000	2.21	7.96	4.95
	1200	2.21	7.96	4.95
	1500	2.20	7.92	4.92
	2000	2.19	7.88	4.90
	3000	2.19	7.88	4.90
	∞	2.17	7.81	4.85
0.4 mm. / 100 km. = 0.1 in. / 395 mi.	155.33	8.68	31.25	19.42
	200	5.89	21.20	13.17
	300	5.12	18.43	11.45
	400	4.87	17.53	10.89
	500	4.74	17.06	10.60
	600	4.66	16.78	10.43
	700	4.61	16.60	10.31
	800	4.57	16.45	10.22
	900	4.54	16.34	10.15
	1000	4.52	16.27	10.11
	1200	4.49	16.16	10.04
	1500	4.46	16.06	9.98
	2000	4.42	15.91	9.89
	3000	4.40	15.84	9.84
	∞	4.34	15.62	9.71
0.6 mm. / 100 km. = 0.1 in. / 263 mi.	233.0	13.02	46.87	29.12
	300	8.83	31.79	19.75
	400	7.90	28.44	17.67
	500	7.52	27.07	16.82
	600	7.30	26.28	16.33
	700	7.16	25.78	16.02
	800	7.06	25.42	15.80
	900	6.99	25.16	15.63
	1000	6.94	24.98	15.52
	1200	6.86	24.70	15.35

0.6 mm. / 100 km. = 0.1 in. / 263 mi.	1500	6.78	24.41	15.17
	2000	6.71	24.16	15.01
	3000	6.64	23.90	14.85
	∞	6.51	23.44	14.57
0.8 mm. / 100 km. = 0.1 in. / 197 mi.	310.66	17.34	62.42	38.79
	400	11.78	42.41	26.35
	500	10.74	38.66	24.04
	600	10.24	36.86	22.90
	700	9.94	35.78	22.23
	800	9.74	35.06	21.79
	900	9.59	34.52	21.45
	1000	9.48	34.13	21.21
	1200	9.31	33.52	20.83
	1500	9.18	33.05	20.54
	2000	9.04	32.54	20.22
	3000	8.91	32.08	19.93
	∞	8.67	31.21	19.39
1.0 mm. / 100 km. = 0.1 in. / 158 mi.	388.33	21.68	78.05	48.50
	400	18.51	66.64	41.41
	500	14.72	52.99	32.93
	600	13.60	48.96	30.42
	700	13.01	46.84	29.11
	800	12.63	45.47	28.25
	900	12.36	44.50	27.65
	1000	12.17	43.81	27.22
	1200	11.90	42.84	26.62
	1500	11.65	41.94	26.06
	2000	11.43	41.15	25.57
	3000	11.24	40.46	25.14
	∞	10.84	39.02	24.25
1.5 mm. / 100 km. = 0.1 in. / 105 mi.	582.5	32.52	117.07	72.75
	600	27.76	99.94	62.10
	700	23.07	83.05	51.60
	800	21.38	76.97	47.83
	900	20.41	73.48	45.66
	1000	19.76	71.14	44.20
	1200	18.94	68.18	42.36
	1500	18.25	65.70	40.82
	2000	17.66	63.58	39.51
	3000	17.14	61.70	38.34
	∞	16.26	58.54	36.38

Latitude 50° (Continued).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	776.66	43.38	156.17	97.04	$\frac{3.0\ mm.}{100\ km.} = \frac{0.1\ in.}{53\ mi}$	1165.0	65.06	234.22	145.54
	800	37.02	133.27	82.81		1200	55.53	199.91	124.22
	900	31.65	113.94	70.80		1500	44.17	159.01	98.80
	1000	29.45	106.02	65.88		2000	39.52	142.27	88.40
	1200	27.21	97.96	60.87		3000	36.51	131.44	81.67
	1500	25.60	92.16	57.27		∞	32.53	117.11	72.77
	2000	24.34	87.62	54.44					
	3000	23.31	83.92	52.15	$\frac{4.0\ mm.}{100\ km.} = \frac{0.1\ in.}{39\ mi.}$	1553.33	86.74	312.26	194.03
	∞	21.69	78.08	48.52		2000	58.90	212.04	131.75
						3000	51.19	184.28	114.51
$\frac{2.5\ mm.}{100\ km.} = \frac{0.1\ in.}{63\ mi.}$	970.8	54.22	195.19	121.29		∞	43.37	156.13	97.01
	1000	46.27	166.57	103.50					
	1200	37.72	135.79	84.38					
	1500	34.05	122.58	76.17					
	2000	31.57	113.65	70.62					
	3000	29.75	107.10	66.55					
	∞	27.11	97.60	60.65					

Latitude 55°.

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$	$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.2\ mm.}{100\ km.} = \frac{0.1\ in.}{789\ mi.}$	67.9	4.06	14.62	9.08	$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	203.7	12.16	43.78	27.20
	100	2.59	9.32	5.79		300	7.77	27.97	17.38
	200	2.24	8.06	5.01		400	7.16	25.78	16.02
	300	2.16	7.78	4.83		500	6.88	24.77	15.39
	400	2.12	7.63	4.74		600	6.71	24.16	15.01
	500	2.10	7.56	4.70		700	6.61	23.80	14.79
	600	2.09	7.52	4.67		800	6.53	23.51	14.61
	700	2.08	7.49	4.65		900	6.48	23.33	14.50
	800	2.07	7.45	4.63		1000	6.43	23.15	14.38
	900	2.07	7.45	4.63		1200	6.37	22.93	14.25
	1000	2.06	7.42	4.61		1500	6.31	22.72	14.12
	1200	2.06	7.42	4.61		2000	6.25	22.50	13.98
	1500	2.05	7.38	4.59		3000	6.19	22.28	13.84
	2000	2.05	7.38	4.59		∞	6.08	21.89	13.60
	3000	2.04	7.34	4.56					
	∞	2.03	7.31	4.54	$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	271.6	16.22	58.39	36.28
						300	12.41	44.68	27.76
$\frac{0.4\ mm.}{100\ km.} = \frac{0.1\ in.}{395\ mi.}$	135.8	8.12	29.23	18.16		400	10.36	37.30	23.18
	200	5.18	18.65	11.59		500	9.68	34.85	21.65
	300	4.66	16.78	10.43		600	9.33	33.59	20.87
	400	4.48	16.13	10.02		700	9.10	32.76	20.36
	500	4.40	15.84	9.84		800	8.95	32.22	20.02
	600	4.32	15.55	9.66		900	8.84	31.82	19.77
	700	4.27	15.37	9.55		1000	8.75	31.50	19.57
	800	4.25	15.30	9.51		1200	8.63	31.07	19.31
	900	4.22	15.19	9.44		1500	8.52	30.67	19.06
	1000	4.20	15.12	9.40		2000	8.41	30.28	18.82
	1200	4.18	15.05	9.35		3000	8.30	29.88	18.57
	1500	4.15	14.94	9.28		∞	8.11	29.20	18.14
	2000	4.13	14.87	9.24					
	3000	4.10	14.76	9.17					
	∞	4.06	14.62	9.08					

Latitude 55° (Continued).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
1.0 *mm.* =	339.5	20.28	73.01	45.37
100 *km.*	400	14.60	52.56	32.66
0.1 *in.*	500	12.95	46.62	28.97
158 *mi.*	600	12.23	44.00	27.34
	700	11.81	42.52	26.42
	800	11.53	41.51	25.79
	900	11.34	40.82	25.36
	1000	11.19	40.28	25.03
	1200	10.98	39.53	24.56
	1500	10.79	38.84	24.13
	2000	10.61	38.20	23.74
	3000	10.44	37.58	23.35
	∞	10.14	36.50	22.68
1.5 *mm.* =	509.2	30.42	109.51	68.05
100 *km.*	600	21.90	78.84	48.99
0.1 *in.*	700	19.99	71.96	44.71
105 *mi.*	800	18.98	68.33	42.46
	900	18.34	66.02	41.02
	1000	17.89	64.40	40.02
	1200	17.30	62.28	38.70
	1500	16.78	60.41	37.54
	2000	16.33	58.79	36.53
	3000	15.92	57.31	35.61
	∞	15.21	54.76	34.03
2.0 *mm.* =	679.0	40.56	146.02	90.73
100 *km.*	700	34.57	124.45	77.33
0.1 *in.*	800	29.20	105.12	65.32
79 *mi.*	900	27.12	97.63	60.66
	1000	25.89	93.20	57.91

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
2.0 *mm.* =	1200	24.45	88.02	54.69
100 *km.*	1500	23.31	83.92	52.15
0.1 *in.*	2000	22.37	80.53	50.04
79 *mi.*	3000	21.58	77.69	48.27
	∞	20.28	73.01	45.37
2.5 *mm.* =	848.7	50.70	182.52	113.41
100 *km.*	900	40.94	147.38	91.58
0.1 *in.*	1000	36.50	131.40	81.65
63 *mi.*	1200	32.90	118.44	73.60
	1500	30.56	110.02	68.36
	2000	28.83	103.79	64.49
	3000	27.45	98.82	61.40
	∞	25.35	91.26	56.71
3.0 *mm.* =	1018.5	60.84	219.02	136.09
100 *km.*	1200	43.80	157.68	97.98
0.1 *in.*	1500	38.84	139.82	86.88
53 *mi.*	2000	35.78	128.81	80.04
	3000	33.56	120.82	75.07
	∞	30.42	109.51	68.05
4.0 *mm.* =	1358.0	81.12	292.03	181.45
100 *km.*	1500	62.04	223.34	138.78
0.1 *in.*	2000	51.78	186.41	115.83
39 *mi.*	3000	46.62	167.83	104.28
	∞	40.56	146.02	90.73

Latitude 60°.

0.2 *mm.* =	60.75	3.82	13.75	8.55
100 *km.*	100	2.36	8.50	5.28
0.1 *in.*	200	2.09	7.52	4.67
789 *mi.*	300	2.03	7.31	4.54
	400	2.00	7.20	4.47
	500	1.98	7.13	4.43
	600	1.97	7.09	4.41
	700	1.96	7.06	4.39
	800	1.96	7.06	4.39
	900	1.95	7.02	4.36
	1000	1.95	7.02	4.36
	1200	1.94	6.98	4.34
	1500	1.94	6.98	4.34
	2000	1.93	6.95	4.32
	3000	1.93	6.95	4.32
	∞	1.91	6.88	4.28

0.4 *mm.* =	121.5	7.60	27.36	17.00
100 *km.*	200	4.72	16.99	10.56
0.1 *in.*	300	4.33	15.59	9.69
395 *mi.*	400	4.18	15.05	9.35
	500	4.10	14.76	9.17
	600	4.05	14.58	9.06
	700	4.02	14.47	8.99
	800	3.99	14.36	8.92
	900	3.98	14.33	8.90
	1000	3.96	14.26	8.86
	1200	3.94	14.18	8.81
	1500	3.92	14.11	8.77
	2000	3.90	14.04	8.72
	3000	3.87	13.93	8.66
	∞	3.80	13.68	8.50

Latitude 60° (*Continued*).

$\frac{\Delta B\ (mm.)}{100\ km.}$	r	$V\frac{m}{s}$	$V\frac{km}{hr}$	$V\frac{mi}{hr}$
$\frac{0.6\ mm.}{100\ km.} = \frac{0.1\ in.}{263\ mi.}$	182.25	11.40	41.04	25.50
	200	8.87	31.93	19.84
	300	7.08	25.49	15.84
	400	6.62	23.83	14.81
	500	6.40	23.04	14.32
	600	6.27	22.57	14.02
	700	6.19	22.28	13.84
	800	6.13	22.07	13.71
	900	6.08	21.89	13.60
	1000	6.05	21.78	13.53
	1200	5.99	21.56	13.40
	1500	5.94	21.38	13.29
	2000	5.89	21.20	13.17
	3000	5.84	21.02	13.06
	∞	5.70	20.52	12.85
$\frac{0.8\ mm.}{100\ km.} = \frac{0.1\ in.}{197\ mi.}$	243.0	15.20	54.72	34.00
	300	10.69	38.48	23.91
	400	9.43	33.95	21.10
	500	8.94	32.18	20.00
	600	8.66	31.18	19.37
	700	8.49	30.56	18.99
	800	8.37	30.13	18.72
	900	8.28	29.81	18.52
	1000	8.21	29.56	18.37
	1200	8.11	29.20	18.14
	1500	8.01	28.84	17.92
	2000	7.92	28.51	17.72
	3000	7.84	28.22	17.54
	∞	7.60	27.36	17.00
$\frac{1.0\ mm.}{100\ km.} = \frac{0.1\ in.}{158\ mi.}$	303.75	19.00	68.40	42.50
	400	12.87	46.33	28.79
	500	11.79	42.44	26.37
	600	11.27	40.57	25.21
	700	10.95	39.42	24.49
	800	10.73	38.63	24.00
	900	10.58	38.09	23.67
	1000	10.46	37.66	23.40
	1200	10.29	37.04	23.02
	1500	10.13	36.47	22.66
	2000	9.99	35.96	22.34
	3000	9.85	35.46	22.03
	∞	9.50	34.20	21.25
$\frac{1.5\ mm.}{100\ km.} = \frac{0.1\ in.}{105\ mi.}$	455.6	28.50	102.60	63.75
	500	22.17	79.81	49.59
	600	19.30	69.48	43.17
	700	18.09	65.12	40.46
	800	17.37	62.53	38.85
	900	16.90	60.84	37.80
	1000	16.56	59.62	37.05
	1200	16.10	57.96	36.02
	1500	15.69	56.48	35.09
	2000	15.31	55.12	34.25
	3000	14.98	53.93	33.51
	∞	14.25	51.30	31.88
$\frac{2.0\ mm.}{100\ km.} = \frac{0.1\ in.}{79\ mi.}$	607.5	37.98	136.73	84.96
	700	28.14	101.30	62.94
	800	25.74	92.66	57.58
	900	24.44	87.98	54.67
	1000	23.59	84.92	52.77
	1200	22.53	81.11	50.40
	1500	21.66	77.98	48.45
	2000	20.92	75.31	46.80
	3000	20.26	72.94	45.32
	∞	18.99	68.36	42.48
$\frac{2.5\ mm.}{100\ km.} = \frac{0.1\ in.}{63\ mi.}$	759.4	47.48	170.93	106.21
	800	39.14	140.90	87.55
	900	34.37	123.73	76.88
	1000	32.18	115.85	71.99
	1200	29.86	107.50	66.80
	1500	28.17	101.41	63.01
	2000	26.83	96.59	60.02
	3000	25.72	92.59	57.53
	∞	23.74	85.46	53.10
$\frac{3.0\ mm.}{100\ km.} = \frac{0.1\ in.}{53\ mi.}$	911.25	56.98	205.13	127.46
	1000	44.34	159.62	99.18
	1200	38.61	139.00	86.37
	1500	35.38	127.37	79.14
	2000	33.12	119.23	74.09
	3000	31.37	112.93	70.17
	∞	28.49	102.56	63.73
$\frac{4.0\ mm.}{100\ km.} = \frac{0.1\ in.}{39\ mi.}$	1215.0	75.96	273.46	169.92
	1500	53.44	192.38	119.54
	2000	47.18	169.85	105.54
	3000	43.32	155.95	96.90
	∞	37.98	136.73	84.96

APPENDIX II

CONSTANTS AND EQUIVALENTS

The following numerical values are taken, chiefly, from the Smithsonian Meteorological Tables, fourth edition, 1918:

Standard Values

Gravity acceleration ....980.665 centimetres per second.
Gram weight ....980.665 dynes.
Atmospheric pressure ....1013250.144+ dynes per square centimetre; that is, the pressure of a mercury column at standard gravity and 0° C. 760 millimetres high.
Bar (meteorological) ....1,000,000 dynes per square centimetre.

Temperature Scales

The temperatures of melting ice, and of boiling water (steam just over boiling water), each at standard atmospheric pressure, are indicated as follows:

Freezing: 32° F.; 0° R.; 0° C.; $273^\circ.13 \pm A_c$.
Boiling: 212° F.; 80° R.; 100° C.; $373^\circ.13 \pm A_c$.

Hence

$$\frac{C^\circ}{5} = \frac{F^\circ - 32^\circ}{9} = \frac{R^\circ}{4},$$

and, on the perfect gas scale, or after correction,

$$A_c^\circ = C^\circ + 273.13 \pm .$$

Linear Equivalents

1 metre	= 39.3700	inches* = 3.280833+ feet.
1 foot	= 0.3048006	metre.
1 kilometre	= 0.621370	mile.
1 mile	= 1.609347	kilometre.

* U. S. statutory equivalent.

Velocity Equivalents

1 metre per second = 2.236932 miles per hour = 196.85 feet per minute.
1 mile per hour = 0.4470409 metres per second.

Weight Equivalents

1 avoirdupois pound	= 453.5924277	grams
1 kilogram	= 2.204622	avoirdupois pounds.
1 gram	= 15.432356	grains.
1 grain	= 0.06479892	gram

Densities (Grams Per Cubic Centimetre)

Mercury, at 0° C.	13.5951
Air, dry, free from carbon dioxide, and at standard atmospheric pressure	0.0012928
Air, dry, containing 3 parts carbon dioxide per 10,000 (normal amount) and at standard atmospheric pressure	0 0012930.
Weight of standard dry air	1.2930 kilograms per cubic metre; 565.039 grains, or 1.29152 ounces, per cubic foot.

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